Pre-Calculus	
domain of $\sqrt{s(x)}$	$s(x)\geq 0$
domain of $\frac{1}{s(x)}$	$\{x\ ;\ s(x)\neq 0\}$
domain of $\ln[s(x)]$	$\{x\ ;\ s(x)>0\}$
zeros or x-intercepts	x such that $y=0$ or $f(x)=0$
y-intercept	y such that $x=0$
symmetry with respect to y-axis (even function)	$f(x)=f(-x)$
symmetry with respect to origin (odd function)	$f(-x)=-f(x)$
symmetry with respect to x-axis	$g(y)=g(-y)$
vertical asymptote	$\lim\limits_{x\to a^{\pm}} f(x)=\infty$ or $\lim\limits_{x\to a^{\pm}} f(x)=-\infty$
horizontal asymptote	$\lim\limits_{x\to\pm\infty} f(x)=a$
point-slope form of a linear equation (tangent line approximation)	$y-y_1=m(x-x_1)$ $y-f(a)=f'(a)(x-a)$ or $y=f'(a)(x-a)+f(a)$
Theorems	
Intermediate Value Theorem	If $f(x)$ is continuous on $[a,b]$ and there exists a number c is in $[a,b]$, then there exists an $f(c)$ such that $f(a)\leq f(c)\leq f(b)$
Extreme Value Theorem	If $f(x)$ is continuous on $[a,b]$, then there exists an absolute maximum and an absolute minimum on the interval $[a,b]$
Mean Value Theorem (Derivatives) Special Case: Rolle's Theorem	If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there exists a number c is in $[a,b]$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$ if $f(a)=f(b)$, then $f'(c)=0$
Mean Value Theorem (Integrals)	If $f(x)$ is continuous on $[a,b]$ and there exists a number c is in (a,b), then $\int_a^b f(x)dx=f(c)(b-a)$
First Fundamental Theorem of Calculus	$\int_a^b f(x)dx=F(b)-F(a)$
Second Fundamental Theorem of Calculus	$\frac{d}{dx}\left[\int_a^x f(t)dt\right]=f(x)$ or $\frac{d}{dx}\left[\int_a^{g(x)} f(t)dt\right]=f(g(x))\cdot g'(x)$

Position, Velocity, Acceleration

position function	$s(t)=\int v(t)\,dt=\int\left[\int a(t)\,dt\right]dt$ (given initial conditions)
velocity function	$s'(t)=v(t)=\int a(t)\,dt$ (given initial condition)
average velocity on $[a,b]$	$\frac{x(b)-x(a)}{b-a}$ or $\frac{1}{b-a}\int_a^b v(t)\,dt$
instantaneous velocity at $t=a$	$s'(a)=v(a)$
acceleration function	$s''(t)=v'(t)=a(t)$
particle moving right	$v(t)>0$
particle moving left	$v(t)<0$
particle at rest	$v(t)=0$
particle changes direction	$v(t)$ changes sign
speed	$\lvert v(t)\rvert$
speed decreases speed increases	$v(t)$ and $a(t)$ have opposite signs $v(t)$ and $a(t)$ have same signs
displacement total distance traveled	$\int_a^b v(t)\,dt=$ postive area + negative area $\int_a^b \lvert v(t)\rvert\,dt=$ postive area + $\lvert$negative area$\rvert$
net change	$\int_a^b F'(x)\,dx=F(b)-F(a)$

Graph Features

slope of a curve $f(x)$ at $x=c$	$f'(c)$
slope of tangent line of $f(x)$ at $x=c$	
critical numbers	$f'(x)=0$ or $f'(x)$ does not exist
$f(x)$ increasing	$f'(x)>0$
$f(x)$ decreasing	$f'(x)<0$
$f(x)$ concave up	$f''(x)>0$ or $f'(x)$ increasing
$f(x)$ concave down	$f''(x)>0$ or $f'(x)$ decreasing
extrema	absolute (closed interval): compare *y*-values of the relative extrema AND the endpoints relative (open interval): compare *y*-values of the critical points

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First Derivative [Extrema] Test	f has a rel. max when f' changes from positive to negative f has a rel. min when f' changes from negative to positive
Second Derivative [Extrema] Test	f has a rel. max when $f'=0$ or undef. and $f''<0$ f has a rel. min when $f'=0$ or undef. and $f''>0$
point of inflection	where f' has extrema and f'' changes sign (change in concavity of f)
Sums, Average, Area, Volume	
left Riemann sum (equal subintervals)	$\int_a^b f(x)\,dx \approx \Delta x\left[f(x_0)+f(x_1)+f(x_2)+...+f(x_{n-1})\right]$
right Riemann sum (equal subintervals)	$\int_a^b f(x)\,dx \approx \Delta x\left[f(x_1)+f(x_2)+f(x_3)+...+f(x_n)\right]$
midpoint rule (equal subintervals)	$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left[f(x_0)+f(x_1)+...+f(x_{n-1})+f(x_n)\right]$
trapezoidal sum (equal subintervals)	$\int_a^b f(x)\,dx \approx \frac{1}{2}\Delta x\left[f(x_0)+2f(x_1)+...+2f(x_{n-1})+f(x_n)\right]$
average value of $f(x)$ on $[a,b]$	$\frac{1}{b-a}\int_a^b f(x)\,dx$
cross-sectional area base = $f(x)$* or radius = $\frac{f(x)*}{2}$ $c=$ multiple of base *in terms of x if perpendicular to x-axis, *in terms of y if perpendicular to y-axis	Square: $\int_a^b (f(x))^2\,dx$ Rectangle: $\int_a^b c(f(x))^2\,dx$ Isoceles Right Triangle (leg base): $\int_a^b \frac{1}{2}(f(x))^2\,dx$ Equilateral Triangle: $\int_a^b \frac{\sqrt{3}}{4}(f(x))^2\,dx$ (not common) Semicircle: $\int_a^b \frac{1}{2}\pi\left(\frac{1}{2}f(x)\right)^2 dx = \int_a^b \frac{\pi}{8}(f(x))^2\,dx$
area between curves (always check for intersection on interval)	$\int_{left}^{right}(top-bottom)\,dx$ or $\int_{lower}^{upper}(right-left)\,dy$
volume	Upper/right function = $R(x)$ or $R(y)$ Lower/left function = $\mathrm{r}(x)$ or $r(y)$
disk Method	$V=\pi\int_{left}^{right}\left[R(x)\right]^2 dx$ $V=\pi\int_{lower}^{upper}\left[R(y)\right]^2 dy$
washer Method	$V=\pi\int_{left}^{right}\left[R(x)^2-r(x)^2\right]dx$ $V=\pi\int_{lower}^{upper}\left[R(y)^2-r(y)^2\right]dy$

If revolving about $x=a$ or $y=b$, always setup $R(x)$ and $r(x)$ as
radius = upper/right line – lower/left line (*just think of outer and inner radius*)
[i.e. $R(x)=f(x)-a$ (f above a) or $a-f(x)$ (a above f); repeat for $r(x)$]

Other Important Rules, Notations, and Definitions	
average rate of change	$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{f(b) - f(a)}{b - a}$
instantaneous rate of change of y with respect to x	$\frac{dy}{dx}$
L' Hôpital's Rule	If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$
special limits	$\lim_{x\to 0} \frac{\sin ax}{ax} = 1$ $\lim_{x\to 0} \frac{\cos ax - 1}{ax} = 0$ $\lim_{x\to \infty} \frac{\sin ax}{ax} = 0$ $\lim_{x\to \infty} \left(1 + \frac{1}{x}\right)^x = e$
derivative notation	$y' = f'(x) = \frac{dy}{dx}$ $y'' = f''(x) = \frac{d^2 y}{dx^2}$ $f^{(n)}(x) = \frac{d^n y}{dx^n}$
continuity	1. $f(a)$ exists (defined) 2. $\lim_{x\to a} f(x)$ exists (left- and right-hand limits equal) 3. $\lim_{x\to a} f(x) = f(a)$
definition of derivative (limit of the difference quotient)	$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ or $f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$ $f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$
change of variables for definite integrals	$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$
derivative of an inverse function	If $f^{-1}(x) = g(x)$, then $g'(x) = \frac{1}{f'(g(x))}$

Derivative Formulas

$\frac{d}{dx}[c] = 0$ $\frac{d}{dx}[x] = 1$ $\frac{d}{dx}[cx] = c$ $\frac{d}{dx}[x^c] = cx^{c-1}$

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\frac{d}{dx}[e^x] = e^x$ $\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$ $\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$

Integration Formulas

$\int dx = x + c$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ $\int \frac{dx}{x} = \ln|x| + c$

$\int e^x\,dx = e^x + c$ $\int \sin x\,dx = -\cos x + c$ $\int \cos x\,dx = \sin x + c$

$\int \tan x\,dx = -\ln|\cos x| + c$ $\int \csc x\,dx = -\ln|\csc x + \cot x| + c$ $\int \sec x\,dx = \ln|\sec x + \tan x| + c$

$\int \cot x\,dx = \ln|\sin x| + c$ $\int \sec^2 x\,dx = \tan x$ $\int \csc^2 x\,dx = -\cot x + c$

$\int \sec x \tan x\,dx = \sec x + c$ $\int \csc x \cot x\,dx = -\csc x + c$ $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + c$

$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + c$ $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + c$