Multiple-Choice Answer Key

The following contains the answers to
the multiple-choice questions in this exam.

Answer Key for AP Calculus BC Practice Exam, Section I

Question 1: D
Question 2: A
Question 3: C
Question 4: D
Question 5: C
Question 6: C
Question 7: D
Question 8: D
Question 9: B
Question 10: C
Question 11: A
Question 12: D
Question 13: B
Question 14: C
Question 15: C
Question 16: A
Question 17: B
Question 18: B
Question 19: C
Question 20: B
Question 21: B
Question 22: B
Question 23: D
Question 24: C
Question 25: B
Question 26: C
Question 27: A
Question 28: A
Question 29: A
Question 30: D
Question 76: C
Question 77: B
Question 78: C
Question 79: B
Question 80: D
Question 81: D
Question 82: A
Question 83: A
Question 84: B
Question 85: C
Question 86: C
Question 87: B
Question 88: A
Question 89: B
Question 90: B

Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

Question 1

(a) $\sqrt{(x'(2))^2 + (y'(2))^2} = 3.272461$ The speed of the particle at time $t = 2$ seconds is 3.272 meters per second.	$2 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{answer with units} \end{cases}$
(b) $s(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(2\cos(2t))^2 + (2t-1)^2}$ $s'(4) = 2.16265$ Since $s'(4) > 0,$ the speed of the particle is increasing at time $t = 4$.	$2 : \begin{cases} 1 : \text{considers } s'(4) \\ 1 : \text{answer with reason} \end{cases}$
(c) $\int_0^5 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 22.381767$ The total distance the particle travels over the time interval $0 \le t \le 5$ seconds is 22.382 (or 22.381) meters.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $x(10) = x(8) + x'(8) \cdot 2 = \sin 16 + x'(8) \cdot 2 = -4.118541$ $y(10) = y(8) + y'(8) \cdot 2 = \left(8^2 - 8\right) + y'(8) \cdot 2 = 86$ The position of the particle at time $t = 10$ seconds is $(-4.119, 86)$ (or $(-4.118, 86)$).	$3 : \begin{cases} 1 : \text{uses position at } t = 8 \\ 1 : \text{uses velocity at } t = 8 \\ 1 : \text{position at } t = 10 \end{cases}$

Question 2

(a) $\int_0^{4.5} a(t)\,dt = 66.532128$ At time $t = 4.5$, tank A contains 66.532 liters of water.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $a(k) = 20.5 \Rightarrow k = 0.892040$ $\int_0^k (20.5 - a(t))\,dt = 10.599191$ At time $t = k$, the difference in the amounts of water in the tanks is 10.599 liters.	$3 : \begin{cases} 1 : \text{sets } a(k) = 20.5 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^{2.416} b(t)\,dt = \int_0^k b(t)\,dt + \int_k^{2.416} b(t)\,dt$ $\int_0^k b(t)\,dt = 20.5 \cdot k = 18.286826$ On $k < t < 2.416$, tank A receives $\int_k^{2.416} a(t)\,dt = 44.497051$ liters of water, which is 14.470 more liters of water than tank B. Therefore, $\int_k^{2.416} b(t)\,dt = \int_k^{2.416} a(t)\,dt - 14.470 = 30.027051.$ $\int_0^k b(t)\,dt + \int_k^{2.416} b(t)\,dt = 48.313876$ At time $t = 2.416$, tank B contains 48.314 (or 48.313) liters of water.	$2 : \begin{cases} 1 : \int_k^{2.416} a(t)\,dt \\ 1 : \text{answer} \end{cases}$
(d) $w'(3.5) - a'(3.5) = -1.14298 < 0$ The difference $w(t) - a(t)$ is decreasing at $t = 3.5$.	$2 : \begin{cases} 1 : w'(3.5) - a'(3.5) < 0 \\ 1 : \text{conclusion} \end{cases}$

Question 3

(a) $\frac{g(5)-g(-5)}{5-(-5)} = \frac{12-(\pi+7)}{10} = \frac{5-\pi}{10}$	3 : { 1 : difference quotient 2 : answer
(b) $g'(x) = f(x)$ $g'(3) = f(3) = 4$ The instantaneous rate of change of g at $x = 3$ is 4.	1 : answer
(c) The graph of g is concave up on $-5 < x < -2$ and $0 < x < 3$, because $g'(x) = f(x)$ is increasing on these intervals.	2 : intervals with justification
(d) $g'(x) = f(x)$ is defined at all x with $-5 < x < 5$. $g'(x) = f(x) = 0$ at $x = -2$ and $x = 1$. Therefore, g has critical points at $x = -2$ and $x = 1$. g has neither a local maximum nor a local minimum at $x = -2$ because g' does not change sign there. g has a local minimum at $x = 1$ because g' changes from negative to positive there.	3 : { 1 : considers $f(x) = 0$ 1 : critical points at $x = -2$ and $x = 1$ 1 : answers with justifications

Question 4

(a) $\int_0^6 f'(x)\,dx \approx 2\cdot 3.5 + 2\cdot 0.8 + 2\cdot 5.8 = 20.2$ $f(6) - f(0) = \int_0^6 f'(x)\,dx$ $f(6) = f(0) + \int_0^6 f'(x)\,dx \approx 20 + 20.2 = 40.2$	3 : { 1 : midpoint sum 1 : Fundamental Theorem of Calculus 1 : answer
(b) Since $f'(x) \le 7$, $\int_0^6 f'(x)\,dx \le 6\cdot 7 = 42.$ $f(6) - f(0) \le 42 \Rightarrow f(6) \le 20 + 42 = 62$ Therefore, the actual value of $f(6)$ could not be 70.	2 : { 1 : integral bound 1 : answer with reasoning
(c) $\int_2^4 f''(x)\,dx = f'(4) - f'(2) = 1.7 - 2 = -0.3$	2 : { 1 : Fundamental Theorem of Calculus 1 : answer
(d) $\lim_{x\to 0}\left(f(x) - 20e^x\right) = 0$ $\lim_{x\to 0}(0.5f(x) - 10) = 0$ Using L'Hospital's Rule, $\lim_{x\to 0}\frac{f(x) - 20e^x}{0.5f(x) - 10} = \lim_{x\to 0}\frac{f'(x) - 20e^x}{0.5f'(x)} = \frac{4 - 20}{0.5(4)} = -8$	2 : { 1 : L'Hospital's Rule 1 : answer

Question 5

(a) $\dfrac{d^2y}{dx^2} = \dfrac{x \cdot 2y\dfrac{dy}{dx} - y^2 \cdot 1}{x^2}$ $= \dfrac{2xy\left(-1 + \dfrac{y^2}{x}\right) - y^2}{x^2} = \dfrac{2y^3 - y^2 - 2xy}{x^2}$	$2 : \begin{cases} 1 : \text{quotient rule} \\ 1 : \dfrac{d^2y}{dx^2} \end{cases}$
(b) $\left.\dfrac{dy}{dx}\right\rvert_{(x, y)=(4, 2)} = -1 + \dfrac{4}{4} = 0$ $\left.\dfrac{d^2y}{dx^2}\right\rvert_{(x, y)=(4, 2)} = \dfrac{2 \cdot 8 - 4 - 16}{16} = -\dfrac{1}{4} < 0$ By the Second Derivative Test, g has a relative maximum at $x = 4$.	$2 : \begin{cases} 1 : \text{considers } \left.\dfrac{dy}{dx}\right\rvert_{(x, y)=(4, 2)} \\ 1 : \text{answer with justification} \end{cases}$
(c) $\left.\dfrac{dy}{dx}\right\rvert_{(x, y)=(1, 2)} = -1 + \dfrac{4}{1} = 3$ $\left.\dfrac{d^2y}{dx^2}\right\rvert_{(x, y)=(1, 2)} = \dfrac{2 \cdot 8 - 4 - 4}{1} = 8$ The second-degree Taylor polynomial for h about $x = 1$ is $T_2(x) = 2 + 3(x - 1) + \dfrac{8}{2!}(x - 1)^2 = 2 + 3(x - 1) + 4(x - 1)^2$.	$3 : \begin{cases} 1 : \text{uses } \left.\dfrac{dy}{dx}\right\rvert_{(x, y)=(1, 2)} \\ \quad \text{and } \left.\dfrac{d^2y}{dx^2}\right\rvert_{(x, y)=(1, 2)} \\ 2 : \text{Taylor polynomial} \end{cases}$
(d) $\lvert h(1.1) - A\rvert \le \dfrac{\max\limits_{1.0 \le x \le 1.1} \lvert h'''(x)\rvert \lvert 1.1 - 1\rvert^3}{3!} \le \dfrac{60}{6} \cdot \dfrac{1}{1000} = \dfrac{10}{1000} = \dfrac{1}{100}$	$2 : \begin{cases} 1 : \text{form of the error bound} \\ 1 : \text{analysis} \end{cases}$

Question 6

(a) $$\int_3^\infty \frac{1}{x^2+9}\,dx = \lim_{b\to\infty}\int_3^b \frac{1}{x^2+9}\,dx = \lim_{b\to\infty}\left(\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\Big|_3^b\right)$$
$$= \lim_{b\to\infty}\left(\frac{1}{3}\tan^{-1}\left(\frac{b}{3}\right) - \frac{1}{3}\tan^{-1}(1)\right) = \frac{\pi}{6} - \frac{\pi}{12} = \frac{\pi}{12}$$

3 : { 1 : antiderivative; 1 : limit expression; 1 : answer }

(b) The function f is continuous, positive, and decreasing on $[3, \infty)$.

By the integral test, since $\int_3^\infty f(x)\,dx$ converges, $\sum_{n=3}^\infty f(n)$ converges.

— OR —

$0 < \frac{1}{n^2+9} < \frac{1}{n^2}$ for $n \ge 3$.

Since the series $\sum_{n=3}^\infty \frac{1}{n^2}$ converges, the series $\sum_{n=3}^\infty f(n) = \sum_{n=3}^\infty \frac{1}{n^2+9}$ converges by the comparison test.

2 : conclusion with conditions

(c) Consider the series $\sum_{n=1}^\infty \frac{1}{\left(e^n \cdot f(n)\right)} = \sum_{n=1}^\infty \frac{n^2+9}{e^n}$.

$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)^2+9}{e^{n+1}}}{\frac{n^2+9}{e^n}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^2+9}{n^2+9}\cdot\frac{1}{e}\right| = \frac{1}{e} < 1$$

By the ratio test, $\sum_{n=1}^\infty \frac{1}{\left(e^n \cdot f(n)\right)}$ converges.

Therefore, $\sum_{n=1}^\infty \frac{(-1)^n}{\left(e^n \cdot f(n)\right)}$ converges absolutely.

4 : { 1 : sets up ratio; 1 : computes limit of ratio; 1 : conclusion of ratio test; 1 : converges absolutely }

Scoring Worksheets

The following provides scoring worksheets and conversion tables used for calculating a composite score of the exam.

2017 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

_______________ × 1.2000 = _______________

Number Correct (out of 45) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 2 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 3 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 4 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 5 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Question 6 _______________ × 1.0000 = _______________
(out of 9) (Do not round)

Sum = _______________
Weighted Section II Score (Do not round)

Composite Score

_______________ + _______________ = _______________

Weighted Section I Score + Weighted Section II Score = Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
65-108	5
54-64	4
39-53	3
24-38	2
0-23	1

2017 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (1-3, 6-7, 9, 11, 13-15, 17-19, 21, 24, 26, 28, 76-79, 81-83, 85-86, 88)

________ × 1.0000 = ________

Number Correct (out of 27) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 2 ________ (out of 9) × 1.0000 = ________ (Do not round)

Question 3 ________ (out of 9) × 1.0000 = ________ (Do not round)

Question 4 ________ (out of 9) × 1.0000 = ________ (Do not round)

Sum = ________ Weighted Section II Score (Do not round)

Composite Score

________ + ________ = ________

Weighted Section I Score + Weighted Section II Score = Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Score
34-54	5
28-33	4
21-27	3
13-20	2
0-12	1

Question Descriptors and Performance Data

The following contains tables showing the content assessed, the correct answer, and how AP students performed on each question.

2017 AP Calculus BC
Question Descriptors and Performance Data

Multiple-Choice Questions

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus 1	Mathematical Practice for AP Calculus 2	Key	% Correct
1	2.1C	2.1C4	Implementing algebraic/computational processes	Building notational fluency	D	89
2	3.3B(a)	3.3B3	Implementing algebraic/computational processes	Building notational fluency	A	81
3	2.1C	2.1C3	Implementing algebraic/computational processes	Building notational fluency	C	88
4	2.3C	2.3C4	Implementing algebraic/computational processes	Connecting concepts	D	85
5	4.1B	4.1B1	Reasoning with definitions and theorems	Building notational fluency	C	58
6	1.2A	1.2A1	Connecting multiple representations	Reasoning with definitions and theorems	C	87
7	3.2C	3.2C2	Reasoning with definitions and theorems	Building notational fluency	D	84
8	3.4D	3.4D3	Reasoning with definitions and theorems	Connecting concepts	D	86
9	2.3B	2.3B1	Implementing algebraic/computational processes	Connecting concepts	B	90
10	2.3F	2.3F2	Implementing algebraic/computational processes	Building notational fluency	C	87
11	2.1C	2.1C5	Implementing algebraic/computational processes	Building notational fluency	A	83
12	3.3B(a)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	D	81
13	2.3F	2.3F1	Connecting multiple representations	Connecting concepts	B	81
14	2.1A	2.1A3	Building notational fluency	Implementing algebraic/computational processes	C	83
15	1.1A(b)	1.1A3	Connecting multiple representations	Connecting concepts	C	54
16	4.2C	4.2C2	Implementing algebraic/computational processes	Connecting concepts	A	60
17	3.3B(b)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	B	61
18	1.1A(b)	1.1A2	Connecting concepts	Implementing algebraic/computational processes	B	80
19	2.3B	2.3B2	Implementing algebraic/computational processes	Connecting concepts	C	69
20	2.1C	2.1C7	Implementing algebraic/computational processes	Connecting concepts	B	77
21	3.2B	3.2B2	Connecting multiple representations	Implementing algebraic/computational processes	B	71
22	3.3B(a)	3.3B5	Implementing algebraic/computational processes	Building notational fluency	B	66
23	4.2B	4.2B5	Implementing algebraic/computational processes	Building notational fluency	D	79
24	3.4B	3.4B1	Connecting concepts	Implementing algebraic/computational processes	C	64
25	4.2C	4.2C1	Connecting concepts	Reasoning with definitions and theorems	B	36
26	2.4A	2.4A1	Reasoning with definitions and theorems	Connecting concepts	C	50
27	3.5B	3.5B2	Implementing algebraic/computational processes	Connecting concepts	A	32
28	2.3C	2.3C2	Connecting concepts	Implementing algebraic/computational processes	A	80

2017 AP Calculus BC
Question Descriptors and Performance Data

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus 1	Mathematical Practice for AP Calculus 2	Key	% Correct
29	4.2B	4.2B2	Implementing algebraic/computational processes	Connecting concepts	A	53
30	4.1A	4.1A6	Reasoning with definitions and theorems	Building notational fluency	D	64
76	2.2A	2.2A1	Reasoning with definitions and theorems	Connecting concepts	C	76
77	3.3B(b)	3.3B2	Implementing algebraic/computational processes	Reasoning with definitions and theorems	B	74
78	1.1D	1.1D1	Building notational fluency	Connecting concepts	C	81
79	3.3A	3.3A3	Connecting multiple representations	Connecting concepts	B	67
80	2.3C	2.3C4	Implementing algebraic/computational processes	Connecting concepts	D	77
81	2.2A	2.2A3	Connecting multiple representations	Connecting concepts	D	65
82	2.2A	2.2A1	Implementing algebraic/computational processes	Connecting concepts	A	41
83	3.4D	3.4D2	Connecting concepts	Connecting multiple representations	A	67
84	3.2D	3.2D2	Connecting concepts	Building notational fluency	B	82
85	3.3A	3.3A3	Connecting multiple representations	Implementing algebraic/computational processes	C	74
86	2.2A	2.2A1	Connecting concepts	Connecting multiple representations	C	48
87	3.4C	3.4C2	Implementing algebraic/computational processes	Reasoning with definitions and theorems	B	65
88	2.2A	2.2A2	Connecting multiple representations	Connecting concepts	A	27
89	3.4D	3.4D1	Implementing algebraic/computational processes	Connecting multiple representations	B	37
90	4.1B	4.1B2	Reasoning with definitions and theorems	Connecting concepts	B	38

Free-Response Questions

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus	Mean
1	2.1C|2.2A|2.3C|3.4C	2.1C7|2.2A1|2.3C4|3.4C2	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Building notational fluency|Communicating	3.9
2	2.3D|3.2C|3.4A|3.4D	2.3D1|3.2C2|3.4A2|3.4D1	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	4.94
3	2.1A|2.2A|3.2C|3.3A	2.1A1|2.2A1|3.2C1|3.3A2,3.3A3	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	5.15
4	1.1C|2.1C|3.2B|3.3B(b)	1.1C3|2.1C2|3.2B2|3.3B2	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	4.1
5	2.1C|2.1D|2.2A|4.2A	2.1C5|2.1D1|2.2A1|4.2A2,4.2A4	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Building notational fluency|Communicating	4.81
6	3.2D|3.3B(b)|4.1A	3.2D2|3.3B5|4.1A4,4.1A6	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Building notational fluency|Communicating	2.85