Chapter 4
Trigonometric Functions

Section 4.1

Check Point Exercises

1. The radian measure of a central angle is the length of the intercepted arc, *s,* divided by the circle's radius, *r*. The length of the intercepted arc is 42 feet: $s = 42$ feet. The circle's radius is 12 feet: $r = 12$ feet. Now use the formula for radian measure to find the radian measure of θ.

$$\theta = \frac{s}{r} = \frac{42 \text{ feet}}{12 \text{ feet}} = 3.5$$

Thus, the radian measure of θ is 3.5

2. **a.** $60° = 60° \cdot \frac{\pi \text{ radians}}{180°} = \frac{60\pi}{180} \text{ radians}$
$= \frac{\pi}{3} \text{ radians}$

b. $270° = 270° \cdot \frac{\pi \text{ radians}}{180°} = \frac{270\pi}{180} \text{ radians}$
$= \frac{3\pi}{2} \text{ radians}$

c. $-300° = -300° \cdot \frac{\pi \text{ radians}}{180°} = \frac{-300\pi}{180} \text{ radians}$
$= -\frac{5\pi}{3} \text{ radians}$

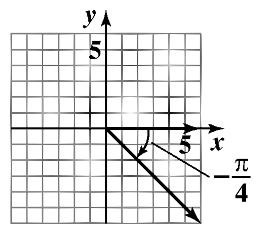
3. **a.** $\frac{\pi}{4} \text{ radians} = \frac{\pi \text{ radians}}{4} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{180^{\text{o}}}{4} = 45^{\text{o}}$

b. $-\frac{4\pi}{3} \text{ radians} = -\frac{4\pi \text{ radians}}{3} \cdot \frac{180^{\text{o}}}{\pi}$
$= -\frac{4 \cdot 180^{\text{o}}}{3} = -240^{\text{o}}$

c. $6 \text{ radians } = 6 \text{ radians} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{6 \cdot 180^{\text{o}}}{\pi} \approx 343.8^{\text{o}}$

d. $-4.7 \text{ radians } = -4.7 \text{ radians} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{-4.7 \cdot 180^{\text{o}}}{\pi} \approx -269.3^{\text{o}}$

4. **a.**



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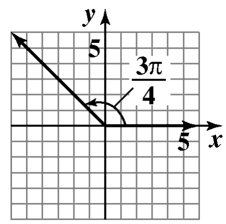

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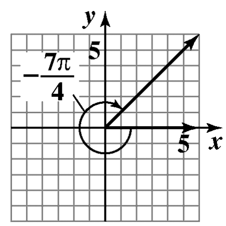

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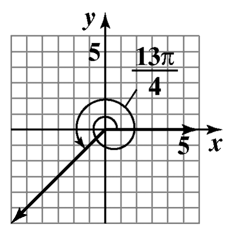

5. **a.** For a 400º angle, subtract 360º to find a positive coterminal angle.
$400^{\text{o}} - 360^{\text{o}} = 40^{\text{o}}$

b. For a –135º angle, add 360º to find a positive coterminal angle.
$-135^{\text{o}} + 360^{\text{o}} = 225^{\text{o}}$

6. **a.** $\frac{13\pi}{5} - 2\pi = \frac{13\pi}{5} - \frac{10\pi}{5} = \frac{3\pi}{5}$

b. $-\frac{\pi}{15} + 2\pi = -\frac{\pi}{15} + \frac{30\pi}{15} = \frac{29\pi}{15}$

7. **a.** $855° - 360° \cdot 2 = 855° - 720° = 135°$

b. $\frac{17\pi}{3} - 2\pi \cdot 2 = \frac{17\pi}{3} - 4\pi$
$= \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$

c. $-\frac{25\pi}{6} + 2\pi \cdot 3 = -\frac{25\pi}{6} + 6\pi$
$= -\frac{25\pi}{6} + \frac{36\pi}{6} = \frac{11\pi}{6}$

d. $17.4 \approx 17.4\left(57^\circ\right) = 991.8^\circ$

For a 991.8° angle, we need to subtract two multiples of 360° to find a positive coterminal angle less than 360°.
$17.4 - 2\pi \cdot 2 = 17.4 - 4\pi \approx 4.8$

8. The formula $s = r\theta$ can only be used when θ is expressed in radians. Thus, we begin by converting 45° to radians. Multiply by $\frac{\pi \text{ radians}}{180°}$.

$$45° = 45° \cdot \frac{\pi \text{ radians}}{180°} = \frac{45}{180}\pi \text{ radians}$$
$$= \frac{\pi}{4} \text{ radians}$$

Now we can use the formula $s = r\theta$ to find the length of the arc. The circle's radius is 6 inches : r = 6 inches. The measure of the central angle in radians is $\frac{\pi}{4} : \theta = \frac{\pi}{4}$. The length of the arc intercepted by this central angle is

$$s = r\theta$$
$$= (6 \text{ inches})\left(\frac{\pi}{4}\right)$$
$$= \frac{6\pi}{4} \text{ inches}$$
$$= \frac{3\pi}{2} \text{ inches}$$
$$\approx 4.71 \text{ inches.}$$

9. We are given ω, the angular speed.
$\omega = 45$ revolutions per minute
We use the formula $v = r\omega$ to find v, the linear speed. Before applying the formula, we must express ω in radians per minute.

$$\omega = \frac{45 \text{ revolutions}}{1 \text{ minute}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$
$$= \frac{90\pi \text{ radians}}{1 \text{ minute}}$$

The angular speed of the propeller is 90π radians per minute. The linear speed is

$$v = r\omega$$
$$= 1.5 \text{ inches} \cdot \frac{90\pi}{1 \text{ minute}}$$
$$= \frac{135\pi \text{ inches}}{\text{minute}}$$
$$= 135\pi \frac{\text{inches}}{\text{minute}}$$
$$\approx 424 \frac{\text{inches}}{\text{minute}}$$

The linear speed is 135π inches per minute, which is approximately 424 inches per minute.

Concept and Vocabulary Check 4.1

1. origin; x-axis

2. counterclockwise; clockwise

3. acute; right; obtuse; straight

4. $\frac{s}{r}$

5. $\frac{\pi}{180°}$

6. $\frac{180°}{\pi}$

7. coterminal; $360°$; 2π

8. $r\theta$

9. false

10. $r\omega$; angular

Exercise Set 4.1

1. obtuse

2. obtuse

3. acute

4. acute

5. straight

6. right

7. $\theta = \frac{s}{r} = \frac{40 \text{ inches}}{10 \text{ inches}} = 4 \text{ radians}$

8. $\theta = \frac{s}{r} = \frac{30 \text{ feet}}{5 \text{ feet}} = 6 \text{ radians}$

9. $\theta = \frac{s}{r} = \frac{8 \text{ yards}}{6 \text{ yards}} = \frac{4}{3} \text{ radians}$

10. $\theta = \frac{s}{r} = \frac{18 \text{ yards}}{8 \text{ yards}} = 2.25 \text{ radians}$

11. $\theta = \frac{s}{r} = \frac{400 \text{ centimeters}}{100 \text{ centimeters}} = 4 \text{ radians}$

12. $\theta = \frac{s}{r} = \frac{600 \text{ centimeters}}{100 \text{ centimeters}} = 6 \text{ radians}$

13. $45° = 45° \cdot \frac{\pi \text{ radians}}{180°} = \frac{45\pi}{180} \text{ radians} = \frac{\pi}{4} \text{ radians}$

14. $18° = 18° \cdot \frac{\pi \text{ radians}}{180°} = \frac{18\pi}{180} \text{ radians} = \frac{\pi}{10} \text{ radians}$

15. $135° = 135° \cdot \frac{\pi \text{ radians}}{180°} = \frac{135\pi}{180} \text{ radians} = \frac{3\pi}{4} \text{ radians}$

16. $150° = 150° \cdot \frac{\pi \text{ radians}}{180°} = \frac{150\pi}{180} \text{ radians} = \frac{5\pi}{6} \text{ radians}$

17. $300° = 300° \cdot \frac{\pi \text{ radians}}{180°} = \frac{300\pi}{180} \text{ radians} = \frac{5\pi}{3} \text{ radians}$

18. $330° = 330° \cdot \frac{\pi \text{ radians}}{180°} = \frac{330\pi}{180} \text{ radians} = \frac{11\pi}{6} \text{ radians}$

19. $-225° = -225° \cdot \frac{\pi \text{ radians}}{180°} = -\frac{225\pi}{180} \text{ radians} = -\frac{5\pi}{4} \text{ radians}$

20. $-270° = -270° \cdot \frac{\pi \text{ radians}}{180°} = -\frac{270\pi}{180} \text{ radians} = -\frac{3\pi}{2} \text{ radians}$

21. $\frac{\pi}{2} \text{ radians} = \frac{\pi \text{ radians}}{2} \cdot \frac{180^{o}}{\pi \text{ radians}} = \frac{180^{o}}{2} = 90^{o}$

22. $\frac{\pi}{9} \text{ radians} = \frac{\pi \text{ radians}}{9} \cdot \frac{180^{o}}{\pi \text{ radians}} = \frac{180^{o}}{9} = 20^{o}$

23. $\frac{2\pi}{3} \text{ radians} = \frac{2\pi \text{ radians}}{3} \cdot \frac{180^{o}}{\pi \text{ radians}} = \frac{2 \cdot 180^{o}}{3} = 120^{o}$

24. $\frac{3\pi \text{ radians}}{4} \cdot \frac{180^{0}}{\pi \text{ radians}} = \frac{3 \cdot 180^{o}}{4} = 135^{o}$

25. $\frac{7\pi}{6} \text{ radians} = \frac{7\pi \text{ radians}}{6} \cdot \frac{180^{o}}{\pi \text{ radians}} = \frac{7 \cdot 180^{o}}{6} = 210^{o}$

26. $\frac{11\pi \text{ radians}}{6} \cdot \frac{180^{o}}{\pi \text{ radians}} = \frac{11 \cdot 180^{o}}{6} = 330^{o}$

27. $-3\pi \text{ radians} = -3\pi \text{ radians} \cdot \dfrac{180^\circ}{\pi \text{ radians}}$
$= -3 \cdot 180^\circ$
$= -540^\circ$

28. $-4\pi \text{ radians } \cdot \dfrac{180^\circ}{\pi \text{ radians}} = -4 \cdot 180^\circ = -720^\circ$

29. $18° = 18° \cdot \dfrac{\pi \text{ radians}}{180°}$
$= \dfrac{18\pi}{180} \text{ radians}$
$\approx 0.31 \text{ radians}$

30. $76° = 76° \cdot \dfrac{\pi \text{ radians}}{180°}$
$= \dfrac{76\pi}{180} \text{ radians}$
$\approx 1.33 \text{ radians}$

31. $-40° = -40° \cdot \dfrac{\pi \text{ radians}}{180°}$
$= -\dfrac{40\pi}{180} \text{ radians}$
$\approx -0.70 \text{ radians}$

32. $-50° = -50° \cdot \dfrac{\pi \text{ radians}}{180°}$
$= -\dfrac{50\pi}{180} \text{ radians}$
$\approx -0.87 \text{ radians}$

33. $200° = 200° \cdot \dfrac{\pi \text{ radians}}{180°}$
$= \dfrac{200\pi}{180} \text{ radians}$
$\approx 3.49 \text{ radians}$

34. $250° = 250° \cdot \dfrac{\pi \text{ radians}}{180°}$
$= \dfrac{250\pi}{180} \text{ radians}$
$\approx 4.36 \text{ radians}$

35. $2 \text{ radians} = 2 \text{ radians} \cdot \dfrac{180^\circ}{\pi \text{ radians}}$
$= \dfrac{2 \cdot 180^\circ}{\pi}$
$\approx 114.59^\circ$

36. $3 \text{ radians} \cdot \dfrac{180^\circ}{\pi \text{ radians}} = \dfrac{3 \cdot 180^\circ}{\pi} \approx 171.89^\circ$

37. $\dfrac{\pi}{13} \text{ radians} = \dfrac{\pi \text{ radians}}{13} \cdot \dfrac{180^\circ}{\pi \text{ radians}}$
$= \dfrac{180^\circ}{13}$
$\approx 13.85^\circ$

38. $\dfrac{\pi}{17} \text{radians} \cdot \dfrac{180^\circ}{\pi \text{radians}} = \dfrac{180^\circ}{17} \approx 10.59^\circ$

39. $-4.8 \text{ radians} = -4.8 \text{ radians} \cdot \dfrac{180^\circ}{\pi \text{ radians}}$
$= \dfrac{-4.8 \cdot 180^\circ}{\pi}$
$\approx -275.02^\circ$

40. $-5.2 \text{ radians} \cdot \dfrac{180^\circ}{\pi \text{radians}} = \dfrac{-5.2 \cdot 180^\circ}{\pi}$
$\approx -297.94^\circ$

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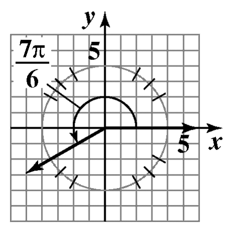

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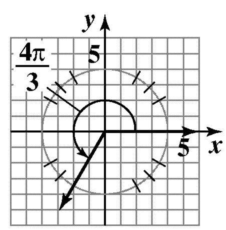

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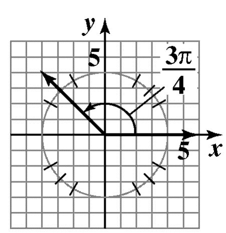

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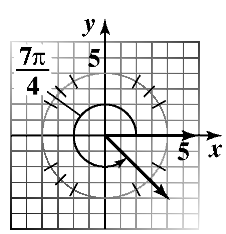

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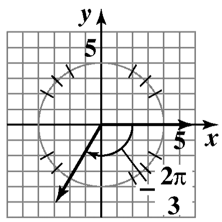

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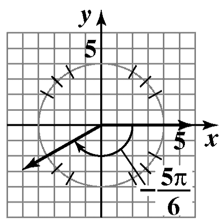

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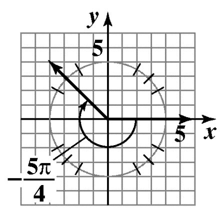

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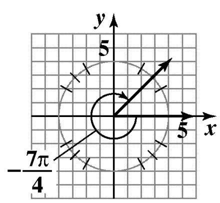

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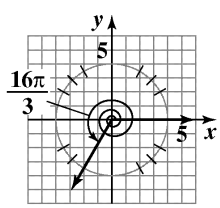

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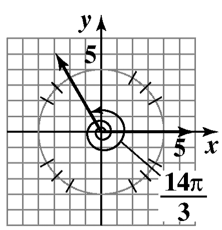

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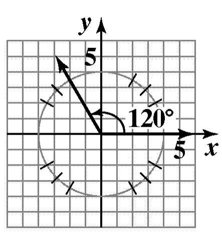

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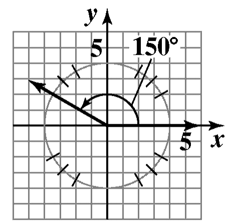

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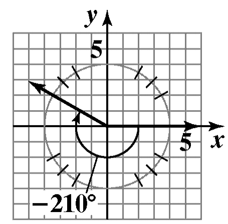

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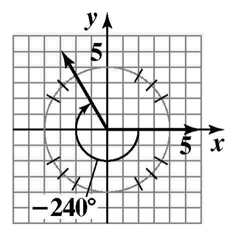

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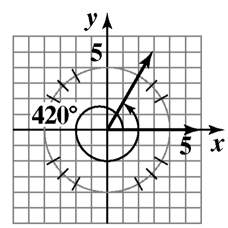

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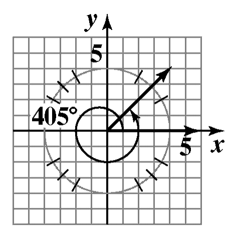


57. $395° - 360° = 35°$

58. $415° - 360° = 55°$

59. $-150° + 360° = 210°$

60. $-160° + 360° = 200°$

61. $-765° + 360° \cdot 3 = -765° + 1080° = 315°$

62. $-760° + 360° \cdot 3 = -760° + 1080° = 320°$

63. $\frac{19\pi}{6} - 2\pi = \frac{19\pi}{6} - \frac{12\pi}{6} = \frac{7\pi}{6}$

64. $\frac{17\pi}{5}-2\pi=\frac{17\pi}{5}-\frac{10\pi}{5}=\frac{7\pi}{5}$

65. $\frac{23\pi}{5}-2\pi\cdot 2=\frac{23\pi}{5}-4\pi=\frac{23\pi}{5}-\frac{20\pi}{5}=\frac{3\pi}{5}$

66. $\frac{25\pi}{6}-2\pi\cdot 2=\frac{25\pi}{6}-4\pi=\frac{25\pi}{6}-\frac{24\pi}{6}=\frac{\pi}{6}$

67. $-\frac{\pi}{50}+2\pi=-\frac{\pi}{50}+\frac{100\pi}{50}=\frac{99\pi}{50}$

68. $-\frac{\pi}{40}+2\pi=-\frac{\pi}{40}+\frac{80\pi}{40}=\frac{79\pi}{40}$

69. $-\frac{31\pi}{7}+2\pi\cdot 3=-\frac{31\pi}{7}+6\pi$

$=-\frac{31\pi}{7}+\frac{42\pi}{7}=\frac{11\pi}{7}$

70. $-\frac{38\pi}{9}+2\pi\cdot 3=-\frac{38\pi}{9}+6\pi=-\frac{38\pi}{9}+\frac{54\pi}{9}=\frac{16\pi}{9}$

71. $r=12$ inches, $\theta=45°$

Begin by converting 45° to radians, in order to use the formula $s=r\theta$.

$45°=45°\cdot\frac{\pi\text{ radians}}{180°}=\frac{\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=12\cdot\frac{\pi}{4}=3\pi$ inches ≈ 9.42 inches

72. $r=16$ inches, $\theta=60°$

Begin by converting 60° to radians, in order to use the formula $s=r\theta$.

$60°=60°\cdot\frac{\pi\text{ radians}}{180°}=\frac{\pi}{3}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=16\cdot\frac{\pi}{3}=\frac{16\pi}{3}$ inches ≈ 16.76 inches

73. $r=8$ feet, $\theta=225°$

Begin by converting 225° to radians, in order to use the formula $s=r\theta$.

$225°=225°\cdot\frac{\pi\text{ radians}}{180°}=\frac{5\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=8\cdot\frac{5\pi}{4}=10\pi$ feet ≈ 31.42 feet

74. $r=9$ yards, $\theta=315°$

Begin by converting 315° to radians, in order to use the formula $s=r\theta$.

$315°=315°\cdot\frac{\pi\text{ radians}}{180°}=\frac{7\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=9\cdot\frac{7\pi}{4}=\frac{63\pi}{4}$ yards ≈ 49.48 yards

75. 6 revolutions per second

$=\frac{6\text{ revolutions}}{1\text{ second}}\cdot\frac{2\pi\text{ radians}}{1\text{ revolutions}}=\frac{12\pi\text{ radians}}{1\text{ seconds}}$

$=12\pi$ radians per second

76. 20 revolutions per second

$=\frac{20\text{ revolutions}}{1\text{ second}}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}=\frac{40\pi\text{ radians}}{1\text{ second}}$

$=40\pi$ radians per second

77. $-\frac{4\pi}{3}$ and $\frac{2\pi}{3}$

78. $-\frac{7\pi}{6}$ and $\frac{5\pi}{6}$

79. $-\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

80. $-\frac{\pi}{4}$ and $\frac{7\pi}{4}$

81. $-\frac{\pi}{2}$ and $\frac{3\pi}{2}$

82. $-\pi$ and π

83. $\frac{55}{60}\cdot 2\pi=\frac{11\pi}{6}$

84. $\frac{35}{60}\cdot 2\pi=\frac{7\pi}{6}$

85. 3 minutes and 40 seconds equals 220 seconds.

$\frac{220}{60}\cdot 2\pi=\frac{22\pi}{3}$

86. 4 minutes and 25 seconds equals 265 seconds.

$\frac{265}{60}\cdot 2\pi=\frac{53\pi}{6}$

87. First, convert to degrees.

$$\frac{1}{6}\text{ revolution} = \frac{1}{6}\text{ revolution}\cdot\frac{360°}{1\text{ revolution}} = \frac{1}{6}\cdot 360° = 60°$$

Now, convert 60° to radians.

$$60° = 60°\cdot\frac{\pi\text{ radians}}{180°} = \frac{60\pi}{180}\text{ radians} = \frac{\pi}{3}\text{ radians}$$

Therefore, $\frac{1}{6}$ revolution is equivalent to 60° or $\frac{\pi}{3}$ radians.

88. First, convert to degrees.

$$\frac{1}{3}\text{revolutions} = \frac{1}{3}\text{ revolutions}\cdot\frac{360^{\text{o}}}{1\text{ revolution}} = \frac{1}{3}\cdot 360^{\text{o}} = 120^{\text{o}}$$

Now, convert 120° to radians.

$$120° = 120°\cdot\frac{\pi\text{ radians}}{180°} = \frac{120\pi}{180}\text{ radians} = \frac{2\pi}{3}\text{ radians}$$

Therefore, $\frac{1}{3}$ revolution is equivalent to 120° or $\frac{2\pi}{3}$ radians.

89. The distance that the tip of the minute hand moves is given by its arc length, s. Since $s = r\theta$, we begin by finding r and θ. We are given that $r = 8$ inches. The minute hand moves from 12 to 2 o'clock, or $\frac{1}{6}$ of a complete revolution. The formula $s = r\theta$ can only be used when θ is expressed in radians. We must convert $\frac{1}{6}$ revolution to radians.

$$\frac{1}{6}\text{ revolution} = \frac{1}{6}\text{ revolution}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}} = \frac{\pi}{3}\text{ radians}$$

The distance the tip of the minute hand moves is

$$s = r\theta = (8\text{ inches})\left(\frac{\pi}{3}\right) = \frac{8\pi}{3}\text{ inches} \approx 8.38\text{ inches.}$$

90. The distance that the tip of the minute hand moves is given by its arc length, s. Since $s = r\theta$, we begin by finding r and θ. We are given that $r = 6$ inches. The minute hand moves from 12 to 4 o'clock, or $\frac{1}{3}$ of a complete revolution. The formula $s = r\theta$ can only be used when θ is expressed in radians. We must convert $\frac{1}{3}$ revolution to radians.

$$\frac{1}{3}\text{ revolution} = \frac{1}{3}\text{revolution}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}} = \frac{2\pi}{3}\text{ radians}$$

The distance the tip of the minute hand moves is

$$s = r\theta = (6\text{ inches})\left(\frac{2\pi}{3}\right) = \frac{12\pi}{3}\text{ inches} = 4\pi\text{ inches} \approx 12.57\text{ inches.}$$

91. The length of each arc is given by $s = r\theta$. We are given that $r = 24$ inches and $\theta = 90°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$90° = 90°\cdot\frac{\pi\text{ radians}}{180°} = \frac{90\pi}{180}\text{ radians} = \frac{\pi}{2}\text{ radians}$$

The length of each arc is

$$s = r\theta = (24\text{ inches})\left(\frac{\pi}{2}\right) = 12\pi\text{ inches} \approx 37.70\text{ inches.}$$

92. The distance that the wheel moves is given by $s = r\theta$. We are given that $r = 80$ centimeters and $\theta = 60°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$60° = 60°\cdot\frac{\pi\text{ radians}}{180°} = \frac{60\pi}{180}\text{ radians} = \frac{\pi}{3}\text{ radians}$$

The length that the wheel moves is

$$s = r\theta = (80\text{ centimeters})\left(\frac{\pi}{3}\right) = \frac{80\pi}{3}\text{ centimeters} \approx 83.78\text{ centimeters.}$$

93. Recall that $\theta = \frac{s}{r}$. We are given that s = 8000 miles and r = 4000 miles.

$$\theta = \frac{s}{r} = \frac{8000 \text{ miles}}{4000 \text{ miles}} = 2 \text{ radians}$$

Now, convert 2 radians to degrees.

$$2 \text{ radians} = 2 \text{ radians} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}} \approx 114.59^{\text{o}}$$

94. Recall that $\theta = \frac{s}{r}$. We are given that s = 10,000 miles and r = 4000 miles.

$$\theta = \frac{s}{r} = \frac{10{,}000 \text{ miles}}{4000 \text{ miles}} = 2.5 \text{ radians}$$

Now, convert 2.5 radians to degrees.

$$2.5 \text{ radians} \cdot \frac{180^{\text{o}}}{2\pi \text{ radians}} \approx 143.24^{\text{o}}$$

95. Recall that $s = r\theta$. We are given that r = 4000 miles and $\theta = 30°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$30° = 30° \cdot \frac{\pi \text{ radians}}{180°} = \frac{30\pi}{180} \text{ radians}$$
$$= \frac{\pi}{6} \text{ radians}$$
$$s = r\theta = (4000 \text{ miles})\left(\frac{\pi}{6}\right) \approx 2094 \text{ miles}$$

To the nearest mile, the distance from *A* to *B* is 2094 miles.

96. Recall that $s = r\theta$. We are given that r = 4000 miles and $\theta = 10°$. We can only use the formula $s = r\theta$ when θ is expressed in radians.

$$10° = 10° \cdot \frac{\pi \text{ radians}}{180°} = \frac{10\pi}{180} \text{ radians}$$
$$= \frac{\pi}{18} \text{ radians}$$
$$s = r\theta = (4000 \text{ miles})\left(\frac{\pi}{18}\right) \approx 698 \text{ miles}$$

To the nearest mile, the distance from *A* to *B* is 698 miles.

97. Linear speed is given by $v = r\omega$. We are given that $\omega = \frac{\pi}{12}$ radians per hour and r = 4000 miles. Therefore,

$$v = r\omega = (4000 \text{ miles})\left(\frac{\pi}{12}\right)$$
$$= \frac{4000\pi}{12} \text{ miles per hour}$$
$$\approx 1047 \text{ miles per hour}$$

The linear speed is about 1047 miles per hour.

98. Linear speed is given by $v = r\omega$. We are given that r = 25 feet and the wheel rotates at 2 revolutions per minute. We need to convert 2 revolutions per minute to radians per minute.
2 revolutions per minute

$$= 2 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$
$$= 4\pi \text{ radians per minute}$$
$$v = r\omega = (25 \text{ feet})(4\pi) \approx 314 \text{ feet per minute}$$

The linear speed of the Ferris wheel is about 314 feet per minute.

99. Linear speed is given by $v = r\omega$. We are given that r = 12 feet and the wheel rotates at 20 revolutions per minute.
20 revolutions per minute

$$= 20 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$
$$= 40\pi \text{ radians per minute}$$
$$v = r\omega = (12 \text{ feet})(40\pi)$$
$$\approx 1508 \text{ feet per minute}$$

The linear speed of the wheel is about 1508 feet per minute.

100. Begin by converting 2.5 revolutions per minute to radians per minute.
2.5 revolutions per minute

$$= 2.5 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$
$$= 5\pi \text{ radians per minute}$$

The linear speed of the animals in the outer rows is
$$v = r\omega = (20 \text{ feet})(5\pi) \approx 100 \text{ feet per minute}$$
The linear speed of the animals in the inner rows is
$$v = r\omega = (10 \text{ feet})(5\pi) \approx 50 \text{ feet per minute}$$
The difference is $100\pi - 50\pi = 50\pi$ feet per minute or about 157 feet per minute.

101. – 112. Answers may vary.

113. $30°15'10" = 30.25°$

114. $65°45'20" = 65.76°$

115. $30.42° = 30°25'12"$

116. $50.42° = 50°25'12"$

117. does not make sense; Explanations will vary. Sample explanation: Angles greater than π will exceed a straight angle.

118. does not make sense; Explanations will vary. Sample explanation: It is possible for π to be used in an angle measured using degrees.

119. makes sense

120. does not make sense; Explanations will vary. Sample explanation: That will not be possible if the angle is a multiple of 2π.

121. A right angle measures 90° and

$90° = \frac{\pi}{2}$ radians ≈ 1.57 radians.

If $\theta = \frac{3}{2}$ radians = 1.5 radians, θ is smaller than a right angle.

122. $s = r\theta$

Begin by changing $\theta = 20°$ to radians.

$$20° = 20° \cdot \frac{\pi}{180°} = \frac{\pi}{9} \text{ radians}$$

$$100 = \frac{\pi}{9} r$$

$$r = \frac{900}{\pi} \approx 286 \text{ miles}$$

To the nearest mile, a radius of 286 miles should be used.

123. $s = r\theta$

Begin by changing $\theta = 26°$ to radians.

$$26° = 26° \cdot \frac{\pi}{180°} = \frac{13\pi}{90} \text{ radians}$$

$$s = 4000 \cdot \frac{13\pi}{90}$$

$$\approx 1815 \text{ miles}$$

To the nearest mile, Miami, Florida is 1815 miles north of the equator.

124. $\log_3(x+5) = 2$

$$3^2 = x+5$$
$$9 = x+5$$
$$4 = x$$

Check:

$$\log_3(x+5) = 2$$
$$\log_3(4+5) = 2$$
$$\log_3 9 = 2$$
$$2 = 2$$

The solution set is $\{4\}$.

125. $x^2 + 4x + 6 = 0$

$a = 1, b = 4, c = 6$

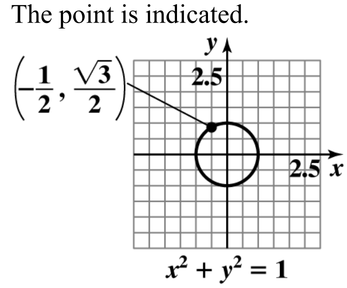
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(6)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16-24}}{2}$$
$$= \frac{-4 \pm \sqrt{-8}}{2}$$
$$= \frac{-4 \pm 2i\sqrt{2}}{2}$$
$$= \frac{2(-2 \pm i\sqrt{2})}{2}$$
$$= -2 \pm i\sqrt{2}$$

The solution set is $\{-2 \pm i\sqrt{2}\}$.

126. $f(x) = \begin{cases} x^2 + 2x - 1 & \text{if } x \ge 2 \\ 3x + 1 & \text{if } x < 2 \end{cases}$

$$f(5) - f(-5) = (5^2 + 2 \cdot 5 - 1) - (3(-5) + 1)$$
$$= (25 + 10 - 1) - (-15 + 1)$$
$$= 34 - (-14)$$
$$= 48$$

127. The point is indicated.



128. domain: $\{x \mid -1 \le x \le 1\}$ or $[-1,1]$

range: $\{y \mid -1 \le y \le 1\}$ or $[-1,1]$

129. $x = -\frac{1}{2}; \quad y = \frac{\sqrt{3}}{2}$

$$\frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Section 4.2

Check Point Exercises

1. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin t = y = \frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

$\csc t = \frac{1}{y} = 2$

$\sec t = \frac{1}{x} = \frac{2\sqrt{3}}{3}$

$\cot t = \frac{x}{y} = \sqrt{3}$

2. The point P on the unit circle that corresponds to $t = \pi$ has coordinates (–1, 0). Use $x = -1$ and $y = 0$ to find the values of the trigonometric functions.

$\sin \pi = y = 0$

$\cos \pi = x = -1$

$\tan \pi = \frac{y}{x} = \frac{0}{-1} = 0$

$\sec \pi = \frac{1}{x} = \frac{1}{-1} = -1$

$\cot \pi = \frac{x}{y} = \frac{-1}{0} = \text{undefined}$

$\csc \pi = \frac{1}{y} = \frac{1}{0} = \text{undefined}$

3. $t = \frac{\pi}{4}, P\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\csc \frac{\pi}{4} = \frac{1}{y} = \sqrt{2}$

$\sec \frac{\pi}{4} = \frac{1}{x} = \sqrt{2}$

$\cot \frac{\pi}{4} = \frac{x}{y} = \frac{\frac{1}{y}}{\frac{1}{\sqrt{2}}} = 1$

4. **a.** $\sec\left(-\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

b. $\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

5. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}}$

$= \frac{2}{3} \cdot \frac{3}{\sqrt{5}} = \frac{2}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}$

$= \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2}{\sqrt{5}}} = \frac{\sqrt{5}}{2}$

6. $\sin t = \frac{1}{2}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{1}{2}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{1}{4}$

$\cos t = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Because $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive.

7. **a.** $\cot\frac{5\pi}{4}=\cot\left(\frac{\pi}{4}+\pi\right)=\cot\frac{\pi}{4}=1$

b. $\cos\left(-\frac{9\pi}{4}\right)=\cos\left(-\frac{9\pi}{4}+4\pi\right)$
$=\cos\frac{7\pi}{4}$
$=\frac{\sqrt{2}}{2}$

8. **a.** $\sin\frac{\pi}{4}\approx 0.7071$

b. $\csc 1.5\approx 1.0025$

Concept and Vocabulary Check 4.2

1. intercepted are
2. cosine; sine
3. sine; cosine; $(-\infty,\infty)$
4. 1; -1; $[-1,1]$
5. $\frac{\sqrt{2}}{2}$; $\frac{\sqrt{2}}{2}$; 1
6. cos t; sec t; even
7. $-\sin t$; $-\csc t$; $-\tan t$; $-\cot t$; odd
8. $\sin t$; $\cos t$; $\tan t$
9. $\tan t$; $\cot t$
10. 1; $\sec^2 t$; $\csc^2 t$
11. periodic; period
12. $\sin t$; $\cos t$; periodic; 2π
13. $\tan t$; $\cot t$; periodic; π

Exercise Set 4.2

1. The point P on the unit circle has coordinates $\left(-\frac{15}{17},\frac{8}{17}\right)$. Use $x=-\frac{15}{17}$ and $y=\frac{8}{17}$ to find the values of the trigonometric functions.

$\sin t=y=\frac{8}{17}$

$\cos t=x=-\frac{15}{17}$

$\tan t=\frac{y}{x}=\frac{\frac{8}{17}}{-\frac{15}{17}}=-\frac{8}{15}$

$\csc t=\frac{1}{y}=\frac{17}{8}$

$\sec t=\frac{1}{x}=-\frac{17}{15}$

$\cot t=\frac{x}{y}=-\frac{15}{8}$

2. The point P on the unit circle has coordinates $\left(-\frac{5}{13},-\frac{12}{13}\right)$ Use $x=-\frac{5}{13}$ and $y=-\frac{12}{13}$ to find the values of the trigonometric functions.

$\sin t=y=-\frac{12}{13}$

$\cos t=x=-\frac{5}{13}$

$\tan t=\frac{y}{x}=\frac{-\frac{12}{13}}{-\frac{5}{13}}=\frac{12}{5}$

$\csc t=\frac{1}{y}=-\frac{13}{12}$

$\sec t=\frac{1}{x}=-\frac{13}{5}$

$\cot t=\frac{x}{y}=\frac{5}{12}$

3. The point P on the unit circle that corresponds to $t=-\frac{\pi}{4}$ has coordinates $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$. Use $x=\frac{\sqrt{2}}{2}$ and $y=-\frac{\sqrt{2}}{2}$ to find the values of the trigonometric functions.

$\sin t = y = -\frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

$\csc t = \frac{1}{y} = -\sqrt{2}$

$\sec t = \frac{1}{x} = \sqrt{2}$

$\cot t = \frac{x}{y} = -1$

4. The point P on the unit circle that corresponds to $t=\frac{3\pi}{4}$ has coordinates $\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$. Use $x=-\frac{\sqrt{2}}{2}$ and $y=\frac{\sqrt{2}}{2}$ to find the values of the trigonometric functions.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = -\frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$

$\csc t = \frac{1}{y} = \sqrt{2}$

$\sec t = \frac{1}{x} = -\sqrt{2}$

$\cot t = \frac{x}{y} = -1$

5. $\sin\frac{\pi}{6} = \frac{1}{2}$

6. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

7. $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

8. $\cos\frac{2\pi}{3} = -\frac{1}{2}$

9. $\tan\pi = \frac{0}{-1} = 0$

10. $\tan 0 = \frac{0}{1} = 0$

11. $\csc\frac{7\pi}{6} = \frac{1}{-\frac{1}{2}} = -2$

12. $\csc\frac{4\pi}{3} = \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{-2\sqrt{3}}{3}$

13. $\sec\frac{11\pi}{6} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

14. $\sec\frac{5\pi}{3} = \frac{1}{\frac{1}{2}} = 2$

15. $\sin\frac{3\pi}{2} = -1$

16. $\cos\frac{3\pi}{2} = 0$

17. $\sec\frac{3\pi}{2} = \text{undefined}$

18. $\tan\frac{3\pi}{2} = \text{undefined}$

19. **a.** $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

b. $\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

20. **a.** $\cos\frac{\pi}{3} = \frac{1}{2}$

b. $\cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

21. **a.** $\sin\frac{5\pi}{6} = \frac{1}{2}$

b. $\sin\left(-\frac{5\pi}{6}\right) = -\sin\frac{5\pi}{6} = -\frac{1}{2}$

22. **a.** $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

b. $\sin\left(-\frac{2\pi}{3}\right) = -\sin\frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

23. **a.** $\tan\frac{5\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

b. $\tan\left(-\frac{5\pi}{3}\right) = -\tan\frac{5\pi}{3} = \sqrt{3}$

24. **a.** $\tan\frac{11\pi}{6} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

b. $\tan\left(-\frac{11\pi}{6}\right) = -\tan\frac{11\pi}{6} = \frac{\sqrt{3}}{3}$

25. $\sin t = \frac{8}{17}, \cos t = \frac{15}{17}$

$\tan t = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$

$\csc t = \frac{17}{8}$

$\sec t = \frac{17}{15}$

$\cot t = \frac{15}{8}$

26. $\sin t = \frac{3}{5}, \cos t = \frac{4}{5}$

$\tan t = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

$\csc t = \frac{5}{3}$

$\sec t = \frac{5}{4}$

$\cot t = \frac{4}{3}$

27. $\sin t = \frac{1}{3}, \cos t = \frac{2\sqrt{2}}{3}$

$\tan t = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{\sqrt{2}}{4}$

$\csc t = 3$

$\sec t = \frac{3\sqrt{2}}{4}$

$\cot t = 2\sqrt{2}$

28. $\sin t = \frac{2}{3}, \cos t = \frac{\sqrt{5}}{3}$

$\tan t = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2\sqrt{5}}{5}$

$\csc t = \frac{3}{2}$

$\sec t = \frac{3\sqrt{5}}{5}$

$\cot t = \frac{\sqrt{5}}{2}$

29. $\sin t = \frac{6}{7}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{6}{7}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{36}{49}$

$\cos t = \sqrt{\frac{13}{49}} = \frac{\sqrt{13}}{7}$

Because $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive.

30. $\sin t = \frac{7}{8}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{7}{8}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{49}{64}$

$\cos t = \sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{8}$

Because $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive.

31. $\sin t = \frac{\sqrt{39}}{8}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{\sqrt{39}}{8}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{39}{64}$

$\cos t = \sqrt{\frac{25}{64}} = \frac{5}{8}$

Because $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive.

32. $\sin t = \frac{\sqrt{21}}{5}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{\sqrt{21}}{5}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{21}{25}$

$\cos t = \sqrt{\frac{4}{25}} = \frac{2}{5}$

Because $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive.

33. $\sin 1.7 \csc 1.7 = \sin 1.7\left(\frac{1}{\sin 1.7}\right) = 1$

34. $\cos 2.3 \sec 2.3 = \cos 2.3\left(\frac{1}{\cos 2.3}\right) = 1$

35. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{2} = 1$ by the Pythagorean identity.

36. $\sin^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{3} = 1$ because $\sin^2 t + \cos^2 t = 1$.

37. $\sec^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{3} = 1$ because $1 + \tan^2 t = \sec^2 t$.

38. $\csc^2 \frac{\pi}{6} - \cot^2 \frac{\pi}{6} = 1$ because $1 + \cot^2 t = \csc^2 t$.

39. $\cos \frac{9\pi}{4} = \cos\left(\frac{\pi}{4} + 2\pi\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

40. $\csc \frac{9\pi}{4} = \csc\left(\frac{\pi}{4} + 2\pi\right) = \csc \frac{\pi}{4} = \sqrt{2}$

41. $\sin\left(-\frac{9\pi}{4}\right) = \sin\left(-\frac{9\pi}{4} + 4\pi\right) = \sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$

42. $\sec\left(-\frac{9\pi}{4}\right) = \sec\left(-\frac{9\pi}{4} + 4\pi\right) = \sec \frac{7\pi}{4} = \sqrt{2}$

43. $\tan \frac{5\pi}{4} = \tan\left(\frac{\pi}{4} + \pi\right) = \tan \frac{\pi}{4} = 1$

44. $\cot \frac{5\pi}{4} = \cot\left(\frac{\pi}{4} + \pi\right) = \cot \frac{\pi}{4} = 1$

45. $\cot\left(-\frac{5\pi}{4}\right) = \cot\left(\frac{3\pi}{4} - 2\pi\right) = \cot \frac{3\pi}{4} = -1$

46. $\tan\left(-\frac{9\pi}{4}\right) = \tan\left(-\frac{9\pi}{4} + 3\pi\right) = \tan \frac{3\pi}{4} = -1$

47. $-\tan\left(\frac{\pi}{4} + 15\pi\right) = -\tan \frac{\pi}{4} = -1$

48. $-\cot\left(\frac{\pi}{4} + 17\pi\right) = -\cot \frac{\pi}{4} = -1$

49. $\sin\left(-\frac{\pi}{4} - 1000\pi\right) = \sin\left(-\frac{\pi}{4} + 2\pi\right)$

$= \sin \frac{7\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

50. $\sin\left(-\frac{\pi}{4} - 2000\pi\right) = \sin\left(-\frac{\pi}{4} + 2\pi\right)$

$= \sin \frac{7\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

51. $\cos\left(-\frac{\pi}{4}-1000\pi\right)=\cos\left(-\frac{\pi}{4}+2\pi\right)$
$=\cos\frac{7\pi}{4}$
$=\frac{\sqrt{2}}{2}$

52. $\cos\left(-\frac{\pi}{4}-2000\pi\right)=\cos\left(-\frac{\pi}{4}+2\pi\right)$
$=\cos\frac{7\pi}{4}$
$=\frac{\sqrt{2}}{2}$

53. **a.** $\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

b. $\sin\frac{11\pi}{4}=\sin\left(\frac{3\pi}{4}+2\pi\right)=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

54. **a.** $\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

b. $\cos\frac{11\pi}{4}=\cos\left(\frac{3\pi}{4}+2\pi\right)=\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

55. **a.** $\cos\frac{\pi}{2}=0$

b. $\cos\frac{9\pi}{2}=\cos\left(\frac{\pi}{2}+4\pi\right)$
$=\cos\left[\frac{\pi}{2}+2(2\pi)\right]$
$=\cos\frac{\pi}{2}$
$=0$

56. **a.** $\sin\frac{\pi}{2}=1$

b. $\sin\frac{9\pi}{2}=\sin\left(\frac{\pi}{2}+4\pi\right)=\sin\frac{\pi}{2}=1$

57. **a.** $\tan\pi=\frac{0}{-1}=0$

b. $\tan 17\pi=\tan(\pi+16\pi)$
$=\tan[\pi+8(2\pi)]$
$=\tan\pi$
$=0$

58. **a.** $\cot\frac{\pi}{2}=\frac{0}{1}=0$

b. $\cot\frac{15\pi}{2}=\cot\left(\frac{\pi}{2}+7\pi\right)=\cot\frac{\pi}{2}=0$

59. **a.** $\sin\frac{7\pi}{4}=-\frac{\sqrt{2}}{2}$

b. $\sin\frac{47\pi}{4}=\sin\left(\frac{7\pi}{4}+10\pi\right)$
$=\sin\left[\frac{7\pi}{4}+5(2\pi)\right]$
$=\sin\frac{7\pi}{4}$
$=-\frac{\sqrt{2}}{2}$

60. **a.** $\cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2}$

b. $\cos\frac{47\pi}{4}=\cos\left(\frac{7\pi}{4}+10\pi\right)=\cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2}$

61. $\sin 0.8\approx 0.7174$

62. $\cos 0.6\approx 0.8253$

63. $\tan 3.4\approx 0.2643$

64. $\tan 3.7\approx 0.6247$

65. $\csc 1\approx 1.1884$

66. $\sec 1\approx 1.8508$

67. $\cos\frac{\pi}{10}\approx 0.9511$

68. $\sin\frac{3\pi}{10}\approx 0.8090$

69. $\cot\frac{\pi}{12}\approx 3.7321$

70. $\cot\frac{\pi}{18}\approx 5.6713$

71. $\sin(-t)-\sin t=-\sin t-\sin t=-2\sin t=-2a$

72. $\tan(-t)-\tan t=-\tan t-\tan t=-2\tan t=-2c$

73. $4\cos(-t)-\cos t = 4\cos t - \cos t = 3\cos t = 3b$

74. $3\cos(-t)-\cos t = 3\cos t - \cos t = 2\cos t = 2b$

75. $\sin(t+2\pi)-\cos(t+4\pi)+\tan(t+\pi)$
$= \sin(t)-\cos(t)+\tan(t)$
$= a-b+c$

76. $\sin(t+2\pi)+\cos(t+4\pi)-\tan(t+\pi)$
$= \sin(t)+\cos(t)-\tan(t)$
$= a+b-c$

77. $\sin(-t-2\pi)-\cos(-t-4\pi)-\tan(-t-\pi)$
$= -\sin(t+2\pi)-\cos(t+4\pi)+\tan(t+\pi)$
$= -\sin(t)-\cos(t)+\tan(t)$
$= -a-b+c$

78. $\sin(-t-2\pi)+\cos(-t-4\pi)-\tan(-t-\pi)$
$= -\sin(t+2\pi)+\cos(t+4\pi)+\tan(t+\pi)$
$= -\sin(t)+\cos(t)+\tan(t)$
$= -a+b+c$

79. $\cos t+\cos(t+1000\pi)-\tan t-\tan(t+999\pi)$
$\quad -\sin t+4\sin(t-1000\pi)$
$= \cos t+\cos t-\tan t-\tan t-\sin t+4\sin t$
$= 2\cos t-2\tan t+3\sin t$
$= 3a+2b-2c$

80. $-\cos t+7\cos(t+1000\pi)+\tan t+\tan(t+999\pi)$
$\quad +\sin t+\sin(t-1000\pi)$
$= -\cos t+7\cos t+\tan t+\tan t+\sin t+\sin t$
$= 6\cos t+2\tan t+2\sin t$
$= 2a+6b+2c$

81. a. $H = 12+8.3\sin\left[\frac{2\pi}{365}(80-80)\right]$
$= 12+8.3\sin 0 = 12+8.3(0)$
$= 12$
There are 12 hours of daylight in Fairbanks on March 21.

b. $H = 12+8.3\sin\left[\frac{2\pi}{365}(172-80)\right]$
$\approx 12+8.3\sin 1.5837$
≈ 20.3
There are about 20.3 hours of daylight in Fairbanks on June 21.

c. $H = 12+8.3\sin\left[\frac{2\pi}{365}(355-80)\right]$
$\approx 12+8.3\sin 4.7339$
≈ 3.7
There are about 3.7 hours of daylight in Fairbanks on December 21.

82. a. $H = 12+24\sin\left[\frac{2\pi}{365}(80-80)\right]$
$12+24\sin 0 = 12+24(0) = 12$
There are 12 hours of daylight in San Diego on March 21.

b. $H = 12+24\sin\left[\frac{2\pi}{365}(172-80)\right]$
$\approx 12+24\sin 1.5837$
≈ 14.3998
There are about 14.4 hours of daylight in San Diego on June 21.

c. $H = 12+24\sin\left[\frac{2\pi}{365}(355-80)\right]$
$\approx 12+24\sin 4.7339$
≈ 9.6
There are about 9.6 hours of daylight in San Diego on December 21.

83. a. For $t = 7$,
$E = \sin\frac{\pi}{14}\cdot 7 = \sin\frac{\pi}{2} = 1$

For $t = 14$,
$E = \sin\frac{\pi}{14}\cdot 14 = \sin\pi = 0$

For $t = 21$,
$E = \sin\frac{\pi}{14}\cdot 21 = \sin\frac{3\pi}{2} = -1$

For $t = 28$,
$E = \sin\frac{\pi}{14}\cdot 28 = \sin 2\pi = \sin 0 = 0$

For $t = 35$,
$E = \sin\frac{\pi}{14}\cdot 35 = \sin\frac{5\pi}{2} = \sin\frac{\pi}{2} = 1$

Observations may vary.

b. Because $E(35) = E(7) = 1$, the period is $35 - 7 = 28$ or 28 days.

84. **a.** At 6 A.M., $t = 0$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 0$$
$$= 10 + 4\sin 0 = 10 + 4\cdot 0 = 10$$

The height is 10 feet.
At 9 A.M., $t = 3$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 3$$
$$= 10 + 4\sin\frac{\pi}{2} = 10 + 4(1) = 14$$

The height is 14 feet.
At noon, $t = 6$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 6$$
$$= 10 + 4\sin\pi = 10 + 4\cdot 0 = 10$$

The height is 10 feet.
At 6 P.M., $t = 12$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 12$$
$$= 10 + 4\sin 2\pi = 10 + 4\cdot 0 = 10$$

The height is 10 feet.
At midnight, $t = 18$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 18$$
$$= 10 + 4\sin 3\pi = 10 + 4\sin\pi$$
$$= 10 + 4\cdot 0 = 10$$

The height is 10 feet.
At 3 A.M., $t = 21$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 21$$
$$= 10 + 4\sin\frac{7\pi}{2} = 10 + 4\sin\frac{3\pi}{2}$$
$$= 10 + 4(-1) = 6$$

The height is 6 feet.

b. The sine function has a minimum at $\frac{3\pi}{2}$. Thus, we find a low tide at $\frac{\pi}{6}t = \frac{3\pi}{2}$ or $t = 9$. This value of t corresponds to 3 P.M. For $t = 9$,

$$h = 10 + 4\sin\frac{\pi}{6}\cdot 9$$
$$= 10 + 4\sin\frac{3\pi}{2} = 10 + 4(-1) = 6$$

The height is 6 feet. From part *a*, the height at 3 A.M. is also 6 feet. Thus, low tide is at 3 A.M. and 3 P.M.

The sine function has a maximum at $\frac{\pi}{2}$. Thus, we find a high tide at $\frac{\pi}{6}t = \frac{\pi}{2}$ or $t = 3$. This value of t corresponds to 9 a.m. From part *a*, the height at 9 A.M. is 14 feet. Because the sine has a period of 2π we also find a maximum at $\frac{5\pi}{2}$.

We find another high tide at $\frac{\pi}{6}t = \frac{5\pi}{2}$ or $t = 15$. This value of t corresponds to 9 P.M. Thus, high tide is at 9 A.M. and 9 P.M.

c. The period of the sine function is 2π or on the interval $[0, 2\pi]$. The cycle of the sine function starts at $\frac{\pi}{6}t = \frac{5\pi}{2}$ or $t = 0$, and ends at $\frac{\pi}{6}t = 2\pi$ or $t = 12$. Thus, the period is 12 hours, which means high and low tides occur every 12 hours.

85. – 96. Answers may vary.

97. makes sense

98. does not make sense; Explanations will vary. Sample explanation: $\sin t$ cannot be less than -1. Note that $-\frac{\sqrt{10}}{2} \approx -1.58 < -1$.

99. does not make sense; Explanations will vary. Sample explanation: Cosine is not an odd function.

100. makes sense

101. t is in the third quadrant therefore $\sin t < 0$, $\tan t > 0$, and $\cot t > 0$.
Thus, only choice (c) is true.

102. $f(x) = \sin x$ and $f(a) = \frac{1}{4}$

$$f(a) + f(a + 2\pi) + f(a + 4\pi) + f(a + 6\pi)$$
$$= 4f(a) = 4\left(\frac{1}{4}\right) = 1$$ because $\sin x$ has a period of 2π.

103. $f(x) = \sin x$ and $f(a) = \frac{1}{4}$

$$f(a) + 2f(-a) = f(a) - 2f(a)$$
$$= \frac{1}{4} - 2\left(\frac{1}{4}\right)$$
$$= -\frac{1}{4}$$

$f(-a) = -f(a)$ because $\sin(-x) = -\sin x$.
Sine is an odd function.

104. The height is given by
$h = 45 + 40\sin(t - 90°)$
$$h(765°) = 45 + 40\sin(765° - 90°)$$
$$\approx 16.7$$
You are about 16.7 feet above the ground.

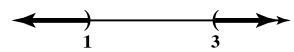
105.
$$f(x) = 3x^2 - x + 5$$
$$f(x+h) = 3(x+h)^2 - (x+h) + 5$$
$$= 3(x^2 + 2xh + h^2) - x - h + 5$$
$$= 3x^2 + 6xh + 3h^2 - x - h + 5$$
$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - x - h + 5 - (3x^2 - x + 5)}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - x - h + 5 - 3x^2 + x - 5}{h}$$
$$= \frac{6xh + 3h^2 - h}{h}$$
$$= \frac{h(6x + 3h - 1)}{h}$$
$$= 6x + 3h - 1, \quad h \neq 0$$

106.
$$x^2 - 4x > -3$$
$$x^2 - 4x + 3 > 0$$
$$(x-1)(x-3) > 0$$
$x = 1$ or $x = 3$

T	F	T
1	3	

Test 0: $0^2 - 4(0) > -3$
$0 > -3$ True
Test 2: $2^2 - 4(2) > -3$
$-4 > -3$ False
Test 4: $4^2 - 4(4) > -3$
$0 > -3$ True

The solution set is $(-\infty, 1) \cup (3, \infty)$.



107. a. $A_0 = 25.5$. Since 2010 is 30 years after 1980, when $t = 30$, $A = 40.3$.
$$A = A_0 e^{kt}$$
$$40.3 = 25.5e^{k(30)}$$
$$\frac{40.3}{25.5} = e^{30k}$$
$$\ln\left(\frac{40.3}{25.5}\right) = \ln e^{30k}$$
$$\ln\left(\frac{40.3}{25.5}\right) = 30k$$
$$k = \frac{\ln\left(\frac{40.3}{25.5}\right)}{30} \approx 0.01526$$
Thus, the growth function is $A = 25.5e^{0.01526t}$.

b.
$$A = 25.5e^{0.01526t}$$
$$80 = 25.5e^{0.01526t}$$
$$\frac{80}{25.5} = e^{0.01526t}$$
$$\ln\left(\frac{80}{25.5}\right) = \ln e^{0.01526t}$$
$$\ln\left(\frac{80}{25.5}\right) = 0.01526t$$
$$t = \frac{\ln\left(\frac{80}{25.5}\right)}{0.01526} \approx 75$$
The elderly U.S. population will reach 80 million approximately 75 years after 1980, or 2055.

108. First find the hypotenuse.
$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144$$
$$c^2 = 169$$
$$c = 13$$
Next write the ratio.
$$\frac{a}{c} = \frac{5}{13}$$

109. First find the hypotenuse.

$c^2 = a^2 + b^2$

$c^2 = 1^2 + 1^2$

$c^2 = 1 + 1$

$c^2 = 2$

$c = \sqrt{2}$

Next write the ratio and simplify.

$$\frac{a}{c} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

110. $$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2}$$

Since $c^2 = a^2 + b^2$, continue simplifying by substituting c^2 for $a^2 + b^2$.

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{\overbrace{a^2 + b^2}^{c^2}}{c^2} = \frac{c^2}{c^2} = 1$$

Section 4.3

Checkpoint Exercises

1. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find c.

$a = 3, b = 4$

$c^2 = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$

$c = \sqrt{25} = 5$

Referring to these lengths as opposite, adjacent, and hypotenuse, we have

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

2. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find b.

$a^2 + b^2 = c^2$

$1^2 + b^2 = 5^2$

$1 + b^2 = 25$

$b^2 = 24$

$b = \sqrt{24} = 2\sqrt{6}$

Note that side a is opposite θ and side b is adjacent to θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{6}}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{1} = 5$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{6}}{1} = 2\sqrt{6}$$

3. Apply the definitions of these three trigonometric functions.

$$\csc 45° = \frac{\text{length of hypotenuse}}{\text{length of side opposite } 45°}$$
$$= \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45° = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to } 45°}$$
$$= \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45° = \frac{\text{length of side adjacent to } 45°}{\text{length of side opposite } 45°}$$
$$= \frac{1}{1} = 1$$

4. $$\tan 60° = \frac{\text{length of side opposite } 60°}{\text{length of side adjacent to } 60°}$$
$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\tan 30° = \frac{\text{length of side opposite } 30°}{\text{length of side adjacent to } 30°}$$
$$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$$

5. **a.** $\sin 46^{\circ} = \cos(90^{\circ} - 46^{\circ}) = \cos 44^{\circ}$

b. $$\cot \frac{\pi}{12} = \tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$$
$$= \tan\left(\frac{6\pi}{12} - \frac{\pi}{12}\right)$$
$$= \tan \frac{5\pi}{12}$$

6. Because we have a known angle, an unknown opposite side, and a known adjacent side, we select the tangent function.

$$\tan 24^{\circ} = \frac{a}{750}$$
$$a = 750 \tan 24^{\circ}$$
$$a \approx 750(0.4452) \approx 333.9$$

The distance across the lake is approximately 333.9 yards.

7. $\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{14}{10}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	**Many Graphing Calculators**
$\boxed{\text{TAN}^{-1}}$ $\boxed{(}$ 14 $\boxed{\div}$ 10 $\boxed{)}$ $\boxed{\text{ENTER}}$	$\boxed{\text{TAN}}$ $\boxed{(}$ 14 ÷ 10 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display should show approximately 54. Thus, the angle of elevation of the sun is approximately 54°.

Concept and Vocabulary Check 4.3

1. $\sin\theta = \frac{a}{c}$; $\csc\theta = \frac{c}{a}$; $\cos\theta = \frac{b}{c}$; $\sec\theta = \frac{c}{b}$, $\tan\theta = \frac{a}{b}$; $\cot\theta = \frac{b}{a}$

2. opposite; adjacent to; hypotenuse

3. true

4. $\sin\theta$; $\tan\theta$; $\sec\theta$

Exercise Set 4.3

1. $c^2 = 9^2 + 12^2 = 225$

$c = \sqrt{225} = 15$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{15} = \frac{3}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{12} = \frac{3}{4}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{15}{9} = \frac{5}{3}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{15}{12} = \frac{5}{4}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{12}{9} = \frac{4}{3}$

2. $c^2 = 6^2 + 8^2 = 100$

$c = \sqrt{100} = 10$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{6}{10} = \dfrac{3}{5}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{8}{10} = \dfrac{4}{5}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{6}{8} = \dfrac{3}{4}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{10}{6} = \dfrac{5}{3}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{10}{8} = \dfrac{5}{4}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{8}{6} = \dfrac{4}{3}$

3. $a^2 + 21^2 = 29^2$

$a^2 = 841 - 441 = 400$

$a = \sqrt{400} = 20$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{20}{29}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{21}{29}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{20}{21}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{29}{20}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{29}{21}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{21}{20}$

4. $a^2 + 15^2 = 17^2$

$a^2 = 289 - 225 = 64$

$a = \sqrt{64} = 8$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{8}{17}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{15}{17}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{8}{15}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{17}{8}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{17}{15}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{15}{8}$

5. $10^2 + b^2 = 26^2$

$b^2 = 676 - 100 = 576$

$b = \sqrt{576} = 24$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{10}{26} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{24}{26} = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{10}{24} = \dfrac{5}{12}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{26}{10} = \dfrac{13}{5}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{26}{24} = \dfrac{13}{12}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{24}{10} = \dfrac{12}{5}$

6. $a^2+40^2=41^2$

$a^2=1681-1600=81$

$a=\sqrt{81}=9$

$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}=\frac{9}{41}$

$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}=\frac{40}{41}$

$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}=\frac{9}{40}$

$\csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}=\frac{41}{9}$

$\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}=\frac{41}{40}$

$\cot\theta=\frac{\text{adjacent}}{\text{opposite}}=\frac{40}{9}$

7. $21^2+b^2=35^2$

$b^2=1225-441=784$

$b=\sqrt{784}=28$

$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}=\frac{28}{35}=\frac{4}{5}$

$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}=\frac{21}{35}=\frac{3}{5}$

$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}=\frac{28}{21}=\frac{4}{3}$

$\csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}=\frac{35}{28}=\frac{5}{4}$

$\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}=\frac{35}{21}=\frac{5}{3}$

$\cot\theta=\frac{\text{adjacent}}{\text{opposite}}=\frac{21}{28}=\frac{3}{4}$

8. $a^2+24^2=25^2$

$a^2=625-576=49$

$a=\sqrt{49}=7$

$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}=\frac{24}{25}$

$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}=\frac{7}{25}$

$\tan\theta=\frac{\text{opposite}}{\text{adjacent}}=\frac{24}{7}$

$\csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}=\frac{25}{24}$

$\sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}=\frac{25}{7}$

$\cot\theta=\frac{\text{adjacent}}{\text{opposite}}=\frac{7}{24}$

9. $\cos 30°=\frac{\text{length of side adjacent to } 30°}{\text{length of hypotenuse}}$

$=\frac{\sqrt{3}}{2}$

10. $\tan 30°=\frac{\text{length of side opposite } 30°}{\text{length of side adjacent to } 30°}$

$=\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

11. $\sec 45°=\frac{\text{length of hypotenuse}}{\text{length of side adjacent to } 45°}$

$=\frac{\sqrt{2}}{1}=\sqrt{2}$

12. $\csc 45°=\frac{\text{length of hypotenuse}}{\text{length of side opposite } 45°}$

$=\frac{\sqrt{2}}{1}=\sqrt{2}$

13. $\tan\frac{\pi}{3}=\tan 60°$

$=\frac{\text{length of side opposite } 60°}{\text{length of side adjacent to } 60°}$

$=\frac{\sqrt{3}}{1}=\sqrt{3}$

14. $\cot\frac{\pi}{3} = \cot 60° = \frac{\text{length of side adjacent to } 60°}{\text{length of side opposite } 60°}$

$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

15. $\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = \sin 45° - \cos 45°$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

16. $\tan\frac{\pi}{4} + \csc\frac{\pi}{6} = \tan 45° + \csc 30°$

$= \frac{1}{1} + \frac{2}{1} = 1 + 2 = 3$

17. $\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \tan\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - 1$

$= \frac{\sqrt{6}}{4} - 1$

$= \frac{\sqrt{6}-4}{4}$

18. $\cos\frac{\pi}{3}\sec\frac{\pi}{3} - \cot\frac{\pi}{3} = 1 - \frac{\sqrt{3}}{3} = \frac{3-\sqrt{3}}{3}$

19. $2\tan\frac{\pi}{3} + \cos\frac{\pi}{4}\tan\frac{\pi}{6} = 2\left(\sqrt{3}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)$

$= 2\sqrt{3} + \frac{\sqrt{6}}{6}$

$= \frac{12\sqrt{3}+\sqrt{6}}{6}$

20. $6\tan\frac{\pi}{4} + \sin\frac{\pi}{3}\sec\frac{\pi}{6} = 6(1) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2\sqrt{3}}{3}\right)$

$= 6 + \frac{6}{6}$

$= 7$

21. $\sin 7° = \cos(90° - 7°) = \cos 83°$

22. $\sin 19° = \cos\left(90° - 19°\right) = \cos 71°$

23. $\csc 25° = \sec(90° - 25°) = \sec 65°$

24. $\csc 35° = \sec(90° - 35°) = \sec 55°$

25. $\tan\frac{\pi}{9} = \cot\left(\frac{\pi}{2} - \frac{\pi}{9}\right)$

$= \cot\left(\frac{9\pi}{18} - \frac{2\pi}{18}\right)$

$= \cot\frac{7\pi}{18}$

26. $\tan\frac{\pi}{7} = \cot\left(\frac{\pi}{2} - \frac{\pi}{7}\right) = \cot\left(\frac{7\pi}{14} - \frac{2\pi}{14}\right) = \cot\frac{5\pi}{14}$

27. $\cos\frac{2\pi}{5} = \sin\left(\frac{\pi}{2} - \frac{2\pi}{5}\right)$

$= \sin\left(\frac{5\pi}{10} - \frac{4\pi}{10}\right)$

$= \sin\frac{\pi}{10}$

28. $\cos\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) = \sin\left(\frac{4\pi}{8} - \frac{3\pi}{8}\right) = \sin\frac{\pi}{8}$

29. $\tan 37° = \frac{a}{250}$

$a = 250\tan 37°$

$a \approx 250(0.7536) \approx 188$ cm

30. $\tan 61° = \frac{a}{10}$

$a = 10\tan 61°$

$a \approx 10(1.8040) \approx 18$ cm

31. $\cos 34° = \frac{b}{220}$

$b = 220\cos 34°$

$b \approx 220(0.8290) \approx 182$ in.

32. $\sin 34° = \frac{a}{13}$

$a = 13\sin 34°$

$a \approx 13(0.5592) \approx 7$ m

33. $\sin 23° = \frac{16}{c}$

$c = \frac{16}{\sin 23°} \approx \frac{16}{0.3907} \approx 41$ m

34. $\tan 44° = \dfrac{23}{b}$

$b = \dfrac{23}{\tan 44°} \approx \dfrac{23}{0.9657} \approx 24 \text{ yd}$

35.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
0.2974 [SIN^{-1}]	[SIN^{-1}] 0.2974 [ENTER]	17

If $\sin\theta = 0.2974$, then $\theta \approx 17°$.

36.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
0.877 [COS^{-1}]	[COS^{-1}] 0.877 [ENTER]	29

If $\cos\theta = 0.877$, then $\theta \approx 29°$.

37.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
4.6252 [TAN^{-1}]	[TAN^{-1}] 4.6252 [ENTER]	78

If $\tan\theta = 4.6252$, then $\theta \approx 78°$.

38.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
26.0307 [TAN^{-1}]	[TAN^{-1}] 26.0307 [ENTER]	88

If $\tan\theta = 26.0307$, then $\theta \approx 88°$.

39.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
0.4112 [COS^{-1}]	[COS^{-1}] 0.4112 [ENTER]	1.147

If $\cos\theta = 0.4112$, then $\theta \approx 1.147$ radians.

40.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
0.9499 [SIN^{-1}]	[SIN^{-1}] 0.9499 [ENTER]	1.253

If $\sin\theta = 0.9499$, then $\theta = 1.253$ radians.

41.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
0.4169 [TAN^{-1}]	[TAN^{-1}] 0.4169 [ENTER]	0.395

If $\tan\theta = 0.4169$, then $\theta \approx 0.395$ radians.

42.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
0.5117 [TAN^{-1}]	[TAN^{-1}] 0.5117 [ENTER]	0.473

If $\tan\theta = 0.5117$, then $\theta = 0.473$

43. $\dfrac{\tan\frac{\pi}{3}}{2}-\dfrac{1}{\sec\frac{\pi}{6}}=\dfrac{\sqrt{3}}{2}-\dfrac{1}{\frac{1}{\cos\frac{\pi}{6}}}$

$=\dfrac{\sqrt{3}}{2}-\dfrac{1}{\frac{1}{\frac{\sqrt{3}}{2}}}$

$=\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{3}}{2}$

$=0$

44. $\dfrac{1}{\cot\frac{\pi}{4}}-\dfrac{2}{\csc\frac{\pi}{6}}=\dfrac{1}{\frac{1}{\tan\frac{\pi}{4}}}-\dfrac{2}{\frac{1}{\sin\frac{\pi}{6}}}$

$=\dfrac{1}{\frac{1}{1}}-\dfrac{2}{\frac{1}{1/2}}$

$=\dfrac{1}{1}-\dfrac{2}{2}$

$=1-1$

$=0$

45. $1+\sin^2 40°+\sin^2 50°$

$=1+\sin^2(90°-50°)+\sin^2 50°$

$=1+\cos^2 50°+\sin^2 50°$

$=1+1$

$=2$

46. $1-\tan^2 10°+\csc^2 80°$

$=1-\cot^2 80°+\csc^2 80°$

$=1+\csc^2 80°-\cot^2 80°$

$=1+1$

$=2$

47. $\csc 37°\sec 53°-\tan 53°\cot 37°$

$=\sec 53°\sec 53°-\tan 53°\tan 53°$

$=\sec^2 53°-\tan^2 53°$

$=1$

48. $\cos 12°\sin 78°+\cos 78°\sin 12°$

$=\sin 78°\sin 78°+\cos 78°\cos 78°$

$=\sin^2 78°+\cos^2 78°$

$=1$

49. $f(\theta)=2\cos\theta-\cos 2\theta$

$f\left(\dfrac{\pi}{6}\right)=2\cos\dfrac{\pi}{6}-\cos\left(2\cdot\dfrac{\pi}{6}\right)$

$=2\left(\dfrac{\sqrt{3}}{2}\right)-\cos\left(\dfrac{\pi}{3}\right)$

$=\dfrac{2\sqrt{3}}{2}-\dfrac{1}{2}$

$=\dfrac{2\sqrt{3}-1}{2}$

50. $f(\theta)=2\sin\theta-\sin\dfrac{\theta}{2}$

$f\left(\dfrac{\pi}{3}\right)=2\sin\dfrac{\pi}{3}-\sin\dfrac{\frac{\pi}{3}}{2}$

$=2\left(\dfrac{\sqrt{3}}{2}\right)-\sin\left(\dfrac{\pi}{6}\right)$

$=\dfrac{2\sqrt{3}}{2}-\dfrac{1}{2}$

$=\dfrac{2\sqrt{3}-1}{2}$

51. $\tan\left(\dfrac{\pi}{2}-\theta\right)=\cot\theta=\dfrac{1}{4}$

52. $\csc\left(\dfrac{\pi}{2}-\theta\right)=\sec\theta=\dfrac{1}{\cos\theta}=\dfrac{1}{\frac{1}{3}}=3$

53. $\tan 40°=\dfrac{a}{630}$

$a=630\tan 40°$

$a\approx 630(0.8391)\approx 529$

The distance across the lake is approximately 529 yards.

54. $\tan 40°=\dfrac{h}{35}$

$h=35\tan 40°$

$h\approx 35(0.8391)\approx 29$

The tree's height is approximately 29 feet.

55. $\tan\theta = \frac{125}{172}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
125 ÷ 172 = TAN^{-1}	TAN^{-1} (125 ÷ 172) ENTER

The display should show approximately 36. Thus, the angle of elevation of the sun is approximately 36°.

56. $\tan\frac{555}{1320}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
555 ÷ 1320 = TAN^{-1}	TAN^{-1} (555 ÷ 1320) ENTER

The display should show approximately 23. Thus, the angle of elevation is approximately 23°.

57. $\sin 10° = \frac{500}{c}$

$c = \frac{500}{\sin 10°} \approx \frac{500}{0.1736} \approx 2880$

The plane has flown approximately 2880 feet.

58. $\sin 5° = \frac{a}{5000}$

$a = 5000 \sin 5° \approx 5000(0.0872) = 436$

The driver's increase in altitude was approximately 436 feet.

59. $\cos\theta = \frac{60}{75}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
60 ÷ 75 = COS^{-1}	COS^{-1} (60 ÷ 75) ENTER

The display should show approximately 37. Thus, the angle between the wire and the pole is approximately 37°.

60. $\cos\theta = \frac{55}{80}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
55 ÷ 80 = COS^{-1}	COS^{-1} (55 ÷ 80) ENTER

The display should show approximately 47. Thus, the angle between the wire and the pole is approximately 47°.

61. – 67. Answers may vary.

68.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin\theta$	0.3894	0.2955	0.1987	0.0998	0.0099998	9.999998×10^{-4}	9.99999998×10^{-5}	1×10^{-5}
$\frac{\sin\theta}{\theta}$	0.9736	0.9851	0.9933	0.9983	0.99998	0.9999998	0.999999998	1

$\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0.

69.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\cos\theta$	0.92106	0.95534	0.98007	0.99500	0.99995	0.9999995	0.999999995	1
$\frac{\cos\theta-1}{\theta}$	–0.19735	–0.148878	–0.099667	–0.04996	–0.005	–0.0005	–0.00005	0

$\frac{\cos\theta-1}{\theta}$ approaches 0 as θ approaches 0.

70. does not make sense; Explanations will vary. Sample explanation: An increase in the size of a triangle does not affect the ratios of the sides.

71. does not make sense; Explanations will vary. Sample explanation: This value is irrational. Irrational numbers are rounded on calculators.

72. does not make sense; Explanations will vary. Sample explanation: The sine and cosine functions are not reciprocal functions of each other.

73. makes sense

74. false; Changes to make the statement true will vary. A sample change is: $\frac{\tan 45°}{\tan 15°} \neq \tan\left(\frac{45°}{15°}\right)$

75. true

76. false; Changes to make the statement true will vary. A sample change is: $\sin 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \neq 1$

77. true

78. In a right triangle, the hypotenuse is greater than either other side. Therefore both $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$ must be less than 1 for an acute angle in a right triangle.

79. Use a calculator in degree mode to generate the following table. Then use the table to describe what happens to the tangent of an acute angle as the angle gets close to 90°.

θ	60	70	80	89	89.9	89.99	89.999	89.9999
tanθ	1.7321	2.7475	5.6713	57	573	5730	57,296	572,958

As θ approaches 90°, tanθ increases without bound. At 90°, tanθ is undefined.

80. a. Let a = distance of the ship from the lighthouse.

$$\tan 35° = \frac{250}{a}$$

$$a = \frac{250}{\tan 35°} \approx \frac{250}{0.7002} \approx 357$$

The ship is approximately 357 feet from the lighthouse.

b. Let b = the plane's height above the lighthouse.

$$\tan 22^\circ = \frac{b}{357}$$
$$b = 357\tan 22^\circ \approx 357(0.4040) \approx 144$$
$$144 + 250 = 394$$

The plane is approximately 394 feet above the water.

81. $|2x-3| = 7$

$$2x-3=7 \quad \text{or} \quad 2x-3=-7$$
$$2x=10 \qquad 2x=-4$$
$$x=5 \qquad x=-2$$

The solution set is {–2, 5}.

82. Write the equation in slope-intercept form:

$$x+10y-13=0$$
$$10y=-x+13$$
$$y=-\frac{1}{10}x+\frac{13}{10}$$

The slope of this line is $-\frac{1}{10}$ thus the slope of any line perpendicular to this line is 10.

Use $m=10$ and the point (1, –1) to write the equation.

$$y-y_1=m(x-x_1)$$
$$y-(-1)=10(x-1)$$
$$y+1=10(x-1)$$
$$y+1=10x-10$$
$$y=10x-11$$

83. $\log_4\left(x^2-9\right)-\log_4(x+3)=\log_4 64$

$$\log_4\left(x^2-9\right)-\log_4(x+3)-\log_4 64=0$$
$$\log_4\left(\frac{x^2-9}{x+3}\right)=\log_4 64$$
$$\frac{x^2-9}{x+3}=64$$
$$x^2-9=64(x+3)$$
$$x^2-9=64x+192$$
$$x^2-64x-201=0$$
$$(x-67)(x+3)=0$$

$$x-67=0 \quad \text{or} \quad x+3=0$$
$$x=67 \qquad x=-3$$

–3 is rejected. The solution set is {67}.

84. **a.** $\frac{y}{r}$

b. First find r: $r=\sqrt{x^2+y^2}$

$r=\sqrt{(-3)^2+4^2}$

$r=5$

$\frac{y}{r}=\frac{4}{5}$, which is positive.

85. **a.** $\frac{x}{r}$

b. First find r: $r=\sqrt{x^2+y^2}$

$r=\sqrt{(-3)^2+5^2}$

$r=\sqrt{34}$

$\frac{x}{r}=\frac{-3}{\sqrt{34}}=\frac{-3}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}}=\frac{-3\sqrt{34}}{34}$,

which is negative.

86. **a.** $\theta'=360^{\circ}-345^{\circ}=15^{\circ}$

b. $\theta'=\pi-\frac{5\pi}{6}=\frac{6\pi}{6}-\frac{5\pi}{6}=\frac{\pi}{6}$

Section 4.4

Checkpoint Exercises

1. $r=\sqrt{x^2+y^2}$

$r=\sqrt{1^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta=\frac{y}{r}=\frac{-3}{\sqrt{10}}=-\frac{3\sqrt{10}}{10}$

$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$

$\tan\theta=\frac{y}{x}=\frac{-3}{1}=-3$

$\csc\theta=\frac{r}{y}=\frac{\sqrt{10}}{-3}=-\frac{\sqrt{10}}{3}$

$\sec\theta=\frac{r}{x}=\frac{\sqrt{10}}{1}=\sqrt{10}$

$\cot\theta=\frac{x}{y}=\frac{1}{-3}=-\frac{1}{3}$

2. **a.** $\theta=0°=0$ radians

The terminal side of the angle is on the positive *x*-axis. Select the point

$P=(1,0)$: $x=1, y=0, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 0°=\cos 0=\frac{x}{r}=\frac{1}{1}=1$

$\csc 0°=\csc 0=\frac{r}{y}=\frac{1}{0}$, undefined

b. $\theta=90°=\frac{\pi}{2}$ radians

The terminal side of the angle is on the positive *y*-axis. Select the point

$P=(0,1)$: $x=0, y=1, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 90°=\cos\frac{\pi}{2}=\frac{x}{r}=\frac{0}{1}=0$

$\csc 90°=\csc\frac{\pi}{2}=\frac{r}{y}=\frac{1}{1}=1$

c. $\theta=180°=\pi$ radians

The terminal side of the angle is on the negative *x*-axis. Select the point

$P=(-1,0)$: $x=-1, y=0, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 180°=\cos\pi=\frac{x}{r}=\frac{-1}{1}=-1$

$\csc 180°=\csc\pi=\frac{r}{y}=\frac{1}{0}$, undefined

d. $\theta=270°=\frac{3\pi}{2}$ radians

The terminal side of the angle is on the negative *y*-axis. Select the point

$P=(0,-1)$: $x=0, y=-1, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 270°=\cos\frac{3\pi}{2}=\frac{x}{r}=\frac{0}{1}=0$

$\csc 270°=\csc\frac{3\pi}{2}=\frac{r}{y}=\frac{1}{-1}=-1$

3. Because $\sin\theta < 0,\ \theta$ cannot lie in quadrant I; all the functions are positive in quadrant I. Furthermore, θ cannot lie in quadrant II; $\sin\theta$ is positive in quadrant II. Thus, with $\sin\theta < 0,\ \theta$ lies in quadrant III or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which cosine is negative and the sine is negative, we conclude that θ lies in quadrant III.

4. Because the tangent is negative and the cosine is negative, θ lies in quadrant II. In quadrant II, *x* is negative and *y* is positive. Thus,

$$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3}$$

$x = -3,\ y = 1$

Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Now that we know *x, y,* and *r,* we can find $\sin\theta$ and $\sec\theta$.

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

5. a. Because 210° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.

b. Because $\frac{7\pi}{4}$ lies between $\frac{3\pi}{2} = \frac{6\pi}{4}$ and $2\pi = \frac{8\pi}{4}$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

c. Because –240° lies between –180° and –270°, it is in quadrant II. The reference angle is $\theta = 240 - 180 = 60°$.

d. Because 3.6 lies between $\pi \approx 3.14$ and $\frac{3\pi}{2} \approx 4.71$, it is in quadrant III. The reference angle is $\theta' = 3.6 - \pi \approx 0.46$.

6. a. $665° - 360° = 305°$
This angle is in quadrant IV, thus the reference angle is $\theta' = 360° - 305° = 55°$.

b. $\frac{15\pi}{4} - 2\pi = \frac{15\pi}{4} - \frac{8\pi}{4} = \frac{7\pi}{4}$
This angle is in quadrant IV, thus the reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

c. $-\frac{11\pi}{3} + 2\cdot 2\pi = -\frac{11\pi}{3} + \frac{12\pi}{3} = \frac{\pi}{3}$
This angle is in quadrant I, thus the reference angle is $\theta' = \frac{\pi}{3}$.

7. a. 300° lies in quadrant IV. The reference angle is $\theta' = 360° - 300° = 60°$.

$$\sin 60° = \frac{\sqrt{3}}{2}$$

Because the sine is negative in quadrant IV,

$$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}.$$

b. $\frac{5\pi}{4}$ lies in quadrant III. The reference angle is

$$\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}.$$

$$\tan\frac{\pi}{4} = 1$$

Because the tangent is positive in quadrant III,

$$\tan\frac{5\pi}{4} = +\tan\frac{\pi}{4} = 1.$$

c. $-\frac{\pi}{6}$ lies in quadrant IV. The reference angle is

$$\theta' = \frac{\pi}{6}.$$

$$\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$$

Because the secant is positive in quadrant IV,

$$\sec\left(-\frac{\pi}{6}\right) = +\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}.$$

8. **a.** $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ lies in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

The function value for the reference angle is $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

Because the cosine is negative in quadrant II,
$\cos\frac{17\pi}{6} = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

b. $\frac{-22\pi}{3} + 8\pi = \frac{-22\pi}{3} + \frac{24\pi}{3} = \frac{2\pi}{3}$ lies in quadrant II. The reference angle is
$\theta' = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

The function value for the reference angle is
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Because the sine is positive in quadrant II,
$\sin\frac{-22\pi}{3} = \sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Concept and Vocabulary Check 4.4

1. $\sin\theta = \frac{y}{r}$; $\csc\theta = \frac{r}{y}$; $\cos\theta = \frac{x}{r}$; $\sec\theta = \frac{r}{x}$,
$\tan\theta = \frac{y}{x}$; $\cot\theta = \frac{x}{y}$

2. $\tan\theta$; $\sec\theta$; $\cot\theta$; $\csc\theta$; $\tan\theta$; $\cot\theta$

3. $\sin\theta$; $\csc\theta$;

4. $\tan\theta$; $\cot\theta$;

5. $\cos\theta$; $\sec\theta$

6. terminal; x

7. **(a)** $180° - \theta$; **(b)** $\theta - 180°$; **(c)** $360° - \theta$

Exercise Set 4.4

1. We need values for *x, y,* and *r.* Because $P = (-4, 3)$ is a point on the terminal side of θ, $x = -4$ and $y = 3$. Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ No w that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{3}{5}$

$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$

$\tan\theta = \frac{y}{x} = \frac{3}{-4} = -\frac{3}{4}$

$\csc\theta = \frac{r}{y} = \frac{5}{3}$

$\sec\theta = \frac{r}{x} = \frac{5}{-4} = -\frac{5}{4}$

$\cot\theta = \frac{x}{y} = \frac{-4}{3} = -\frac{4}{3}$

2. We need values for *x, y,* and *r,* Because $P = (-12, 5)$ is a point on the terminal side of θ, $x = -12$ and $y = 5$. Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + 5^2} = \sqrt{144+25}$
$= \sqrt{169} = 13$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{5}{13}$

$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$

$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$

$\csc\theta = \frac{r}{y} = \frac{13}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$

$\cot\theta = \frac{x}{y} = \frac{-12}{5} - \frac{12}{5}$

3. We need values for *x, y,* and *r*. Because $P = (2, 3)$ is a point on the terminal side of θ, $x = 2$ and $y = 3$.
Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$
Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} = \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$
$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{3}{2}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{2}$$
$$\cot\theta = \frac{x}{y} = \frac{2}{3}$$

4. We need values for *x, y,* and *r,* Because $P = (3, 7)$ is a point on the terminal side of θ, $x = 3$ and $y = 7$. Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$
Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{58}} = \frac{7}{\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$$
$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{58}} = \frac{3}{\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$$
$$\tan\theta = \frac{y}{x} = \frac{7}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{58}}{7}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{3}{7}$$

5. We need values for *x, y,* and *r*. Because $P = (3, -3)$ is a point on the terminal side of θ, $x = 3$ and $y = -3$.
Furthermore, $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9}$
$= \sqrt{18} = 3\sqrt{2}$
Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = -\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$
$$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{3} = -1$$
$$\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$$
$$\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{3} = \sqrt{2}$$
$$\cot\theta = \frac{x}{y} = \frac{3}{-3} = -1$$

6. We need values for *x, y,* and *r,* Because $P = (5, -5)$ is a point on the terminal side of θ, $x = 5$ and $y = -5$.
Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{5 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$
$= 5\sqrt{2}$
Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{5\sqrt{2}} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$
$$\cos\theta = \frac{x}{r} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$
$$\tan\theta = \frac{y}{x} = \frac{-5}{5} = -1$$
$$\csc\theta = \frac{r}{y} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$$
$$\sec\theta = \frac{r}{x} = \frac{5\sqrt{2}}{5} = \sqrt{2}$$
$$\cot\theta = \frac{x}{y} = \frac{5}{-5} = -1$$

7. We need values for *x, y,* and *r.* Because $P = (-2, -5)$ is a point on the terminal side of θ, $x = -2$ and $y = -5$. Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$ No w that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} = \frac{-5}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$
$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = \frac{-2}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$
$$\tan\theta = \frac{y}{x} = \frac{-5}{-2} = \frac{5}{2}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$
$$\cot\theta = \frac{x}{y} = \frac{-2}{-5} = \frac{2}{5}$$

8. We need values for *x, y,* and *r,* Because $P = (-1, -3)$ is a point on the terminal side of θ, $x = -1$ and $y = -3$. Furthermore,
$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$
Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{10}} = \frac{-3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$
$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{10}} = \frac{-1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{-1} = 3$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-1} = -\sqrt{10}$$
$$\cot\theta = \frac{x}{y} = \frac{-1}{-3} = \frac{1}{3}$$

9. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1,\ y = 0,\ r = 1$ Apply the definition of the cosine function.
$$\cos\pi = \frac{x}{r} = \frac{-1}{1} = -1$$

10. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1, y = 0,\ r = 1$
Apply the definition of the tangent function.
$$\tan\pi = \frac{y}{x} = \frac{0}{-1} = 0$$

11. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1,\ y = 0,\ r = 1$ Apply the definition of the secant function.
$$\sec\pi = \frac{r}{x} = \frac{1}{-1} = -1$$

12. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1, y = 0,\ r = 1$
Apply the definition of the cosecant function.
$$\csc\pi = \frac{r}{y} = \frac{1}{0}, \text{ undefined}$$

13. $\theta = \frac{3\pi}{2}$ radians
The terminal side of the angle is on the negative *y*-axis. Select the point $P = (0, -1)$: $x = 0,\ y = -1,\ r = 1$ Apply the definition of the tangent function. $\tan\frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0}$, undefined

14. $\theta = \frac{3\pi}{2}$ radians
The terminal side of the angle is on the negative *y*-axis. Select the point $P = (0, -1)$: $x = 0, y = -1,\ r = 1$
Apply the definition of the cosine function.
$$\cos\frac{3\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$$

15. $\theta = \frac{\pi}{2}$ radians
The terminal side of the angle is on the positive *y*-axis. Select the point $P = (0, 1)$: $x = 0,\ y = 1,\ r = 1$ Apply the definition of the cotangent function. $\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

16. $\theta = \frac{\pi}{2}$ radians
The terminal side of the angle is on the positive *y*-axis. Select the point $P = (0, 1)$: $x = 0, y = 1,\ r = 1$
Apply the definition of the tangent function.
$$\tan\frac{\pi}{2} = \frac{y}{x} = \frac{1}{0}, \text{ undefined}$$

17. Because $\sin\theta > 0$, θ cannot lie in quadrant III or quadrant IV; the sine function is negative in those quadrants. Thus, with $\sin\theta > 0$, θ lies in quadrant I or quadrant II. We are also given that $\cos\theta > 0$. Because quadrant I is the only quadrant in which the cosine is positive and sine is positive, we conclude that θ lies in quadrant I.

18. Because $\sin\theta < 0$, θ cannot lie in quadrant I or quadrant II; the sine function is positive in those two quadrants. Thus, with $\sin\theta < 0$, θ lies in quadrant III or quadrant IV. We are also given that $\cos\theta > 0$. Because quadrant IV is the only quadrant in which the cosine is positive and the sine is negative, we conclude that θ lies in quadrant IV.

19. Because $\sin\theta < 0$, θ cannot lie in quadrant I or quadrant II; the sine function is positive in those two quadrants. Thus, with $\sin\theta < 0$, θ lies in quadrant III or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which the cosine is positive and the sine is negative, we conclude that θ lies in quadrant III.

20. Because $\tan\theta < 0$, θ cannot lie in quadrant I or quadrant III; the tangent function is positive in those two quadrants. Thus, with $\tan\theta < 0$, θ lies in quadrant II or quadrant IV. We are also given that $\sin\theta < 0$. Because quadrant IV is the only quadrant in which the sine is negative and the tangent is negative, we conclude that θ lies in quadrant IV.

21. Because $\tan\theta < 0$, θ cannot lie in quadrant I or quadrant III; the tangent function is positive in those quadrants. Thus, with $\tan\theta < 0$, θ lies in quadrant II or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant II is the only quadrant in which the cosine is negative and the tangent is negative, we conclude that θ lies in quadrant II.

22. Because $\cot\theta > 0$, θ cannot lie in quadrant II or quadrant IV; the cotangent function is negative in those two quadrants. Thus, with $\cot\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\sec\theta < 0$. Because quadrant III is the only quadrant in which the secant is negative and the cotangent is positive, we conclude that θ lies in quadrant III.

23. In quadrant III x is negative and y is negative. Thus, $\cos\theta = -\frac{3}{5} = \frac{x}{r} = \frac{-3}{5}$, $x = -3$, $r = 5$. Furthermore,

$$\begin{aligned} r^2 &= x^2 + y^2 \\ 5^2 &= (-3)^2 + y^2 \\ y^2 &= 25 - 9 = 16 \\ y &= -\sqrt{16} = -4 \end{aligned}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\begin{aligned} \sin\theta &= \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5} \\ \tan\theta &= \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3} \\ \csc\theta &= \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4} \\ \sec\theta &= \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3} \\ \cot\theta &= \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4} \end{aligned}$$

24. In quadrant III, x is negative and y is negative. Thus, $\sin\theta = -\frac{12}{13} = \frac{y}{r} = \frac{-12}{13}$, $y = -12$, $r = 13$.

Furthermore,

$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-12)^2 &= 13^2 \\ x^2 &= 169 - 144 = 25 \\ x &= -\sqrt{25} = -5 \end{aligned}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\begin{aligned} \cos\theta &= \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13} \\ \tan\theta &= \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5} \\ \csc\theta &= \frac{r}{y} = \frac{13}{-12} = -\frac{13}{12} \\ \sec\theta &= \frac{r}{x} = \frac{13}{-5} = -\frac{13}{5} \\ \cot\theta &= \frac{x}{y} = \frac{-5}{-12} = \frac{5}{12} \end{aligned}$$

25. In quadrant II x is negative and y is positive. Thus,

$\sin\theta = \frac{5}{13} = \frac{y}{r}$, $y = 5$, $r = 13$. Furthermore,

$$x^2 + y^2 = r^2$$
$$x^2 + 5^2 = 13^2$$
$$x^2 = 169 - 25 = 144$$
$$x = -\sqrt{144} = -12$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$
$$\csc\theta = \frac{r}{y} = \frac{13}{5}$$
$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$
$$\cot\theta = \frac{x}{y} = \frac{-12}{5} = -\frac{12}{5}$$

26. In quadrant IV, x is positive and y is negative. Thus,

$\cos\theta = \frac{4}{5} = \frac{x}{r}$, $x = 4$, $r = 5$.

Furthermore, $x^2 + y^2 = r^2$

$$4^2 + y^2 = 5^2$$
$$y^2 = 25 - 16 = 9$$
$$y = -\sqrt{9} = -3$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{4} = -\frac{3}{4}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{4}$$
$$\cot\theta = \frac{x}{y} = \frac{4}{-3} = -\frac{4}{3}$$

27. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV x is positive and y is negative. Thus,

$\cos\theta = \frac{8}{17} = \frac{x}{r}$, $x = 8$,

$r = 17$. Furthermore

$$x^2 + y^2 = r^2$$
$$8^2 + y^2 = 17^2$$
$$y^2 = 289 - 64 = 225$$
$$y = -\sqrt{225} = -15$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$$
$$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$$
$$\csc\theta = \frac{r}{y} = \frac{17}{-15} = -\frac{17}{15}$$
$$\sec\theta = \frac{r}{x} = \frac{17}{8}$$
$$\cot\theta = \frac{x}{y} = \frac{8}{-15} = -\frac{8}{15}$$

28. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV, x is positive and y is negative. Thus,

$\cos\theta = \frac{1}{3} = \frac{x}{r}$, $x = 1$, $r = 3$. Furthermore,

$$x^2 + y^2 = r^2$$
$$1^2 + y^2 = 3^2$$
$$y^2 = 9 - 1 = 8$$
$$y = -\sqrt{8} = -2\sqrt{2}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$$
$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{1} = -2\sqrt{2}$$
$$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{3}{1} = 3$$
$$\cot\theta = \frac{x}{y} = \frac{1}{-2\sqrt{2}} = \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

29. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{2}{3} = \frac{y}{x} = \frac{2}{-3}$, $x = -3$, $y = 2$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} = \frac{-3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{2} = -\frac{3}{2}$$

30. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3}$, $y = 1$, $x = -3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{1} = -3$$

31. Because the tangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus, $\tan\theta = \frac{4}{3} = \frac{y}{x} = \frac{-4}{-3}$,

$x = -3$, $y = -4$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16}$
$= \sqrt{25} = 5$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

32. Because the tangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\tan\theta = \frac{5}{12} = \frac{y}{x} = \frac{-5}{-12}$, $x = -12$, $y = -5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144+25}$
$= \sqrt{169} = 13$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\csc\theta = \frac{r}{y} = \frac{13}{-5} = -\frac{13}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$

$$\cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$$

33. Because the secant is negative and the tangent is positive, θ lies in quadrant III. In quadrant III, *x* is negative and *y* is negative. Thus,

$\sec\theta = -3 = \frac{r}{x} = \frac{3}{-1},\ x = -1,\ r = 3$. Furthermore,

$$\begin{aligned} x^2 + y^2 &= r^2 \\ (-1)^2 + y^2 &= 3^2 \\ y^2 &= 9 - 1 = 8 \\ y &= -\sqrt{8} = -2\sqrt{2} \end{aligned}$$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{3} = -\frac{1}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{-1} = 2\sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{-2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

34. Because the cosecant is negative and the tangent is positive, θ lies in quadrant III. In quadrant III, *x* is negative and *y* is negative. Thus,

$\csc\theta = -4 = \frac{r}{y} = \frac{4}{-1},\ y = -1,\ r = 4$. Furthermore,

$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-1)^2 &= 4^2 \\ x^2 &= 16 - 1 = 15 \\ x &= -\sqrt{15} \end{aligned}$$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-1}{4} = -\frac{1}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{15}}{4} = -\frac{\sqrt{15}}{4}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{15}} = \frac{1}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

$$\sec\theta = \frac{r}{x} = \frac{4}{-\sqrt{15}} = -\frac{4}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{15}}{-1} = \sqrt{15}$$

35. Because 160° lies between 90° and 180°, it is in quadrant II. The reference angle is $\theta' = 180° - 160° = 20°$.

36. Because 170° lies between 90° and 180°, it is in quadrant II. The reference angle is $\theta' = 180° - 170° = 10°$.

37. Because 205° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 205° - 180° = 25°$.

38. Because 210° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.

39. Because 355° lies between 270° and 360°, it is in quadrant IV. The reference angle is $\theta' = 360° - 355° = 5°$.

40. Because 351° lies between 270° and 360°, it is in quadrant IV. The reference angle is $\theta' = 360° - 351° = 9°$.

41. Because $\frac{7\pi}{4}$ lies between $\frac{3\pi}{2} = \frac{6\pi}{4}$ and $2\pi = \frac{8\pi}{4}$, it is in quadrant IV. The reference angle is

$$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}.$$

42. Because $\frac{5\pi}{4}$ lies between $\pi = \frac{4\pi}{4}$ and $\frac{3\pi}{2} = \frac{6\pi}{4}$, it is in quadrant III. The reference angle is

$$\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}.$$

43. Because $\frac{5\pi}{6}$ lies between $\frac{\pi}{2} = \frac{3\pi}{6}$ and $\pi = \frac{6\pi}{6}$, it is in quadrant II. The reference angle is

$$\theta' = \pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}.$$

44. Because $\frac{5\pi}{7} = \frac{10\pi}{14}$ lies between $\frac{\pi}{2} = \frac{7\pi}{14}$ and $\pi = \frac{14\pi}{14}$, it is in quadrant II. The reference angle is

$$\theta' = \pi - \frac{5\pi}{7} = \frac{7\pi}{7} - \frac{5\pi}{7} = \frac{2\pi}{7}.$$

45. $-150° + 360° = 210°$
Because the angle is in quadrant III, the reference angle is $\theta' = 210° - 180° = 30°$.

46. $-250° + 360° = 110°$
Because the angle is in quadrant II, the reference angle is $\theta' = 180° - 110° = 70°$.

47. $-335° + 360° = 25°$
Because the angle is in quadrant I, the reference angle is $\theta' = 25°$.

48. $-359° + 360° = 1°$
Because the angle is in quadrant I, the reference angle is $\theta' = 1°$.

49. Because 4.7 lies between $\pi \approx 3.14$ and $\frac{3\pi}{2} \approx 4.71$, it is in quadrant III. The reference angle is $\theta' = 4.7 - \pi \approx 1.56$.

50. Because 5.5 lies between $\frac{3\pi}{2} \approx 4.71$ and $2\pi \approx 6.28$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - 5.5 \approx 0.78$.

51. $565° - 360° = 205°$
Because the angle is in quadrant III, the reference angle is $\theta' = 205° - 180° = 25°$.

52. $553° - 360° = 193°$
Because the angle is in quadrant III, the reference angle is $\theta' = 193° - 180° = 13°$.

53. $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$
Because the angle is in quadrant II, the reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

54. $\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$
Because the angle is in quadrant II, the reference angle is $\theta' = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

55. $\frac{23\pi}{4} - 4\pi = \frac{23\pi}{4} - \frac{16\pi}{4} = \frac{7\pi}{4}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$.

56. $\frac{17\pi}{3} - 4\pi = \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

57. $-\frac{11\pi}{4} + 4\pi = -\frac{11\pi}{4} + \frac{16\pi}{4} = \frac{5\pi}{4}$
Because the angle is in quadrant III, the reference angle is $\theta' = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

58. $-\frac{17\pi}{6} + 4\pi = -\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$
Because the angle is in quadrant III, the reference angle is $\theta' = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$.

59. $-\frac{25\pi}{6} + 6\pi = -\frac{25\pi}{6} + \frac{36\pi}{6} = \frac{11\pi}{6}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$.

60. $-\frac{13\pi}{3} + 6\pi = -\frac{13\pi}{3} + \frac{18\pi}{3} = \frac{5\pi}{3}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

61. 225° lies in quadrant III. The reference angle is $\theta' = 225° - 180° = 45°$.
$\cos 45° = \frac{\sqrt{2}}{2}$
Because the cosine is negative in quadrant III, $\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$.

62. 300° lies in quadrant IV. The reference angle is $\theta' = 360° - 300° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
Because the sine is negative in quadrant IV, $\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$.

63. 210° lies in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.
$\tan 30° = \frac{\sqrt{3}}{3}$
Because the tangent is positive in quadrant III, $\tan 210° = \tan 30° = \frac{\sqrt{3}}{3}$.

64. 240° lies in quadrant III. The reference angle is
$\theta' = 240° - 180° = 60°$.
$\sec 60° = 2$
Because the secant is negative in quadrant III,
$\sec 240° = -\sec 60° - 2$.

65. 420° lies in quadrant I. The reference angle is
$\theta' = 420° - 360° = 60°$.
$\tan 60° = \sqrt{3}$
Because the tangent is positive in quadrant I,
$\tan 420° = \tan 60° = \sqrt{3}$.

66. 405° lies in quadrant I. The reference angle is
$\theta' = 405° - 360° = 45°$.
$\tan 45° = 1$
Because the tangent is positive in quadrant I,
$\tan 405° = \tan 45° = 1$.

67. $\frac{2\pi}{3}$ lies in quadrant II. The reference angle is
$\theta' = \pi - \frac{2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
Because the sine is positive in quadrant II,
$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

68. $\frac{3\pi}{4}$ lies in quadrant II. The reference angle is
$\theta' = \pi - \frac{3\pi}{4} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$.
$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
Because the cosine is negative in quadrant II,
$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

69. $\frac{7\pi}{6}$ lies in quadrant III. The reference angle is
$\theta' = \frac{7\pi}{6} - \pi = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$.
$\csc\frac{\pi}{6} = 2$
Because the cosecant is negative in quadrant III,
$\csc\frac{7\pi}{6} = -\csc\frac{\pi}{6} = -2$.

70. $\frac{7\pi}{4}$ lies in quadrant IV. The reference angle is
$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.
$\cot\frac{\pi}{4} = 1$
Because the cotangent is negative in quadrant IV,
$\cot\frac{7\pi}{4} = -\cot\frac{\pi}{4} = -1$.

71. $\frac{9\pi}{4}$ lies in quadrant I. The reference angle is
$\theta' = \frac{9\pi}{4} - 2\pi = \frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$.
$\tan\frac{\pi}{4} = 1$
Because the tangent is positive in quadrant I,
$\tan\frac{9\pi}{4} = \tan\frac{\pi}{4} = 1$

72. $\frac{9\pi}{2}$ lies on the positive *y*-axis. The reference angle is
$\theta' = \frac{9\pi}{2} - 4\pi = \frac{9\pi}{2} - \frac{8\pi}{2} = \frac{\pi}{2}$.
Because $\tan\frac{\pi}{2}$ is undefined, $\tan\frac{9\pi}{2}$ is also undefined.

73. –240° lies in quadrant II. The reference angle is
$\theta' = 240° - 180° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
Because the sine is positive in quadrant II,
$\sin(-240°) = \sin 60° = \frac{\sqrt{3}}{2}$.

74. –225° lies in quadrant II. The reference angle is
$\theta' = 225° - 180° = 45°$.
$\sin 45° = \frac{\sqrt{2}}{2}$
Because the sine is positive in quadrant II,
$\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$.

75. $-\frac{\pi}{4}$ lies in quadrant IV. The reference angle is $\theta' = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$

Because the tangent is negative in quadrant IV,

$\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

76. $-\frac{\pi}{6}$ lies in quadrant IV. The reference angle is

$\theta = \frac{\pi}{6}$. $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

Because the tangent is negative in quadrant IV,

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

77. $\sec 495° = \sec 135° = -\sqrt{2}$

78. $\sec 510° = \sec 150° = -\frac{2\sqrt{3}}{3}$

79. $\cot\frac{19\pi}{6} = \cot\frac{7\pi}{6} = \sqrt{3}$

80. $\cot\frac{13\pi}{3} = \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

81. $\cos\frac{23\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

82. $\cos\frac{35\pi}{6} = \cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

83. $\tan\left(-\frac{17\pi}{6}\right) = \tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

84. $\tan\left(-\frac{11\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

85. $\sin\left(-\frac{17\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

86. $\sin\left(-\frac{35\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

87. $\sin\frac{\pi}{3}\cos\pi - \cos\frac{\pi}{3}\sin\frac{3\pi}{2}$

$= \left(\frac{\sqrt{3}}{2}\right)(-1) - \left(\frac{1}{2}\right)(-1)$

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{1-\sqrt{3}}{2}$

88. $\sin\frac{\pi}{4}\cos 0 - \sin\frac{\pi}{6}\cos\pi$

$= \left(\frac{\sqrt{2}}{2}\right)(1) - \left(\frac{1}{2}\right)(-1)$

$= \frac{\sqrt{2}}{2} + \frac{1}{2}$

$= \frac{\sqrt{2}+1}{2}$

89. $\sin\frac{11\pi}{4}\cos\frac{5\pi}{6} + \cos\frac{11\pi}{4}\sin\frac{5\pi}{6}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= -\frac{\sqrt{6}+\sqrt{2}}{4}$

90. $\sin\frac{17\pi}{3}\cos\frac{5\pi}{4} + \cos\frac{17\pi}{3}\sin\frac{5\pi}{4}$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

91. $\sin\frac{3\pi}{2}\tan\left(-\frac{15\pi}{4}\right) - \cos\left(-\frac{5\pi}{3}\right)$

$= (-1)(1) - \left(\frac{1}{2}\right)$

$= -1 - \frac{1}{2}$

$= -\frac{2}{2} - \frac{1}{2}$

$= -\frac{3}{2}$

92. $\sin\frac{3\pi}{2}\tan\left(-\frac{8\pi}{3}\right)+\cos\left(-\frac{5\pi}{6}\right)$

$$=(-1)\left(\sqrt{3}\right)+\left(-\frac{\sqrt{3}}{2}\right)$$
$$=-\sqrt{3}-\frac{\sqrt{3}}{2}$$
$$=-\frac{2\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$$
$$=-\frac{3\sqrt{3}}{2}$$

93. $f\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+f\left(\frac{4\pi}{3}\right)+f\left(\frac{\pi}{6}\right)$

$$=\sin\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+\sin\frac{4\pi}{3}+\sin\frac{\pi}{6}$$
$$=\sin\frac{3\pi}{2}+\sin\frac{4\pi}{3}+\sin\frac{\pi}{6}$$
$$=(-1)+\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)$$
$$=-\frac{\sqrt{3}+1}{2}$$

94. $g\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)+g\left(\frac{5\pi}{6}\right)+g\left(\frac{\pi}{6}\right)$

$$=\cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)+\cos\frac{5\pi}{6}+\cos\frac{\pi}{6}$$
$$=\cos\pi+\cos\frac{5\pi}{6}+\cos\frac{\pi}{6}$$
$$=(-1)+\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)$$
$$=-1$$

95. $(h\circ g)\left(\frac{17\pi}{3}\right)=h\left(g\left(\frac{17\pi}{3}\right)\right)$

$$=2\left(\cos\left(\frac{17\pi}{3}\right)\right)$$
$$=2\left(\frac{1}{2}\right)$$
$$=1$$

96. $(h\circ f)\left(\frac{11\pi}{4}\right)=h\left(f\left(\frac{11\pi}{4}\right)\right)$

$$=2\left(\sin\left(\frac{11\pi}{4}\right)\right)$$
$$=2\left(\frac{\sqrt{2}}{2}\right)$$
$$=\sqrt{2}$$

97. The average rate of change is the slope of the line through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$

$$m=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$
$$=\frac{\sin\left(\frac{3\pi}{2}\right)-\sin\left(\frac{5\pi}{4}\right)}{\frac{3\pi}{2}-\frac{5\pi}{4}}$$
$$=\frac{-1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{4}}$$
$$=\frac{-1+\frac{\sqrt{2}}{2}}{\frac{\pi}{4}}$$
$$=\frac{4\left(-1+\frac{\sqrt{2}}{2}\right)}{4\left(\frac{\pi}{4}\right)}$$
$$=\frac{2\sqrt{2}-4}{\pi}$$

98. The average rate of change is the slope of the line through the points $(x_1, g(x_1))$ and $(x_2, g(x_2))$

$$m=\frac{g(x_2)-g(x_1)}{x_2-x_1}$$
$$=\frac{\cos(\pi)-\cos\left(\frac{3\pi}{4}\right)}{\pi-\frac{3\pi}{4}}$$
$$=\frac{-1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{4}}$$
$$=\frac{4\left(-1+\frac{\sqrt{2}}{2}\right)}{4\left(\frac{\pi}{4}\right)}$$
$$=\frac{2\sqrt{2}-4}{\pi}$$

99. $\sin\theta = \frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants I or II.

QI: $\theta = \frac{\pi}{4}$

QII: $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

100. $\cos\theta = \frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants I or IV.

QI: $\theta = \frac{\pi}{3}$

QIV: $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

101. $\sin\theta = -\frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants III or IV.

QIII: $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

QIV: $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

102. $\cos\theta = -\frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or III.

QII: $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

QIII: $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

103. $\tan\theta = -\sqrt{3}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or IV.

QII: $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

QIV: $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

104. $\tan\theta = -\frac{\sqrt{3}}{3}$ when the reference angle is $\frac{\pi}{6}$ and θ is in quadrants II or IV.

QII: $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

QIV: $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

105. – 109. Answers may vary.

110. does not make sense; Explanations will vary. Sample explanation: Sine is defined for all values of the angle.

111. does not make sense; Explanations will vary. Sample explanation: Sine and cosecant have the same sign within any quadrant because they are reciprocals of each other.

112. does not make sense; Explanations will vary. Sample explanation: It is also possible that $y = -3$ and $x = -5$.

113. makes sense

114. $f(x) = \frac{2x^2}{x^2 - 1}$

$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2 - 1} = f(x)$$

y-axis symmetry

y-intercept: $y = \frac{2(0)^2}{0^2 - 1} = \frac{0}{1} = 0$

x-intercept:

$2x^2 = 0$

$x = 0$

vertical asymptote:

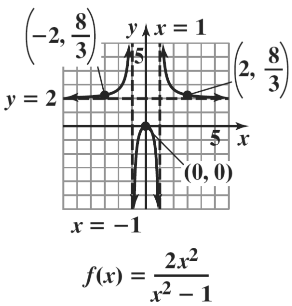
$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

horizontal asymptote:

$n = m$, so $y = \frac{2}{1} = 2$



115. $5^{\log_5 19} + \log_7 7^3 = 19 + 3 = 22$

116. $9e^{3x} - 4 = 32$

$9e^{3x} = 36$

$e^{3x} = 4$

$\ln e^{3x} = \ln 4$

$3x = \ln 4$

$x = \frac{\ln 4}{3} \approx 0.46$

The solution set is $\left\{\frac{\ln 4}{3}\right\}$, approximately 0.46.

117. $y = \frac{1}{2}\cos(4x + \pi)$

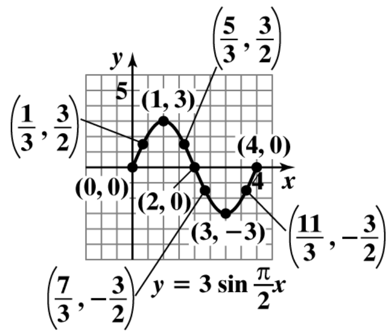
x	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$

118. $y = 4\sin\left(2x - \frac{2\pi}{3}\right)$

x	$\frac{\pi}{3}$	$\frac{7\pi}{12}$	$\frac{5\pi}{6}$	$\frac{13\pi}{12}$	$\frac{4\pi}{3}$
y	0	4	0	–4	0

119. $y = 3\sin\frac{\pi}{2}x$

x	0	$\frac{1}{3}$	1	$\frac{5}{3}$	2	$\frac{7}{3}$	3	$\frac{11}{3}$	4
y	0	$\frac{3}{2}$	3	$\frac{3}{2}$	0	$-\frac{3}{2}$	–3	$-\frac{3}{2}$	0



Mid-Chapter 4 Check Point

1. $10° = 10° \cdot \frac{\pi \text{ radians}}{180°} = \frac{10\pi}{180}$ radians

$= \frac{\pi}{18}$ radians

2. $-105° = -105° \cdot \frac{\pi \text{ radians}}{180°} = -\frac{105\pi}{180}$ radians

$= -\frac{7\pi}{12}$ radians

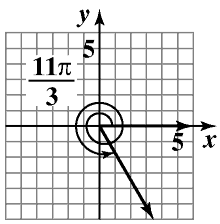
3. $\frac{5\pi}{12}$ radians $= \frac{5\pi \text{ radians}}{12} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = 75^{\circ}$

4. $-\frac{13\pi}{20}$ radians $= -\frac{13\pi \text{ radians}}{20} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$

$= -117^{\circ}$

5. a. $\frac{11\pi}{3} - 2\pi = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$

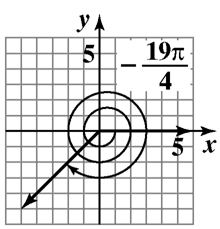
b.



c. Since $\frac{5\pi}{3}$ is in quadrant IV, the reference angle is $2\pi - \frac{5\pi}{3} = \frac{6\pi}{3} - \frac{5\pi}{3} = \frac{\pi}{3}$

6. a. $-\frac{19\pi}{4} + 6\pi = -\frac{19\pi}{4} + \frac{24\pi}{4} = \frac{5\pi}{4}$

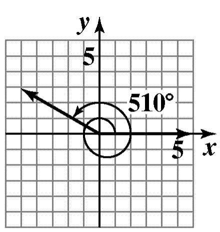
b.



c. Since $\frac{5\pi}{4}$ is in quadrant III, the reference angle is $\frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$

7. a. $510° - 360° = 150°$

b.



c. Since $150°$ is in quadrant II, the reference angle is $180° - 150° = 30°$

8. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{\left(-\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-\frac{4}{5}}{1} = -\frac{4}{5}$

$\cos\theta = \frac{x}{r} = \frac{-\frac{3}{5}}{1} = -\frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$

$\csc\theta = \frac{r}{y} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$

$\sec\theta = \frac{r}{x} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$

$\cot\theta = \frac{x}{y} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$

9. Use the Pythagorean theorem to find *b*.

$a^2 + b^2 = c^2$

$5^2 + b^2 = 6^2$

$25 + b^2 = 36$

$b^2 = 11$

$b = \sqrt{11}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{6}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{11}}{6}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5\sqrt{11}}{11}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{6}{5}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{11}}{5}$

10. $r = \sqrt{x^2 + y^2}$

$r = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$

$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\tan\theta = \frac{y}{x} = \frac{-2}{3} = -\frac{2}{3}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{-2} = -\frac{\sqrt{13}}{2}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{-2} = -\frac{3}{2}$

11. Because the tangent is negative and the cosine is negative, θ is in quadrant II. In quadrant II, *x* is negative and *y* is positive. Thus,

$\tan\theta = -\frac{3}{4} = \frac{x}{y}, \quad x = -4,\ y = 3$. Furthermore,

$r^2 = x^2 + y^2$

$r^2 = (-3)^2 + 4^2$

$r^2 = 9 + 16 = 25$

$r = 5$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{3}{5}$

$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$

$\csc\theta = \frac{r}{y} = \frac{5}{3}$

$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{4}$

$\cot\theta = \frac{x}{y} = \frac{-3}{4} = -\frac{4}{3}$

12. Since $\cos\theta = \frac{3}{7} = \frac{x}{r}, \quad x = 3,\ r = 7$. Furthermore,

$x^2 + y^2 = r^2$

$3^2 + y^2 = 7^2$

$9 + y^2 = 49$

$y^2 = 40$

$y = \pm\sqrt{40} = \pm 2\sqrt{10}$

Because the cosine is positive and the sine is negative, θ is in quadrant IV. In quadrant IV, *x* is positive and *y* is negative.

Therefore $y = -2\sqrt{10}$

Use *x, y,* and *r* to find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{10}}{7} = -\frac{2\sqrt{10}}{7}$

$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{10}}{3} = -\frac{2\sqrt{10}}{3}$

$\csc\theta = \frac{r}{y} = \frac{7}{-2\sqrt{10}} = -\frac{7\sqrt{10}}{20}$

$\sec\theta = \frac{r}{x} = \frac{7}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{-2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$

13. $\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent } \theta}$

$\tan 41° = \frac{a}{60}$

$a = 60\tan 41°$

$a \approx 52$ cm

14. $\cos\theta = \frac{\text{side adjacent } \theta}{\text{hypotenuse}}$

$\cos 72° = \frac{250}{c}$

$c = \frac{250}{\cos 72°}$

$c \approx 809$ m

15. Since $\cos\theta = \frac{1}{6} = \frac{x}{r}$, $x = 1$, $r = 6$. Furthermore,

$$x^2 + y^2 = r^2$$
$$1^2 + y^2 = 6^2$$
$$1 + y^2 = 36$$
$$y^2 = 35$$
$$y = \pm\sqrt{35}$$

Since θ is acute, $y = +\sqrt{35} = \sqrt{35}$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta = \frac{y}{x} = \frac{\sqrt{35}}{1} = \sqrt{35}$$

16. $\tan 30° = \frac{\sqrt{3}}{3}$

17. $\cot 120° = \frac{1}{\tan 120°} = \frac{1}{-\tan 60°} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

18. $\cos 240° = -\cos 60° = -\frac{1}{2}$

19. $\sec\frac{11\pi}{6} = \frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

20. $\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} = 1$

21. $\sin\left(-\frac{2\pi}{3}\right) = \sin\left(-\frac{2\pi}{3} + 2\pi\right)$

$$= \sin\frac{4\pi}{3} = -\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{3}}{2}$$

22. $\csc\left(\frac{22\pi}{3}\right) = \csc\left(\frac{22\pi}{3} - 6\pi\right) = \csc\frac{4\pi}{3}$

$$= \frac{1}{\sin\frac{4\pi}{3}} = \frac{1}{-\sin\frac{\pi}{3}} = \frac{1}{-\frac{\sqrt{3}}{2}}$$
$$= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

23. $\cos 495° = \cos(495° - 360°) = \cos 135°$

$$= -\cos 45° = -\frac{\sqrt{2}}{2}$$

24. $\tan\left(-\frac{17\pi}{6}\right) = \tan\left(-\frac{17\pi}{6} + 4\pi\right) = \tan\frac{7\pi}{6}$

$$= \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

25. $\sin^2\frac{\pi}{2} - \cos\pi = (1)^2 - (-1) = 1 + 1 = 2$

26. $\cos\left(\frac{5\pi}{6} + 2\pi n\right) + \tan\left(\frac{5\pi}{6} + n\pi\right)$

$$= \cos\frac{5\pi}{6} + \tan\frac{5\pi}{6} = -\cos\frac{\pi}{6} - \tan\frac{\pi}{6}$$
$$= -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{3} = -\frac{3\sqrt{3}}{6} - \frac{2\sqrt{3}}{6}$$
$$= -\frac{5\sqrt{3}}{6}$$

27. Begin by converting from degrees to radians.

$$36° = 36° \cdot \frac{\pi \text{ radians}}{180°} = \frac{\pi}{5} \text{ radians}$$
$$s = r\theta = 40 \cdot \frac{\pi}{5} = 8\pi \approx 25.13 \text{ cm}$$

28. Linear speed is given by $v = r\omega$. It is given that $r = 10$ feet and the merry-go-round rotates at 8 revolutions per minute. Convert 8 revolutions per minute to radians per minute.

8 revolutions per minute

$$= 8 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$
$$= 16\pi \text{ radians per minute}$$
$$v = r\omega = (10)(16\pi) = 160\pi \approx 502.7 \text{ feet per minute}$$

The linear speed of the horse is about 502.7 feet per minute.

29. $\sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}}$

$$\sin 6° = \frac{h}{5280}$$
$$h = 5280 \sin 6°$$
$$h \approx 551.9 \text{ feet}$$

30. $\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent } \theta}$

$$\tan\theta = \frac{50}{60}$$
$$\theta = \tan^{-1}\left(\frac{50}{60}\right)$$
$$\theta \approx 40°$$

Section 4.5

Check Point Exercises

1. The equation $y = 3\sin x$ is of the form $y = A\sin x$ with $A = 3$. Thus, the amplitude is $|A| = |3| = 3$ The period for both $y = 3\sin x$ and $y = \sin x$ is 2π. We find the three x–intercepts, the maximum point, and the minimum point on the interval $[0, 2\pi]$ by dividing the period, 2π, by 4, $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$, then by adding quarter-periods to generate x-values for each of the key points. The five x-values are

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

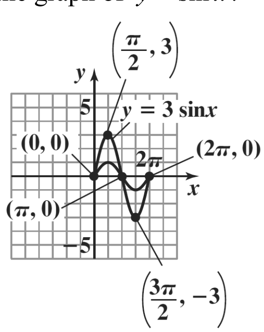
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 3\sin x$	coordinates
0	$y = 3\sin 0 = 3 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = 3\sin x = 3 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 3\sin\frac{3\pi}{2} = 3(-1) = -3$	$\left(\frac{3\pi}{2}, -3\right)$
2π	$y = 3\sin 2\pi = 3 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



2. The equation $y = -\frac{1}{2}\sin x$ is of the form $y = A\sin x$ with $A = -\frac{1}{2}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$. The period for both $y = -\frac{1}{2}\sin x$ and $y = \sin x$ is 2π.

Find the x–values for the five key points by dividing the period, 2π, by 4, $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$, then by adding quarter- periods. The five x-values are

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

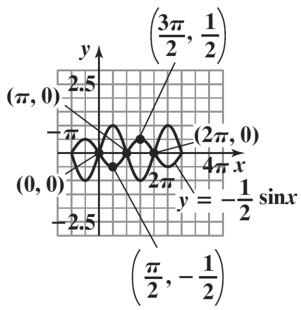
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -\frac{1}{2}\sin x$	coordinates
0	$y = -\frac{1}{2}\sin 0 = -\frac{1}{2} \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -\frac{1}{2}\sin\frac{\pi}{2} = -\frac{1}{2} \cdot 1 = -\frac{1}{2}$	$\left(\frac{\pi}{2}, -\frac{1}{2}\right)$
π	$y = -\frac{1}{2}\sin\pi = -\frac{1}{2} \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -\frac{1}{2}\sin\frac{3\pi}{2} = -\frac{1}{2}(-1) = \frac{1}{2}$	$\left(\frac{3\pi}{2}, \frac{1}{2}\right)$
2π	$y = -\frac{1}{2}\sin 2\pi = -\frac{1}{2} \cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$. Extend the pattern of each graph to the left and right as desired.



3. The equation $y = 2\sin\frac{1}{2}x$ is of the form

$y = A\sin Bx$ with $A = 2$ and $B = \frac{1}{2}$.

The amplitude is $|A| = |2| = 2$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

Find the x–values for the five key points by dividing the period, 4π, by 4, $\frac{\text{period}}{4} = \frac{4\pi}{4} = \pi$, then by adding quarter-periods.

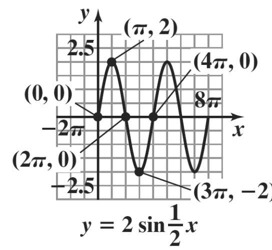
The five x-values are

$x = 0$
$x = 0 + \pi = \pi$
$x = \pi + \pi = 2\pi$
$x = 2\pi + \pi = 3\pi$
$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{2}x$	coordinates
0	$y = 2\sin\left(\frac{1}{2}\cdot 0\right)$ $= 2\sin 0$ $= 2\cdot 0 = 0$	(0, 0)
π	$y = 2\sin\left(\frac{1}{2}\cdot \pi\right)$ $= 2\sin\frac{\pi}{2} = 2\cdot 1 = 2$	$(\pi, 2)$
2π	$y = 2\sin\left(\frac{1}{2}\cdot 2\pi\right)$ $= 2\sin\pi = 2\cdot 0 = 0$	$(2\pi, 0)$
3π	$y = 2\sin\left(\frac{1}{2}\cdot 3\pi\right)$ $= 2\sin\frac{3\pi}{2}$ $= 2\cdot(-1) = -2$	$(3\pi, -2)$
4π	$y = 2\sin\left(\frac{1}{2}\cdot 4\pi\right)$ $= 2\sin 2\pi = 2\cdot 0 = 0$	$(4\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function. Extend the pattern of the graph another full period to the right.



4. The equation $y = 3\sin\left(2x - \frac{\pi}{3}\right)$ is of the form

$y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \frac{\pi}{3}$. The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{3}\cdot\frac{1}{2} = \frac{\pi}{6}$.

Find the x-values for the five key points by dividing the period, π, by 4, $\frac{\text{period}}{4} = \frac{\pi}{4}$, then by adding quarter-periods to the value of x where the cycle begins, $x = \frac{\pi}{6}$.

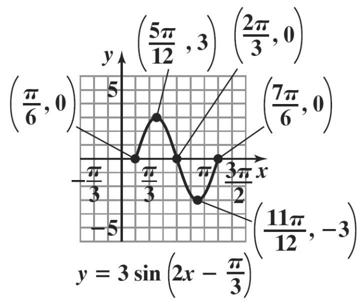
The five x-values are

$x = \frac{\pi}{6}$
$x = \frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{12}$
$x = \frac{5\pi}{12} + \frac{\pi}{4} = \frac{5\pi}{12} + \frac{3\pi}{12} = \frac{8\pi}{12} = \frac{2\pi}{3}$
$x = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{8\pi}{12} + \frac{3\pi}{12} = \frac{11\pi}{12}$
$x = \frac{11\pi}{12} + \frac{\pi}{4} = \frac{11\pi}{12} + \frac{3\pi}{12} = \frac{14\pi}{12} = \frac{7\pi}{6}$

Evaluate the function at each value of x.

x	$y=3\sin\left(2x-\frac{\pi}{3}\right)$	coordinates
$\frac{\pi}{6}$	$y=3\sin\left(2\cdot\frac{\pi}{6}-\frac{\pi}{3}\right)$ $=3\sin 0=3\cdot 0=0$	$\left(\frac{\pi}{6}, 0\right)$
$\frac{5\pi}{12}$	$y=3\sin\left(2\cdot\frac{5\pi}{12}-\frac{\pi}{3}\right)$ $=3\sin\frac{3\pi}{6}=3\sin\frac{\pi}{2}$ $=3\cdot 1=3$	$\left(\frac{5\pi}{12}, 3\right)$
$\frac{2\pi}{3}$	$y=3\sin\left(2\cdot\frac{2\pi}{3}-\frac{\pi}{3}\right)$ $=3\sin\frac{3\pi}{3}=3\sin\pi$ $=3\cdot 0=0$	$\left(\frac{2\pi}{3}, 0\right)$
$\frac{11\pi}{12}$	$y=3\sin\left(2\cdot\frac{11\pi}{12}-\frac{\pi}{3}\right)$ $=3\sin\frac{9\pi}{6}=3\sin\frac{3\pi}{2}$ $=3(-1)=-3$	$\left(\frac{11\pi}{12}, -3\right)$
$\frac{7\pi}{6}$	$y=3\sin\left(2\cdot\frac{7\pi}{6}-\frac{\pi}{3}\right)$ $=3\sin\frac{6\pi}{3}=3\sin 2\pi$ $=3\cdot 0=0$	$\left(\frac{7\pi}{6}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given graph.



5. The equation $y=-4\cos\pi x$ is of the form $y=A\cos Bx$ with $A=-4$, and $B=\pi$.

Thus, the amplitude is $|A|=|-4|=4$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{\pi}=2$.

Find the x-values for the five key points by dividing the period, 2, by 4, $\frac{\text{period}}{4}=\frac{2}{4}=\frac{1}{2}$, then by adding quarter periods to the value of x where the cycle begins. The five x-values are

$x=0$

$x=0+\frac{1}{2}=\frac{1}{2}$

$x=\frac{1}{2}+\frac{1}{2}=1$

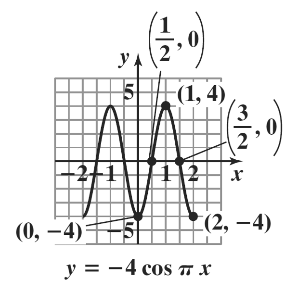
$x=1+\frac{1}{2}=\frac{3}{2}$

$x=\frac{3}{2}+\frac{1}{2}=2$

Evaluate the function at each value of x.

x	$y=-4\cos\pi x$	coordinates
0	$y=-4\cos(\pi\cdot 0)$ $=-4\cos 0=-4$	(0, –4)
$\frac{1}{2}$	$y=-4\cos\left(\pi\cdot\frac{1}{2}\right)$ $=-4\cos\frac{\pi}{2}=0$	$\left(\frac{1}{2}, 0\right)$
1	$y=-4\cos(\pi\cdot 1)$ $=-4\cos\pi=4$	(1, 4)
$\frac{3}{2}$	$y=-4\cos\left(\pi\cdot\frac{3}{2}\right)$ $=-4\cos\frac{3\pi}{2}=0$	$\left(\frac{3}{2}, 0\right)$
2	$y=-4\cos(\pi\cdot 2)$ $=-4\cos 2\pi=-4$	(2, –4)

Connect the five key points with a smooth curve and graph one complete cycle of the given function. Extend the pattern of the graph another full period to the left.



6. $y=\frac{3}{2}\cos(2x+\pi)=\frac{3}{2}\cos(2x-(-\pi))$

The equation is of the form $y=A\cos(Bx-C)$ with $A=\frac{3}{2}$, $B=2$, and $C=-\pi$.

Thus, the amplitude is $|A|=\left|\frac{3}{2}\right|=\frac{3}{2}$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$.

The phase shift is $\frac{C}{B}=\frac{-\pi}{2}=-\frac{\pi}{2}$.

Find the x-values for the five key points by dividing the period, π, by 4, $\frac{\text{period}}{4} = \frac{\pi}{4}$, then by adding quarter-periods to the value of x where the cycle begins, $x = -\frac{\pi}{2}$.

The five x-values are

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

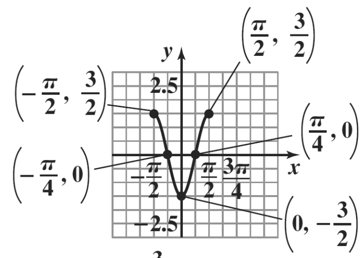
$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \frac{3}{2}\cos(2x + \pi)$	coordinates
$-\frac{\pi}{2}$	$y = \frac{3}{2}\cos(-\pi + \pi)$ $= \frac{3}{2} \cdot 1 = \frac{3}{2}$	$\left(-\frac{\pi}{2}, \frac{3}{2}\right)$
$-\frac{\pi}{4}$	$y = \frac{3}{2}\cos\left(-\frac{\pi}{2} + \pi\right)$ $= \frac{3}{2} \cdot 0 = 0$	$\left(-\frac{\pi}{4}, 0\right)$
0	$y = \frac{3}{2}\cos(0 + \pi)$ $= \frac{3}{2} \cdot -1 = -\frac{3}{2}$	$\left(0, -\frac{3}{2}\right)$
$\frac{\pi}{4}$	$y = \frac{3}{2}\cos\left(\frac{\pi}{2} + \pi\right)$ $= \frac{3}{2} \cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = \frac{3}{2}\cos(\pi + \pi)$ $= \frac{3}{2} \cdot 1 = \frac{3}{2}$	$\left(\frac{\pi}{2}, \frac{3}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given graph.



$y = \frac{3}{2}\cos(2x + \pi)$

7. The graph of $y = 2\cos x + 1$ is the graph of $y = 2\cos x$ shifted one unit upwards. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

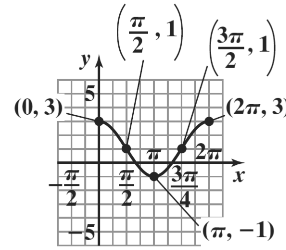
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 2\cos x + 1$	coordinates
0	$y = 2\cos 0 + 1$ $= 2 \cdot 1 + 1 = 3$	$(0, 3)$
$\frac{\pi}{2}$	$y = 2\cos\frac{\pi}{2} + 1$ $= 2 \cdot 0 + 1 = 1$	$\left(\frac{\pi}{2}, 1\right)$
π	$y = 2\cos\pi + 1$ $= 2 \cdot (-1) + 1 = -1$	$(\pi, -1)$
$\frac{3\pi}{2}$	$y = 2\cos\frac{3\pi}{2} + 1$ $= 2 \cdot 0 + 1 = 1$	$\left(\frac{3\pi}{2}, 1\right)$
2π	$y = 2\cos 2\pi + 1$ $= 2 \cdot 1 + 1 = 3$	$(2\pi, 3)$

By connecting the points with a smooth curve, we obtain one period of the graph.



$y = 2\cos x + 1$

8. A, the amplitude, is the maximum value of y. The graph shows that this maximum value is 4, Thus, $A = 4$. The period is $\frac{\pi}{2}$, and period $= \frac{2\pi}{B}$. Thus,

$$\frac{\pi}{2} = \frac{2\pi}{B}$$
$$\pi B = 4\pi$$
$$B = 4$$

Substitute these values into $y = A\sin Bx$.

The graph is modeled by $y = 4\sin 4x$.

9. Because the hours of daylight ranges from a minimum of 10 hours to a maximum of 14 hours, the curve oscillates about the middle value, 12 hours. Thus, $D = 12$. The maximum number of hours is 2 hours above 12 hours. Thus, $A = 2$. The graph shows that one complete cycle occurs in 12–0, or 12 months. The period is 12. Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

The graph shows that the starting point of the cycle is shifted from 0 to 3. The phase shift, $\frac{C}{B}$, is 3.

$$3 = \frac{C}{B}$$
$$3 = \frac{C}{\frac{\pi}{6}}$$
$$\frac{\pi}{2} = C$$

Substitute these values into $y = A\sin(Bx - C) + D$.

The number of hours of daylight is modeled by

$$y = 2\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12.$$

Concept and Vocabulary Check 4.5

1. $|A|$; $\frac{2\pi}{B}$

2. 3; 4π

3. π; 0; $\frac{\pi}{4}$; $\frac{\pi}{2}$; $\frac{3\pi}{4}$; π

4. $\frac{C}{B}$; right; left

5. $|A|$; $\frac{2\pi}{B}$

6. $\frac{1}{2}$; $\frac{2\pi}{3}$

7. false

8. true

9. true

10. true

Exercise Set 4.5

1. The equation $y = 4\sin x$ is of the form $y = A\sin x$ with $A = 4$. Thus, the amplitude is $|A| = |4| = 4$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

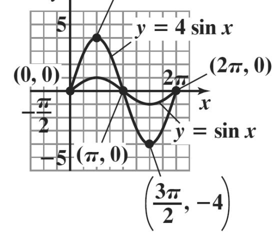
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$$x = 0$$
$$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$$
$$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$
$$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$
$$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$$

Evaluate the function at each value of x.

x	$y = 4\sin x$	coordinates
0	$y = 4\sin 0 = 4 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 4\sin\frac{\pi}{2} = 4 \cdot 1 = 4$	$\left(\frac{\pi}{2}, 4\right)$
π	$y = 4\sin\pi = 4 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 4\sin\frac{3\pi}{2} = 4(-1) = -4$	$\left(\frac{3\pi}{2}, -4\right)$
2π	$y = 4\sin 2\pi = 4 \cdot 0 = 0$	$(2\pi, 0)$

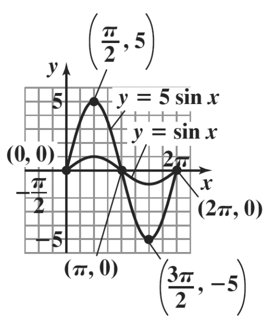
Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



2. The equation $y = 5\sin x$ is of the form $y = A\sin x$ with $A = 5$. Thus, the amplitude is $|A| = |5| = 5$.
The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.
$x = 0$
$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$
$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$
Evaluate the function at each value of x.

x	$y = 5\sin x$	coordinates
0	$y = 5\sin 0 = 5\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 5\sin\frac{\pi}{2} = 5\cdot 1 = 5$	$\left(\frac{\pi}{2}, 5\right)$
π	$y = 5\sin\pi = 5\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 5\sin\frac{3\pi}{2} = 5(-1) = -5$	$\left(\frac{3\pi}{2}, -5\right)$
2π	$y = 5\sin 2\pi = 5\cdot 0 = 0$	$(2\pi, 0)$

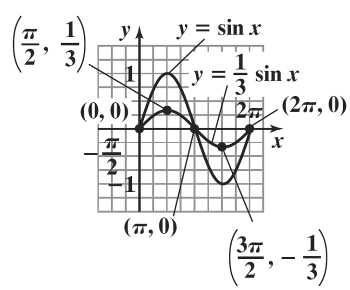
Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



3. The equation $y = \frac{1}{3}\sin x$ is of the form $y = A\sin x$ with $A = \frac{1}{3}$. Thus, the amplitude is $|A| = \left|\frac{1}{3}\right| = \frac{1}{3}$.
The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.
$x = 0$
$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$
$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$
Evaluate the function at each value of x.

x	$y = \frac{1}{3}\sin x$	coordinates
0	$y = \frac{1}{3}\sin 0 = \frac{1}{3}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = \frac{1}{3}\sin\frac{\pi}{2} = \frac{1}{3}\cdot 1 = \frac{1}{3}$	$\left(\frac{\pi}{2}, \frac{1}{3}\right)$
π	$y = \frac{1}{3}\sin\pi = \frac{1}{3}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \frac{1}{3}\sin\frac{3\pi}{2} = \frac{1}{3}(-1) = -\frac{1}{3}$	$\left(\frac{3\pi}{2}, -\frac{1}{3}\right)$
2π	$y = \frac{1}{3}\sin 2\pi = \frac{1}{3}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



4. The equation $y = \frac{1}{4}\sin x$ is of the form $y = A\sin x$ with $A = \frac{1}{4}$. Thus, the amplitude is $|A| = \left|\frac{1}{4}\right| = \frac{1}{4}$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

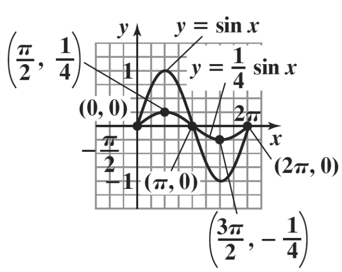
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \frac{1}{4}\sin x$	coordinates
0	$y = \frac{1}{4}\sin 0 = \frac{1}{4}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = \frac{1}{4}\sin\frac{\pi}{2} = \frac{1}{4}\cdot 1 = \frac{1}{4}$	$\left(\frac{\pi}{2}, \frac{1}{4}\right)$
π	$y = \frac{1}{4}\sin\pi = \frac{1}{4}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \frac{1}{4}\sin\frac{3\pi}{2} = \frac{1}{4}(-1) = -\frac{1}{4}$	$\left(\frac{3\pi}{2}, -\frac{1}{4}\right)$
2π	$y = \frac{1}{4}\sin 2\pi = \frac{1}{4}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



5. The equation $y = -3\sin x$ is of the form $y = A\sin x$ with $A = -3$. Thus, the amplitude is $|A| = |-3| = 3$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

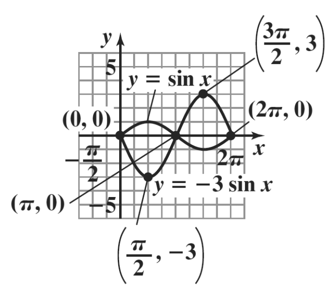
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -3\sin x$	coordinates
0	$y = -3\sin x$ $= -3\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -3\sin\frac{\pi}{2}$ $= -3\cdot 1 = -3$	$\left(\frac{\pi}{2}, -3\right)$
π	$y = -3\sin\pi$ $= -3\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -3\sin\frac{3\pi}{2}$ $= -3(-1) = 3$	$\left(\frac{3\pi}{2}, 3\right)$
2π	$y = -3\sin 2\pi$ $= -3\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



6. The equation $y = -4\sin x$ is of the form $y = A\sin x$ with $A = -4$. Thus, the amplitude is $|A| = |-4| = 4$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

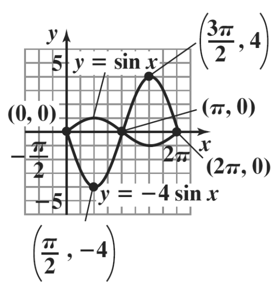
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -4\sin x$	coordinates
0	$y = -4\sin 0 = -4\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -4\sin\frac{\pi}{2} = -4\cdot 1 = -4$	$\left(\frac{\pi}{2}, -4\right)$
π	$y = -4\sin\pi = -4\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -4\sin\frac{3\pi}{2} = -4(-1) = 4$	$\left(\frac{3\pi}{2}, 4\right)$
2π	$y = -4\sin 2\pi = -4\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



7. The equation $y = \sin 2x$ is of the form $y = A\sin Bx$ with $A = 1$ and $B = 2$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

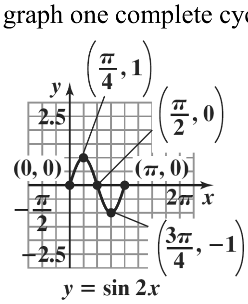
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	$y = \sin 2x$	coordinates
0	$y = \sin 2\cdot 0 = \sin 0 = 0$	(0, 0)
$\frac{\pi}{4}$	$y = \sin\left(2\cdot\frac{\pi}{4}\right)$ $= \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{2}$	$y = \sin\left(2\cdot\frac{\pi}{2}\right)$ $= \sin\pi = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2\cdot\frac{3\pi}{4}\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{3\pi}{4}, -1\right)$
π	$y = \sin(2\cdot\pi)$ $= \sin 2\pi = 0$	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



8. The equation $y = \sin 4x$ is of the form $y = A\sin Bx$ with $A = 1$ and $B = 4$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

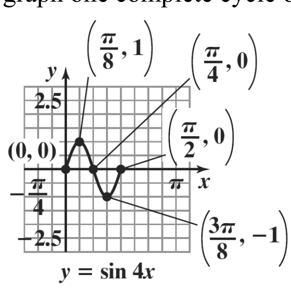
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin 4x$	coordinates
0	$y = \sin(4\cdot 0) = \sin 0 = 0$	(0, 0)
$\frac{\pi}{8}$	$y = \sin\left(4\cdot\frac{\pi}{8}\right) = \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{8}, 1\right)$
$\frac{\pi}{4}$	$y = \sin\left(4\cdot\frac{\pi}{4}\right) = \sin\pi = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{3\pi}{8}$	$y = \sin\left(4\cdot\frac{3\pi}{8}\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{3\pi}{8}, -1\right)$
$\frac{\pi}{2}$	$y = \sin 2\pi = 0$	$\left(\frac{\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



9. The equation $y = 3\sin\frac{1}{2}x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = \frac{1}{2}$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi\cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

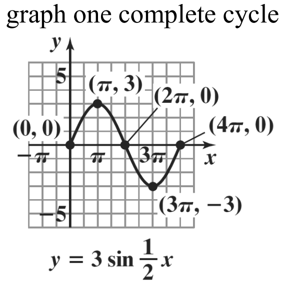
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 3\sin\frac{1}{2}x$	coordinates
0	$y = 3\sin\left(\frac{1}{2}\cdot 0\right)$ $= 3\sin 0 = 3\cdot 0 = 0$	(0, 0)
π	$y = 3\sin\left(\frac{1}{2}\cdot\pi\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$(\pi, 3)$
2π	$y = 3\sin\left(\frac{1}{2}\cdot 2\pi\right)$ $= 3\sin\pi = 3\cdot 0 = 0$	$(2\pi, 0)$
3π	$y = 3\sin\left(\frac{1}{2}\cdot 3\pi\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$(3\pi, -3)$
4π	$y = 3\sin\left(\frac{1}{2}\cdot 4\pi\right)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$(4\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



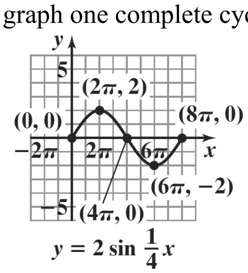
10. The equation $y = 2\sin\frac{1}{4}x$ is of the form $y = A\sin Bx$ with $A = 2$ and $B = \frac{1}{4}$. Thus, the amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$. The quarter-period is $\frac{8\pi}{4} = 2\pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + 2\pi = 2\pi$
$x = 2\pi + 2\pi = 4\pi$
$x = 4\pi + 2\pi = 6\pi$
$x = 6\pi + 2\pi = 8\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{4}x$	coordinates
0	$y = 2\sin\left(\frac{1}{4}\cdot 0\right) = 2\sin 0 = 2\cdot 0 = 0$	(0, 0)
2π	$y = 2\sin\left(\frac{1}{4}\cdot 2\pi\right) = 2\sin\frac{\pi}{2} = 2\cdot 1 = 2$	$(2\pi, 2)$
4π	$y = 2\sin\pi = 2\cdot 0 = 0$	$(4\pi, 0)$
6π	$y = 2\sin\frac{3\pi}{2} = 2(-1) = -2$	$(6\pi, -2)$
8π	$y = 2\sin 2\pi = 2\cdot 0 = 0$	$(8\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



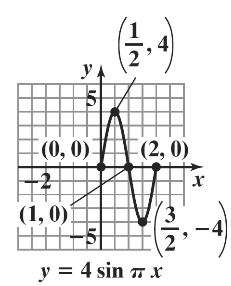
11. The equation $y = 4\sin\pi x$ is of the form $y = A\sin Bx$ with $A = 4$ and $B = \pi$. The amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{1}{2} = \frac{1}{2}$
$x = \frac{1}{2} + \frac{1}{2} = 1$
$x = 1 + \frac{1}{2} = \frac{3}{2}$
$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	$y = 4\sin\pi x$	coordinates
0	$y = 4\sin(\pi\cdot 0) = 4\sin 0 = 4\cdot 0 = 0$	(0, 0)
$\frac{1}{2}$	$y = 4\sin\left(\pi\cdot\frac{1}{2}\right) = 4\sin\frac{\pi}{2} = 4(1) = 4$	$\left(\frac{1}{2}, 4\right)$
1	$y = 4\sin(\pi\cdot 1) = 4\sin\pi = 4\cdot 0 = 0$	(1, 0)
$\frac{3}{2}$	$y = 4\sin\left(\pi\cdot\frac{3}{2}\right) = 4\sin\frac{3\pi}{2} = 4(-1) = -4$	$\left(\frac{3}{2}, -4\right)$
2	$y = 4\sin(\pi\cdot 2) = 4\sin 2\pi = 4\cdot 0 = 0$	(2, 0)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



12. The equation $y = 3\sin 2\pi x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 2\pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

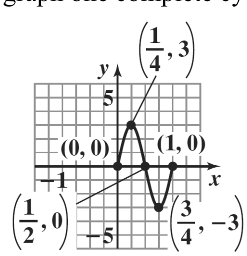
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 3\sin 2\pi x$	coordinates
0	$y = 3\sin(2\pi \cdot 0)$ $= 3\sin 0 = 3 \cdot 0 = 0$	(0, 0)
$\frac{1}{4}$	$y = 3\sin\left(2\pi \cdot \frac{1}{4}\right)$ $= 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$	$\left(\frac{1}{4}, 3\right)$
$\frac{1}{2}$	$y = 3\sin\left(2\pi \cdot \frac{1}{2}\right)$ $= 3\sin \pi = 3 \cdot 0 = 0$	$\left(\frac{1}{2}, 0\right)$
$\frac{3}{4}$	$y = 3\sin\left(2\pi \cdot \frac{3}{4}\right)$ $= 3\sin\frac{3\pi}{2} = 3(-1) = -3$	$\left(\frac{3}{4}, -3\right)$
1	$y = 3\sin(2\pi \cdot 1)$ $= 3\sin 2\pi = 3 \cdot 0 = 0$	(1, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin 2\pi x$

13. The equation $y = -3\sin 2\pi x$ is of the form $y = A\sin Bx$ with $A = -3$ and $B = 2\pi$. The amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

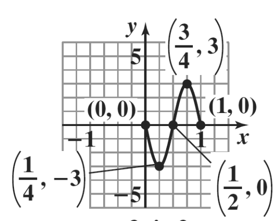
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = -3\sin 2\pi x$	coordinates
0	$y = -3\sin(2\pi \cdot 0)$ $= -3\sin 0$ $= -3 \cdot 0 = 0$	(0, 0)
$\frac{1}{4}$	$y = -3\sin\left(2\pi \cdot \frac{1}{4}\right)$ $= -3\sin\frac{\pi}{2}$ $= -3 \cdot 1 = -3$	$\left(\frac{1}{4}, -3\right)$
$\frac{1}{2}$	$y = -3\sin\left(2\pi \cdot \frac{1}{2}\right)$ $= -3\sin \pi$ $= -3 \cdot 0 = 0$	$\left(\frac{1}{2}, 0\right)$
$\frac{3}{4}$	$y = -3\sin\left(2\pi \cdot \frac{3}{4}\right)$ $= -3\sin\frac{3\pi}{2}$ $= -3(-1) = 3$	$\left(\frac{3}{4}, 3\right)$
1	$y = -3\sin(2\pi \cdot 1)$ $= -3\sin 2\pi$ $= -3 \cdot 0 = 0$	(1, 0)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -3\sin 2\pi x$

14. The equation $y=-2\sin \pi x$ is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

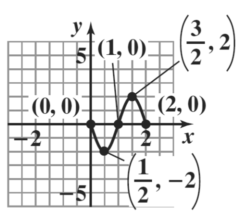
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	$y = -2\sin \pi x$	coordinates
0	$y = -2\sin(\pi \cdot 0)$ $= -2\sin 0 = -2\cdot 0 = 0$	(0, 0)
$\frac{1}{2}$	$y = -2\sin\left(\pi \cdot \frac{1}{2}\right)$ $= -2\sin\frac{\pi}{2} = -2\cdot 1 = -2$	$\left(\frac{1}{2}, -2\right)$
1	$y = -2\sin(\pi \cdot 1)$ $= -2\sin \pi = -2\cdot 0 = 0$	(1, 0)
$\frac{3}{2}$	$y = -2\sin\left(\pi \cdot \frac{3}{2}\right)$ $= -2\sin\frac{3\pi}{2} = -2(-1) = 2$	$\left(\frac{3}{2}, 2\right)$
2	$y = -2\sin(\pi \cdot 2)$ $= -2\sin 2\pi = -2\cdot 0 = 0$	(2, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin \pi x$

15. The equation $y = -\sin\frac{2}{3}x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \frac{2}{3}$.

The amplitude is $|A| = |-1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$.

The quarter-period is $\frac{3\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

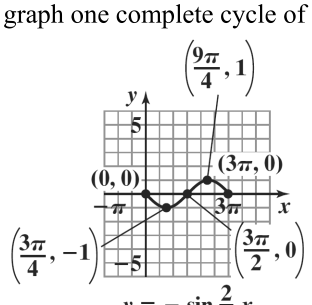
$x = \frac{3\pi}{2} + \frac{3\pi}{4} = \frac{9\pi}{4}$

$x = \frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$

Evaluate the function at each value of x.

x	$y = -\sin\frac{2}{3}x$	coordinates
0	$y = -\sin\left(\frac{2}{3}\cdot 0\right)$ $= -\sin 0 = 0$	(0, 0)
$\frac{3\pi}{4}$	$y = -\sin\left(\frac{2}{3}\cdot\frac{3\pi}{4}\right)$ $= -\sin\frac{\pi}{2} = -1$	$\left(\frac{3\pi}{4}, -1\right)$
$\frac{3\pi}{2}$	$y = -\sin\left(\frac{2}{3}\cdot\frac{3\pi}{2}\right)$ $= -\sin \pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{9\pi}{4}$	$y = -\sin\left(\frac{2}{3}\cdot\frac{9\pi}{4}\right)$ $= -\sin\frac{3\pi}{2}$ $= -(-1) = 1$	$\left(\frac{9\pi}{4}, 1\right)$
3π	$y = -\sin\left(\frac{2}{3}\cdot 3\pi\right)$ $= -\sin 2\pi = 0$	$(3\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -\sin\frac{2}{3}x$

16. The equation $y = -\sin\frac{4}{3}x$ is of the form

$y = A\sin Bx$ with $A = -1$ and $B = \frac{4}{3}$.

The amplitude is $|A| = |-1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{4}{3}} = 2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$.

The quarter-period is $\frac{\frac{3\pi}{2}}{4} = \frac{3\pi}{2} \cdot \frac{1}{4} = \frac{3\pi}{8}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{3\pi}{8} = \frac{3\pi}{4}$

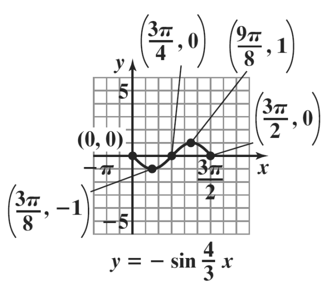
$x = \frac{3\pi}{4} + \frac{3\pi}{8} = \frac{9\pi}{8}$

$x = \frac{9\pi}{8} + \frac{3\pi}{8} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = -\sin\frac{4}{3}x$	coordinates
0	$y = -\sin\left(\frac{4}{3} \cdot 0\right) = -\sin 0 = 0$	$(0, 0)$
$\frac{3\pi}{8}$	$y = -\sin\left(\frac{4}{3} \cdot \frac{3\pi}{8}\right) = -\sin\frac{\pi}{2} = -1$	$\left(\frac{3\pi}{8}, -1\right)$
$\frac{3\pi}{4}$	$y = -\sin\left(\frac{4}{3} \cdot \frac{3\pi}{4}\right) = -\sin\pi = 0$	$\left(\frac{3\pi}{4}, 0\right)$
$\frac{9\pi}{8}$	$y = -\sin\left(\frac{4}{3} \cdot \frac{9\pi}{8}\right) = -\sin\frac{3\pi}{2} = -(-1) = 1$	$\left(\frac{9\pi}{8}, 1\right)$
$\frac{3\pi}{2}$	$y = -\sin\left(\frac{4}{3} \cdot \frac{3\pi}{2}\right) = -\sin 2\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



17. The equation $y = \sin(x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 1$, and $C = \pi$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{1} = \pi$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \pi$. Add quarter-periods to generate x-values for the key points.

$x = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

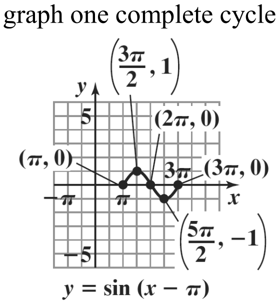
$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} + \frac{\pi}{2} = 3\pi$

Evaluate the function at each value of x.

x	$y = \sin(x - \pi)$	coordinates
π	$y = \sin(\pi - \pi) = \sin 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \sin\left(\frac{3\pi}{2} - \pi\right) = \sin\frac{\pi}{2} = 1$	$\left(\frac{3\pi}{2}, 1\right)$
2π	$y = \sin(2\pi - \pi) = \sin\pi = 0$	$(2\pi, 0)$
$\frac{5\pi}{2}$	$y = \sin\left(\frac{5\pi}{2} - \pi\right) = \sin\frac{3\pi}{2} = -1$	$\left(\frac{5\pi}{2}, -1\right)$
3π	$y = \sin(3\pi - \pi) = \sin 2\pi = 0$	$(3\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



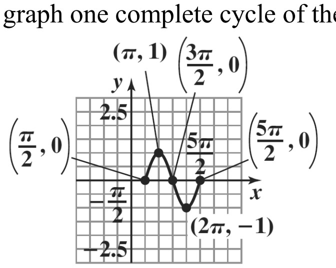
18. The equation $y = \sin\left(x - \frac{\pi}{2}\right)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 1$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$$x = \frac{\pi}{2}$$
$$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$
$$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$
$$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$$
$$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$

Evaluate the function at each value of x.

x	$y = \sin\left(x - \frac{\pi}{2}\right)$	coordinates
$\frac{\pi}{2}$	$y = \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \sin 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = \sin\left(\pi - \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$	$(\pi, 1)$
$\frac{3\pi}{2}$	$y = \sin\ \frac{3\pi}{2} - \frac{\pi}{2}\ = \sin\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = \sin\ 2\pi - \frac{\pi}{2}\ = \sin\frac{3\pi}{2} = -1$	$(2\pi, -1)$
$\frac{5\pi}{2}$	$y = \sin\ \frac{5\pi}{2} - \frac{\pi}{2}\ = \sin 2\pi = 0$	$\left(\frac{5\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = \sin\left(x - \frac{\pi}{2}\right)$

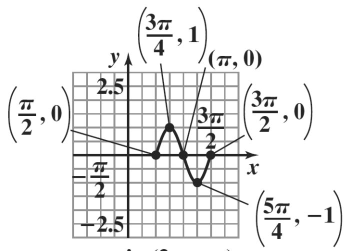
19. The equation $y = \sin(2x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$$x = \frac{\pi}{2}$$
$$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$
$$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$$
$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$
$$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$$

Evaluate the function at each value of x.

x	$y = \sin(2x - \pi)$	coordinates
$\frac{\pi}{2}$	$y = \sin\left(2\cdot\frac{\pi}{2} - \pi\right)$ $= \sin(\pi - \pi)$ $= \sin 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2\cdot\frac{3\pi}{4} - \pi\right)$ $= \sin\left(\frac{3\pi}{2} - \pi\right)$ $= \sin\frac{\pi}{2} = 1$	$\left(\frac{3\pi}{4}, 1\right)$
π	$y = \sin(2\cdot\pi - \pi)$ $= \sin(2\pi - \pi)$ $= \sin\pi = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = \sin\left(2\cdot\frac{5\pi}{4} - \pi\right)$ $= \sin\left(\frac{5\pi}{2} - \pi\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{5\pi}{4}, -1\right)$
$\frac{3\pi}{2}$	$y = \sin\left(2\cdot\frac{3\pi}{2} - \pi\right)$ $= \sin(3\pi - \pi)$ $= \sin 2\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = \sin(2x - \pi)$

20. The equation $y=\sin\left(2x-\frac{\pi}{2}\right)$ is of the form

$y = A\sin(Bx-C)$ with $A = 1$, $B = 2$, and $C=\frac{\pi}{2}$. The amplitude is $|A|=|1|=1$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$.

The phase shift is $\frac{C}{B}=\frac{\frac{\pi}{2}}{2}=\frac{\pi}{2}\cdot\frac{1}{2}=\frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x=\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x=\frac{\pi}{4}$

$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

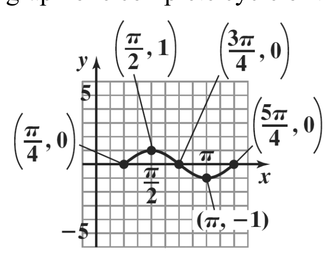
$x=\frac{3\pi}{4}+\frac{\pi}{4}=\pi$

$x=\pi+\frac{\pi}{4}=\frac{5\pi}{4}$

Evaluate the function at each value of x.

x	$y=\sin\left(2x-\frac{\pi}{2}\right)$	coordinates
$\frac{\pi}{4}$	$y=\sin\left(2\cdot\frac{\pi}{4}-\frac{\pi}{2}\right)$ $=\sin\left(\frac{\pi}{2}-\frac{\pi}{2}\right)=\sin 0=0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y=\sin\left(2\cdot\frac{\pi}{2}-\frac{\pi}{2}\right)$ $=\sin\left(\pi-\frac{\pi}{2}\right)=\sin\frac{\pi}{2}=1$	$\left(\frac{\pi}{2}, 1\right)$
$\frac{3\pi}{4}$	$y=\sin\left(2\cdot\frac{3\pi}{4}-\frac{\pi}{2}\right)$ $=\sin\left(\frac{3\pi}{2}-\frac{\pi}{2}\right)$ $=\sin\pi=0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y=\sin\left(2\cdot\pi-\frac{\pi}{2}\right)$ $=\sin\left(2\pi-\frac{\pi}{2}\right)$ $=\sin\frac{3\pi}{2}=-1$	$(\pi, -1)$
$\frac{5\pi}{4}$	$y=\sin\left(2\cdot\frac{5\pi}{4}-\frac{\pi}{2}\right)$ $=\sin\left(\frac{5\pi}{2}-\frac{\pi}{2}\right)$ $=\sin 2\pi=0$	$\left(\frac{5\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y=\sin\left(2x-\frac{\pi}{2}\right)$

21. The equation $y = 3\sin(2x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

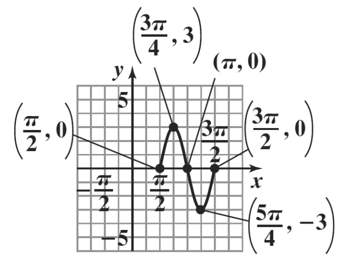
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = 3\sin(2x - \pi)$	coordinates
$\frac{\pi}{2}$	$y = 3\sin\left(2 \cdot \frac{\pi}{2} - \pi\right)$ $= 3\sin(\pi - \pi)$ $= 3\sin 0 = 3 \cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = 3\sin\left(2 \cdot \frac{3\pi}{4} - \pi\right)$ $= 3\sin\left(\frac{3\pi}{2} - \pi\right)$ $= 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$	$\left(\frac{3\pi}{4}, 3\right)$
π	$y = 3\sin(2 \cdot \pi - \pi)$ $= 3\sin(2\pi - \pi)$ $= 3\sin\pi = 3 \cdot 0 = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = 3\sin\left(2 \cdot \frac{5\pi}{4} - \pi\right)$ $= 3\sin\left(\frac{5\pi}{2} - \pi\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$\left(\frac{5\pi}{4}, -3\right)$
$\frac{3\pi}{2}$	$y = 3\sin\left(2 \cdot \frac{3\pi}{2} - \pi\right)$ $= 3\sin(3\pi - \pi)$ $= 3\sin 2\pi = 3 \cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



22. The equation $y = 3\sin\left(2x - \frac{\pi}{2}\right)$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \frac{\pi}{2}$.

The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

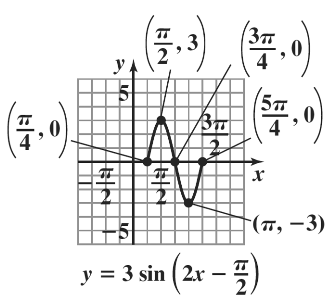
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	$y = 3\sin\left(2x - \frac{\pi}{2}\right)$	coordinates
$\frac{\pi}{4}$	$y = 3\sin\left(2\cdot\frac{\pi}{4} - \frac{\pi}{2}\right)$ $= \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin 0 = 3\cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = 3\sin\left(2\cdot\frac{\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin\left(\pi - \frac{\pi}{2}\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\left(\frac{\pi}{2}, 3\right)$
$\frac{3\pi}{4}$	$y = 3\sin\left(2\cdot\frac{3\pi}{4} - \frac{\pi}{2}\right)$ $= 3\sin\left(\frac{3\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin\pi = 3\cdot 0 = 0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y = 3\sin\left(2\cdot\pi - \frac{\pi}{2}\right)$ $= 3\sin\left(2\pi - \frac{\pi}{2}\right)$ $= 3\sin\frac{3\pi}{2} = 3\cdot(-1) = -3$	$(\pi, -3)$
$\frac{5\pi}{4}$	$y = 3\sin\left(2\cdot\frac{5\pi}{4} - \frac{\pi}{2}\right)$ $= 3\sin\left(\frac{5\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\left(\frac{5\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



23. $y = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right) = \frac{1}{2}\sin\left(x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = \frac{1}{2}\sin\left(x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = \frac{1}{2}$, $B = 1$, and $C = -\frac{\pi}{2}$.

The amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$.

The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

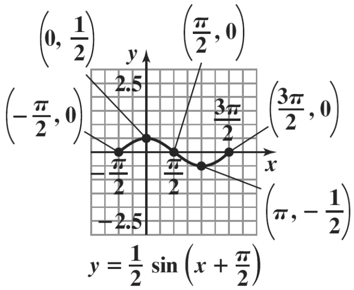
$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{2}$	$y = \frac{1}{2}\sin\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin 0 = \frac{1}{2}\cdot 0 = 0$	$\left(-\frac{\pi}{2}, 0\right)$
0	$y = \frac{1}{2}\sin\left(0 + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin\frac{\pi}{2} = \frac{1}{2}\cdot 1 = \frac{1}{2}$	$\left(0, \frac{1}{2}\right)$
$\frac{\pi}{2}$	$y = \frac{1}{2}\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin\pi = \frac{1}{2}\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = \frac{1}{2}\sin\left(\pi + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin\frac{3\pi}{2}$ $= \frac{1}{2}\cdot(-1) = -\frac{1}{2}$	$\left(\pi, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	$y = \frac{1}{2}\sin\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin 2\pi$ $= \frac{1}{2}\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



24. $y = \frac{1}{2}\sin(x+\pi) = \frac{1}{2}\sin(x-(-\pi))$

The equation $y = \frac{1}{2}\sin(x-(-\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = \frac{1}{2}$, $B = 1$, and $C = -\pi$.

The amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\pi}{1} = -\pi$.

The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\pi$. Add quarter-periods to generate x-values for the key points.

$x = -\pi$

$x = -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

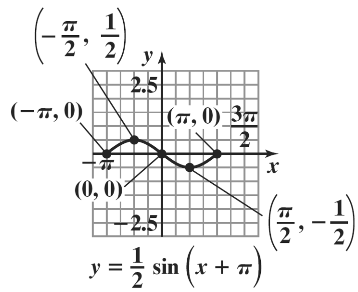
$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

Evaluate the function at each value of x.

x	$y = \frac{1}{2}\sin(x+\pi)$	coordinates
$-\pi$	$y = \frac{1}{2}\sin(-\pi+\pi)$ $= \frac{1}{2}\sin 0 = \frac{1}{2}\cdot 0 = 0$	$(-\pi, 0)$
$-\frac{\pi}{2}$	$y = \frac{1}{2}\sin\left(-\frac{\pi}{2}+\pi\right)$ $= \frac{1}{2}\sin\frac{\pi}{2} = \frac{1}{2}\cdot 1 = \frac{1}{2}$	$\left(-\frac{\pi}{2}, \frac{1}{2}\right)$
0	$y = \frac{1}{2}\sin(0+\pi)$ $= \frac{1}{2}\sin\pi = \frac{1}{2}\cdot 0 = 0$	$(0, 0)$
$\frac{\pi}{2}$	$y = \frac{1}{2}\sin\left(\frac{\pi}{2}+\pi\right)$ $= \frac{1}{2}\sin\frac{3\pi}{2} = \frac{1}{2}\cdot(-1) = -\frac{1}{2}$	$\left(\frac{\pi}{2}, -\frac{1}{2}\right)$
π	$y = \frac{1}{2}\sin(\pi+\pi)$ $= \frac{1}{2}\sin 2\pi = \frac{1}{2}\cdot 0 = 0$	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



25. $y = -2\sin\left(2x + \frac{\pi}{2}\right) = -2\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = -2\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = -2$, $B = 2$, and $C = -\frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2}\cdot\frac{1}{2} = -\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = -\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

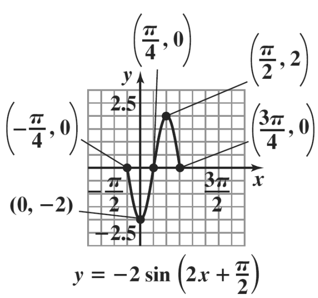
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Evaluate the function at each value of x.

x	$y=-2\sin\left(2x+\frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{4}$	$y=-2\sin\left(2\cdot\left(-\frac{\pi}{4}\right)+\frac{\pi}{2}\right)$ $=-2\sin\left(-\frac{\pi}{2}+\frac{\pi}{2}\right)$ $=-2\sin 0=-2\cdot 0=0$	$\left(-\frac{\pi}{4},0\right)$
0	$y=-2\sin\left(2\cdot 0+\frac{\pi}{2}\right)$ $=-2\sin\left(0+\frac{\pi}{2}\right)$ $=-2\sin\frac{\pi}{2}$ $=-2\cdot 1=-2$	(0, –2)
$\frac{\pi}{4}$	$y=-2\sin\left(2\cdot\frac{\pi}{4}+\frac{\pi}{2}\right)$ $=-2\sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$ $=-2\sin\pi$ $=-2\cdot 0=0$	$\left(\frac{\pi}{4},0\right)$
$\frac{\pi}{2}$	$y=-2\sin\left(2\cdot\frac{\pi}{2}+\frac{\pi}{2}\right)$ $=-2\sin\left(\pi+\frac{\pi}{2}\right)$ $=-2\sin\frac{3\pi}{2}$ $=-2(-1)=2$	$\left(\frac{\pi}{2},2\right)$
$\frac{3\pi}{4}$	$y=-2\sin\left(2\cdot\frac{3\pi}{4}+\frac{\pi}{2}\right)$ $=-2\sin\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)$ $=-2\sin 2\pi$ $=-2\cdot 0=0$	$\left(\frac{3\pi}{4},0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



26. $y=-3\sin\left(2x+\frac{\pi}{2}\right)=-3\sin\left(2x-\left(-\frac{\pi}{2}\right)\right)$

The equation $y=-3\sin\left(2x-\left(-\frac{\pi}{2}\right)\right)$ is of the form

$y=A\sin(Bx-C)$ with $A=-3$, $B=2$, and $C=-\frac{\pi}{2}$.

The amplitude is $|A|=|-3|=3$. The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$. The phase shift is $\frac{C}{B}=\frac{-\frac{\pi}{2}}{2}=-\frac{\pi}{2}\cdot\frac{1}{2}=-\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x=-\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{\pi}{4}$

$x=-\frac{\pi}{4}+\frac{\pi}{4}=0$

$x=0+\frac{\pi}{4}=\frac{\pi}{4}$

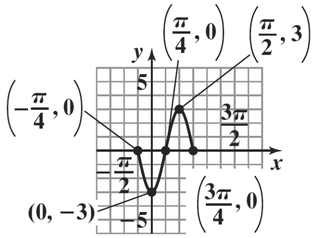
$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

Evaluate the function at each value of x.

x	$y=-3\sin\left(2x+\frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{4}$	$y=-3\sin\left(2\cdot\left(-\frac{\pi}{4}\right)+\frac{\pi}{2}\right)$ $=-3\sin\left(-\frac{\pi}{2}+\frac{\pi}{2}\right)$ $=-3\sin 0=-3\cdot 0=0$	$\left(-\frac{\pi}{4},0\right)$
0	$y=-3\sin\left(2\cdot 0+\frac{\pi}{2}\right)$ $=-3\sin\left(0+\frac{\pi}{2}\right)$ $=-3\sin\frac{\pi}{2}=-3\cdot 1=-3$	(0, –3)
$\frac{\pi}{4}$	$y=-3\sin\left(2\cdot\frac{\pi}{4}+\frac{\pi}{2}\right)$ $=-3\sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$ $=-3\sin\pi=-3\cdot 0=0$	$\left(\frac{\pi}{4},0\right)$
$\frac{\pi}{2}$	$y=-3\sin\left(2\cdot\frac{\pi}{2}+\frac{\pi}{2}\right)$ $=-3\sin\left(\pi+\frac{\pi}{2}\right)$ $=-3\sin\frac{3\pi}{2}=-3\cdot(-1)=3$	$\left(\frac{\pi}{2},3\right)$
$\frac{3\pi}{4}$	$y=-3\sin\left(2\cdot\frac{3\pi}{4}+\frac{\pi}{2}\right)$ $=-3\sin\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)$ $=-3\sin 2\pi=-3\cdot 0=0$	$\left(\frac{3\pi}{4},0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\sin\left(2x + \frac{\pi}{2}\right)$

27. $y = 3\sin(\pi x + 2)$

The equation $y = 3\sin(\pi x - (-2))$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = \pi$, and $C = -2$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The phase shift is $\frac{C}{B} = \frac{-2}{\pi} = -\frac{2}{\pi}$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = -\frac{2}{\pi}$. Add quarter-periods to generate *x*-values for the key points.

$x = -\frac{2}{\pi}$

$x = -\frac{2}{\pi} + \frac{1}{2} = \frac{\pi - 4}{2\pi}$

$x = \frac{\pi - 4}{2\pi} + \frac{1}{2} = \frac{\pi - 2}{\pi}$

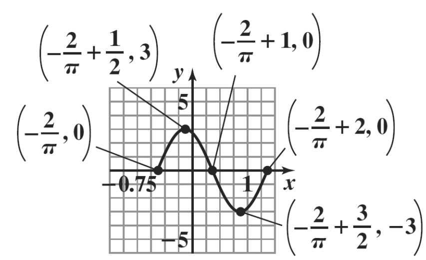
$x = \frac{\pi - 2}{\pi} + \frac{1}{2} = \frac{3\pi - 4}{2\pi}$

$x = \frac{3\pi - 4}{2\pi} + \frac{1}{2} = \frac{2\pi - 2}{\pi}$

Evaluate the function at each value of *x*.

x	$y = 3\sin(\pi x + 2)$	coordinates
$-\frac{2}{\pi}$	$y = 3\sin\left(\pi\left(-\frac{2}{\pi}\right) + 2\right)$ $= 3\sin(-2 + 2)$ $= 3\sin 0 = 3 \cdot 0 = 0$	$\left(-\frac{2}{\pi}, 0\right)$
$\frac{\pi - 4}{2\pi}$	$y = 3\sin\left(\pi\left(\frac{\pi - 4}{2\pi}\right) + 2\right)$ $= 3\sin\left(\frac{\pi - 4}{2} + 2\right)$ $= 3\sin\left(\frac{\pi}{2} - 2 + 2\right)$ $= 3\sin\frac{\pi}{2}$ $= 3 \cdot 1 = 3$	$\left(\frac{\pi - 4}{2\pi}, 3\right)$
$\frac{\pi - 2}{\pi}$	$y = 3\sin\left(\pi\left(\frac{\pi - 2}{\pi}\right) + 2\right)$ $= 3\sin(\pi - 2 + 2)$ $= 3\sin\pi = 3 \cdot 0 = 0$	$\left(\frac{\pi - 2}{\pi}, 0\right)$
$\frac{3\pi - 4}{2\pi}$	$y = 3\sin\left(\pi\left(\frac{3\pi - 4}{2\pi}\right) + 2\right)$ $= 3\sin\left(\frac{3\pi - 4}{2} + 2\right)$ $= 3\sin\left(\frac{3\pi}{2} - 2 + 2\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$\left(\frac{5\pi}{4}, -3\right)$
$\frac{2\pi - 2}{\pi}$	$y = 3\sin\left(\pi\left(\frac{2\pi - 2}{\pi}\right) + 2\right)$ $= 3\sin(2\pi - 2 + 2)$ $= 3\sin 2\pi = 3 \cdot 0 = 0$	$\left(\frac{2\pi - 2}{\pi}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin(\pi x + 2)$

28. $y = 3\sin(2\pi x + 4) = 3\sin(2\pi x - (-4))$

The equation $y = 3\sin(2\pi x - (-4))$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2\pi$, and $C = -4$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4}{2\pi} = -\frac{2}{\pi}$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -\frac{2}{\pi}$. Add quarter-periods to generate *x*-values for the key points.

$x = -\frac{2}{\pi}$

$x = -\frac{2}{\pi} + \frac{1}{4} = \frac{\pi - 8}{4\pi}$

$x = \frac{\pi - 8}{4\pi} + \frac{1}{4} = \frac{\pi - 4}{2\pi}$

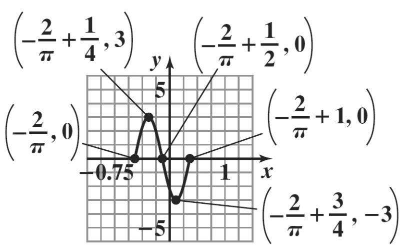
$x = \frac{\pi - 4}{2\pi} + \frac{1}{4} = \frac{3\pi - 8}{4\pi}$

$x = \frac{3\pi - 8}{4\pi} + \frac{1}{4} = \frac{\pi - 2}{\pi}$

Evaluate the function at each value of *x*.

x	$y = 3\sin(2\pi x + 4)$	coordinates
$-\frac{2}{\pi}$	$y = 3\sin\left(2\pi\left(-\frac{2}{\pi}\right)+4\right)$ $= 3\sin(-4+4)$ $= 3\sin 0 = 3\cdot 0 = 0$	$\left(-\frac{2}{\pi},\ 0\right)$
$\frac{\pi-8}{4\pi}$	$y = 3\sin\left(2\pi\left(\frac{\pi-8}{4\pi}\right)+4\right)$ $= 3\sin\left(\frac{\pi-8}{2}+4\right)$ $= 3\sin\left(\frac{\pi}{2}-4+4\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\left(\frac{\pi-8}{4\pi},\ 3\right)$
$\frac{\pi-4}{2\pi}$	$y = 3\sin\left(2\pi\left(\frac{\pi-4}{2\pi}\right)+4\right)$ $= 3\sin(\pi-4+4)$ $= 3\sin\pi = 3\cdot 0 = 0$	$\left(\frac{\pi-4}{2\pi},\ 0\right)$
$\frac{3\pi-8}{4\pi}$	$y = 3\sin\left(2\pi\left(\frac{3\pi-8}{4\pi}\right)+4\right)$ $= 3\sin\left(\frac{3\pi-8}{2}+4\right)$ $= 3\sin\left(\frac{3\pi}{2}-4+4\right)$ $= 3\sin\frac{3\pi}{2} = 3(-1) = -3$	$\left(\frac{3\pi-8}{4\pi},\ -3\right)$
$\frac{\pi-2}{\pi}$	$y = 3\sin\left(2\pi\left(\frac{\pi-2}{\pi}\right)+4\right)$ $= 3\sin(2\pi-4+4)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\left(\frac{\pi-2}{\pi},0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin(2\pi x + 4)$

29. $y = -2\sin(2\pi x + 4\pi) = -2\sin(2\pi x - (-4\pi))$

The equation $y = -2\sin(2\pi x - (-4\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = -2$, $B = 2\pi$, and $C = -4\pi$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate x-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

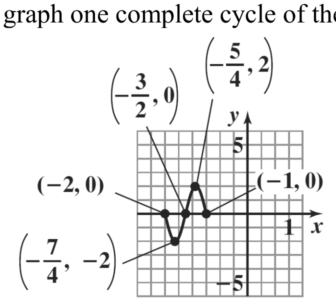
$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of x.

x	$y = -2\sin(2\pi x + 4\pi)$	coordinates
–2	$y = -2\sin(2\pi(-2) + 4\pi)$ $= -2\sin(-4\pi + 4\pi)$ $= -2\sin 0$ $= -2\cdot 0 = 0$	(–2, 0)
$-\frac{7}{4}$	$y = -2\sin\left(2\pi\left(-\frac{7}{4}\right)+4\pi\right)$ $= -2\sin\left(-\frac{7\pi}{2}+4\pi\right)$ $= -2\sin\frac{\pi}{2} = -2\cdot 1 = -2$	$\left(-\frac{7}{4},\ -2\right)$
$-\frac{3}{2}$	$y = -2\sin\left(2\pi\left(-\frac{3}{2}\right)+4\pi\right)$ $= -2\sin(-3\pi + 4\pi)$ $= -2\sin\pi = -2\cdot 0 = 0$	$\left(-\frac{3}{2},\ 0\right)$
$-\frac{5}{4}$	$y = -2\sin\left(2\pi\left(-\frac{5}{4}\right)+4\pi\right)$ $= -2\sin\left(-\frac{5\pi}{2}+4\pi\right)$ $= -2\sin\frac{3\pi}{2}$ $= -2(-1) = 2$	$\left(-\frac{5}{4},\ 2\right)$
–1	$y = -2\sin(2\pi(-1) + 4\pi)$ $= -2\sin(-2\pi + 4\pi)$ $= -2\sin 2\pi$ $= -2\cdot 0 = 0$	(–1, 0)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin(2\pi x + 4\pi)$

30. $y = -3\sin(2\pi x + 4\pi) = -3\sin(2\pi x - (-4\pi))$

The equation $y = -3\sin(2\pi x - (-4\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = -3$, $B = 2\pi$, and $C = -4\pi$. The amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate *x*-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

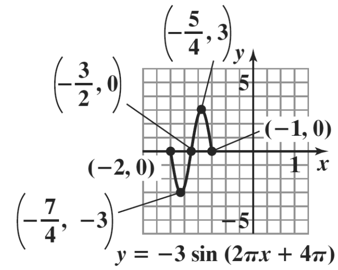
$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of *x*.

x	$y = -3\sin(2\pi x + 4\pi)$	coordinates
-2	$y = -3\sin(2\pi(-2) + 4\pi)$ $= -3\sin(-4\pi + 4\pi)$ $= -3\sin 0 = -3 \cdot 0 = 0$	$(-2, 0)$
$-\frac{7}{4}$	$y = -3\sin\left(2\pi\left(-\frac{7}{4}\right) + 4\pi\right)$ $= -3\sin\left(-\frac{7\pi}{2} + 4\pi\right)$ $= -3\sin\frac{\pi}{2} = -3 \cdot 1 = -3$	$\left(-\frac{7}{4}, -3\right)$
$-\frac{3}{2}$	$y = -3\sin\left(2\pi\left(-\frac{3}{2}\right) + 4\pi\right)$ $= -3\sin(-3\pi + 4\pi)$ $= -3\sin\pi = -3 \cdot 0 = 0$	$\left(-\frac{3}{2}, 0\right)$
$-\frac{5}{4}$	$y = -3\sin\left(2\pi\left(-\frac{5}{4}\right) + 4\pi\right)$ $= -3\sin\left(-\frac{5\pi}{2} + 4\pi\right)$ $= -3\sin\frac{3\pi}{2} = -3(-1) = 3$	$\left(-\frac{5}{4}, 3\right)$
-1	$y = -3\sin(2\pi(-1) + 4\pi)$ $= -3\sin(-2\pi + 4\pi)$ $= -3\sin 2\pi = -3 \cdot 0 = 0$	$(-1, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



31. The equation $y = 2\cos x$ is of the form $y = A\cos x$ with $A = 2$. Thus, the amplitude is $|A| = |2| = 2$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate *x*-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

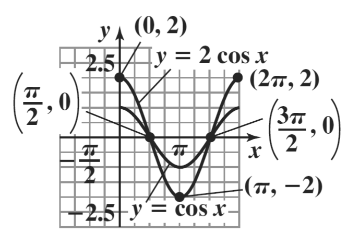
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of *x*.

x	$y = 2\cos x$	coordinates
0	$y = 2\cos 0 = 2 \cdot 1 = 2$	$(0, 2)$
$\frac{\pi}{2}$	$y = 2\cos\frac{\pi}{2} = 2 \cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = 2\cos\pi$ $= 2 \cdot (-1) = -2$	$(\pi, -2)$
$\frac{3\pi}{2}$	$y = 2\cos\frac{3\pi}{2}$ $= 2 \cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = 2\cos 2\pi$ $= 2 \cdot 1 = 2$	$(2\pi, 2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = 2\cos x$.



32. The equation $y = 3\cos x$ is of the form $y = A\cos x$ with $A = 3$. Thus, the amplitude is $|A| = |3| = 3$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

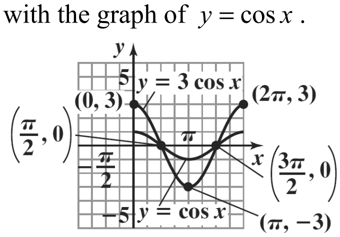
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 3\cos x$	coordinates
0	$y = 3\cos 0 = 3\cdot 1 = 3$	(0, 3)
$\frac{\pi}{2}$	$y = 3\cos\frac{\pi}{2} = 3\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = 3\cos\pi = 3\cdot(-1) = -3$	$(\pi, -3)$
$\frac{3\pi}{2}$	$y = 3\cos\frac{3\pi}{2} = 3\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = 3\cos 2\pi = 3\cdot 1 = 3$	$(2\pi, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



33. The equation $y = -2\cos x$ is of the form $y = A\cos x$ with $A = -2$. Thus, the amplitude is $|A| = |-2| = 2$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

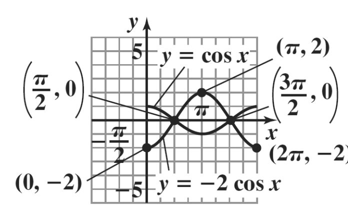
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -2\cos x$	coordinates
0	$y = -2\cos 0$ $= -2\cdot 1 = -2$	(0, –2)
$\frac{\pi}{2}$	$y = -2\cos\frac{\pi}{2}$ $= -2\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = -2\cos\pi$ $= -2\cdot(-1) = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = -2\cos\frac{3\pi}{2}$ $= -2\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = -2\cos 2\pi$ $= -2\cdot 1 = -2$	$(2\pi, -2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



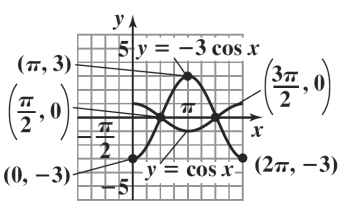
34. The equation $y = -3\cos x$ is of the form $y = A\cos x$ with $A = -3$. Thus, the amplitude is $|A| = |-3| = 3$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$
$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -3\cos x$	coordinates
0	$y = -3\cos 0 = -3\cdot 1 = -3$	(0, –3)
$\frac{\pi}{2}$	$y = -3\cos\frac{\pi}{2} = -3\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = -3\cos\pi = -3\cdot(-1) = 3$	$(\pi, 3)$
$\frac{3\pi}{2}$	$y = -3\cos\frac{3\pi}{2} = -3\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = -3\cos 2\pi = -3\cdot 1 = -3$	$(2\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



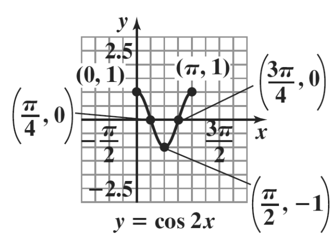
35. The equation $y = \cos 2x$ is of the form $y = A\cos Bx$ with $A = 1$ and $B = 2$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	$y = \cos 2x$	coordinates
0	$y = \cos(2\cdot 0)$ $= \cos 0 = 1$	(0, 1)
$\frac{\pi}{4}$	$y = \cos\left(2\cdot\frac{\pi}{4}\right)$ $= \cos\frac{\pi}{2} = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = \cos\left(2\cdot\frac{\pi}{2}\right)$ $= \cos\pi = -1$	$\left(\frac{\pi}{2}, -1\right)$
$\frac{3\pi}{4}$	$y = \cos\left(2\cdot\frac{3\pi}{4}\right)$ $= \cos\frac{3\pi}{2} = 0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y = \cos(2\cdot\pi)$ $= \cos 2\pi = 1$	$(\pi, 1)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



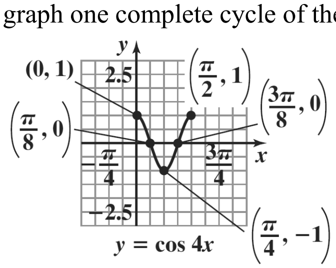
36. The equation $y = \cos 4x$ is of the form $y = A\cos Bx$ with $A = 1$ and $B = 4$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$
$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$
$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \cos 4x$	coordinates
0	$y = \cos(4\cdot 0) = \cos 0 = 1$	(0, 1)
$\frac{\pi}{8}$	$y = \cos\left(4\cdot\frac{\pi}{8}\right) = \cos\frac{\pi}{2} = 0$	$\left(\frac{\pi}{8}, 0\right)$
$\frac{\pi}{4}$	$y = \cos\left(4\cdot\frac{\pi}{4}\right) = \cos\pi = -1$	$\left(\frac{\pi}{4}, -1\right)$
$\frac{3\pi}{8}$	$y = \cos\left(4\cdot\frac{3\pi}{8}\right) = \cos\frac{3\pi}{2} = 0$	$\left(\frac{3\pi}{8}, 0\right)$
$\frac{\pi}{2}$	$y = \cos\left(4\cdot\frac{\pi}{2}\right) = \cos 2\pi = 1$	$\left(\frac{\pi}{2}, 1\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



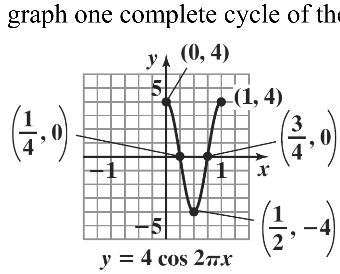
37. The equation $y = 4\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 4$ and $B = 2\pi$. Thus, the amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{1}{4} = \frac{1}{4}$
$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 4\cos 2\pi x$	coordinates
0	$y = 4\cos(2\pi\cdot 0) = 4\cos 0 = 4\cdot 1 = 4$	(0, 4)
$\frac{1}{4}$	$y = 4\cos\left(2\pi\cdot\frac{1}{4}\right) = 4\cos\frac{\pi}{2} = 4\cdot 0 = 0$	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	$y = 4\cos\left(2\pi\cdot\frac{1}{2}\right) = 4\cos\pi = 4\cdot(-1) = -4$	$\left(\frac{1}{2}, -4\right)$
$\frac{3}{4}$	$y = 4\cos\left(2\pi\cdot\frac{3}{4}\right) = 4\cos\frac{3\pi}{2} = 4\cdot 0 = 0$	$\left(\frac{3}{4}, 0\right)$
1	$y = 4\cos(2\pi\cdot 1) = 4\cos 2\pi = 4\cdot 1 = 4$	(1, 4)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



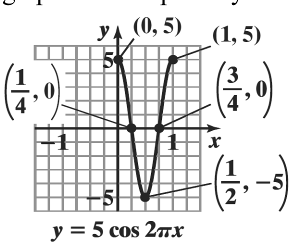
38. The equation $y = 5\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 5$ and $B = 2\pi$. Thus, the amplitude is $|A| = |5| = 5$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{1}{4} = \frac{1}{4}$
$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 5\cos 2\pi x$	coordinates
0	$y = 5\cos(2\pi \cdot 0)$ $= 5\cos 0 = 5 \cdot 1 = 5$	(0, 5)
$\frac{1}{4}$	$y = 5\cos\left(2\pi \cdot \frac{1}{4}\right)$ $= 5\cos\frac{\pi}{2} = 5 \cdot 0 = 0$	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	$y = 5\cos\left(2\pi \cdot \frac{1}{2}\right)$ $= 5\cos\pi = 5 \cdot (-1) = -5$	$\left(\frac{1}{2}, -5\right)$
$\frac{3}{4}$	$y = 5\cos\left(2\pi \cdot \frac{3\pi}{4}\right)$ $= 5\cos\frac{3\pi}{2} = 5 \cdot 0 = 0$	$\left(\frac{3}{4}, 0\right)$
1	$y = 5\cos(2\pi \cdot 1)$ $= 5\cos 2\pi = 5 \cdot 1 = 5$	(1, 5)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



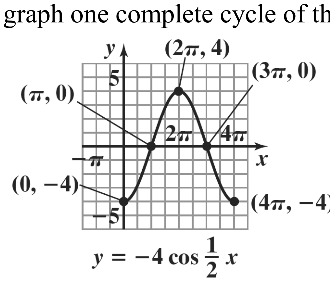
39. The equation $y = -4\cos\frac{1}{2}x$ is of the form $y = A\cos Bx$ with $A = -4$ and $B = \frac{1}{2}$. Thus, the amplitude is $|A| = |-4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \pi = \pi$
$x = \pi + \pi = 2\pi$
$x = 2\pi + \pi = 3\pi$
$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = -4\cos\frac{1}{2}x$	coordinates
0	$y = -4\cos\left(\frac{1}{2} \cdot 0\right)$ $= -4\cos 0$ $= -4 \cdot 1 = -4$	(0, –4)
π	$y = -4\cos\left(\frac{1}{2} \cdot \pi\right)$ $= -4\cos\frac{\pi}{2}$ $= -4 \cdot 0 = 0$	(π, 0)
2π	$y = -4\cos\left(\frac{1}{2} \cdot 2\pi\right)$ $= -4\cos\pi$ $= -4 \cdot (-1) = 4$	(2π, 4)
3π	$y = -4\cos\left(\frac{1}{2} \cdot 3\pi\right)$ $= -4\cos\frac{3\pi}{2}$ $= -4 \cdot 0 = 0$	(3π, 0)
4π	$y = -4\cos\left(\frac{1}{2} \cdot 4\pi\right)$ $= -4\cos 2\pi$ $= -4 \cdot 1 = -4$	(4π, – 4)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



40. The equation $y = -3\cos\frac{1}{3}x$ is of the form

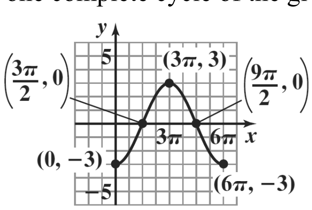
$y = A\cos Bx$ with $A = -3$ and $B = \frac{1}{3}$. Thus, the amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$
$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$
$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	$y = -3\cos\frac{1}{3}x$	coordinates
0	$y = -3\cos\left(\frac{1}{3} \cdot 0\right)$ $= -3\cos 0 = -3 \cdot 1 = -3$	$(0, -3)$
$\frac{3\pi}{2}$	$y = -3\cos\left(\frac{1}{3} \cdot \frac{3\pi}{2}\right)$ $= -3\cos\frac{\pi}{2} = -3 \cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
3π	$y = -3\cos\left(\frac{1}{3} \cdot 3\pi\right)$ $= -3\cos\pi = -3 \cdot (-1) = 3$	$(3\pi, 3)$
$\frac{9\pi}{2}$	$y = -3\cos\left(\frac{1}{3} \cdot \frac{9\pi}{2}\right)$ $= -3\cos\frac{3\pi}{2} = -3 \cdot 0 = 0$	$\left(\frac{9\pi}{2}, 0\right)$
6π	$y = -3\cos\left(\frac{1}{3} \cdot 6\pi\right)$ $= -3\cos 2\pi = -3 \cdot 1 = -3$	$(6\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\cos\frac{1}{3}x$

41. The equation $y = -\frac{1}{2}\cos\frac{\pi}{3}x$ is of the form

$y = A\cos Bx$ with $A = -\frac{1}{2}$ and $B = \frac{\pi}{3}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6$. The quarter-period is $\frac{6}{4} = \frac{3}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

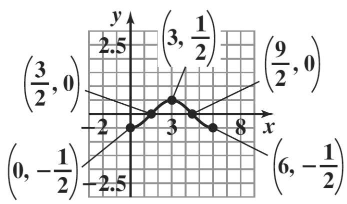
$x = 0$
$x = 0 + \frac{3}{2} = \frac{3}{2}$
$x = \frac{3}{2} + \frac{3}{2} = 3$
$x = 3 + \frac{3}{2} = \frac{9}{2}$
$x = \frac{9}{2} + \frac{3}{2} = 6$

Evaluate the function at each value of x.

x	$y = -\frac{1}{2}\cos\frac{\pi}{3}x$	coordinates
0	$y = -\frac{1}{2}\cos\left(\frac{\pi}{3} \cdot 0\right)$ $= -\frac{1}{2}\cos 0$ $= -\frac{1}{2} \cdot 1 = -\frac{1}{2}$	$\left(0, -\frac{1}{2}\right)$
$\frac{3}{2}$	$y = -\frac{1}{2}\cos\left(\frac{\pi}{3} \cdot \frac{3}{2}\right)$ $= -\frac{1}{2}\cos\frac{\pi}{2}$ $= -\frac{1}{2} \cdot 0 = 0$	$\left(\frac{3}{2}, 0\right)$
3	$y = -\frac{1}{2}\cos\left(\frac{\pi}{3} \cdot 3\right)$ $= -\frac{1}{2}\cos\pi$ $= -\frac{1}{2} \cdot (-1) = \frac{1}{2}$	$\left(3, \frac{1}{2}\right)$
$\frac{9}{2}$	$y = -\frac{1}{2}\cos\left(\frac{\pi}{3} \cdot \frac{9}{2}\right)$ $= -\frac{1}{2}\cos\frac{3\pi}{2}$ $= -\frac{1}{2} \cdot 0 = 0$	$\left(\frac{9}{2}, 0\right)$

6	$y=-\frac{1}{2}\cos\left(\frac{\pi}{3}\cdot 6\right)$ $=-\frac{1}{2}\cos 2\pi$ $=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(6,-\frac{1}{2}\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y=-\frac{1}{2}\cos\frac{\pi}{3}x$

42. The equation $y=-\frac{1}{2}\cos\frac{\pi}{4}x$ is of the form

$y=A\cos Bx$ with $A=-\frac{1}{2}$ and $B=\frac{\pi}{4}$. Thus, the

amplitude is $|A|=\left|-\frac{1}{2}\right|=\frac{1}{2}$. The period is

$\frac{2\pi}{B}=\frac{2\pi}{\frac{\pi}{4}}=2\pi\cdot\frac{4}{\pi}=8$. The quarter-period is $\frac{8}{4}=2$.

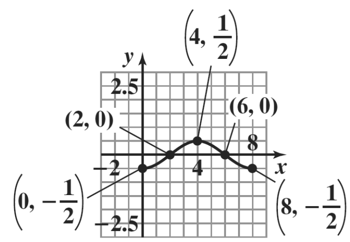
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x=0$
$x=0+2=2$
$x=2+2=4$
$x=4+2=6$
$x=6+2=8$

Evaluate the function at each value of x.

x	$y=-\frac{1}{2}\cos\frac{\pi}{4}x$	coordinates
0	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 0\right)$ $=-\frac{1}{2}\cos 0=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(0,-\frac{1}{2}\right)$
2	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 2\right)$ $=-\frac{1}{2}\cos\frac{\pi}{2}=-\frac{1}{2}\cdot 0=0$	(2, 0)
4	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 4\right)$ $=-\frac{1}{2}\cos\pi=-\frac{1}{2}\cdot(-1)=\frac{1}{2}$	$\left(4,\frac{1}{2}\right)$
6	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 6\right)$ $=-\frac{1}{2}\cos\left(\frac{3\pi}{2}\right)=-\frac{1}{2}\cdot 0=0$	(6, 0)
8	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 8\right)$ $=-\frac{1}{2}\cos 2\pi=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(8,-\frac{1}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y=-\frac{1}{2}\cos\frac{\pi}{4}x$

43. The equation $y=\cos\left(x-\frac{\pi}{2}\right)$ is of the form

$y=A\cos(Bx-C)$ with $A = 1$, and $B = 1$, and

$C=\frac{\pi}{2}$. Thus, the amplitude is $|A|=|1|=1$. The

period is $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$. The phase shift is

$\frac{C}{B}=\frac{\frac{\pi}{2}}{1}=\frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4}=\frac{\pi}{2}$. The

cycle begins at $x=\frac{\pi}{2}$. Add quarter-periods to

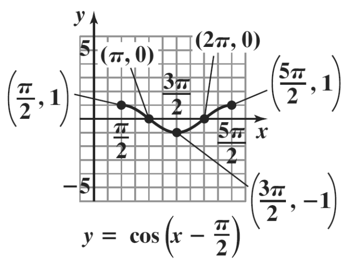
generate x-values for the key points.

$x=\frac{\pi}{2}$
$x=\frac{\pi}{2}+\frac{\pi}{2}=\pi$
$x=\pi+\frac{\pi}{2}=\frac{3\pi}{2}$
$x=\frac{3\pi}{2}+\frac{\pi}{2}=2\pi$
$x=2\pi+\frac{\pi}{2}=\frac{5\pi}{2}$

Evaluate the function at each value of x.

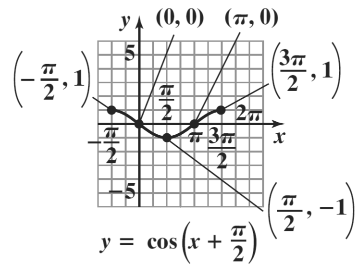
x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 1\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, -1\right)$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\left(\frac{5\pi}{2}, 1\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



x	coordinates
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, 1\right)$
0	$(0, 0)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -1\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 1\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



44. The equation $y = \cos\left(x + \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with A = 1, and B = 1, and $C = -\frac{\pi}{2}$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

45. The equation $y = 3\cos(2x - \pi)$ is of the form $y = A\cos(Bx - C)$ with A = 3, and B = 2, and $C = \pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

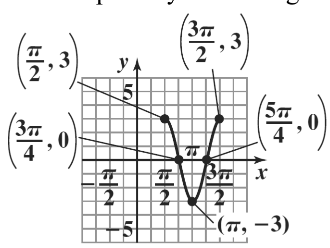
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 3\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, -3)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, 0\right)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 3\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



$y = 3\cos(2x - \pi)$

46. The equation $y = 4\cos(2x - \pi)$ is of the form $y = A\cos(Bx - C)$ with $A = 4$, and $B = 2$, and $C = \pi$. Thus, the amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

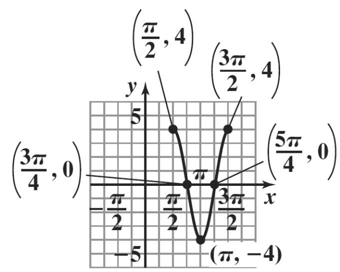
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 4\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, -4)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, 0\right)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 4\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 4\cos(2x - \pi)$

47. $y = \frac{1}{2}\cos\left(3x + \frac{\pi}{2}\right) = \frac{1}{2}\cos\left(3x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = \frac{1}{2}\cos\left(3x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\cos(Bx - C)$ with $A = \frac{1}{2}$, and $B = 3$, and $C = -\frac{\pi}{2}$. Thus, the amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{3}$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{3} = -\frac{\pi}{2} \cdot \frac{1}{3} = -\frac{\pi}{6}$. The quarter-period is $\frac{\frac{2\pi}{3}}{4} = \frac{2\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{6}$. The cycle begins at $x = -\frac{\pi}{6}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{6}$

$x = -\frac{\pi}{6} + \frac{\pi}{6} = 0$

$x = 0 + \frac{\pi}{6} = \frac{\pi}{6}$

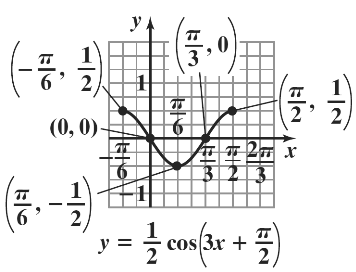
$x = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{6}$	$\left(-\frac{\pi}{6}, \frac{1}{2}\right)$
0	(0, 0)
$\frac{\pi}{6}$	$\left(\frac{\pi}{6}, -\frac{1}{2}\right)$
$\frac{\pi}{3}$	$\left(\frac{\pi}{3}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, \frac{1}{2}\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



48. $y = \frac{1}{2}\cos(2x + \pi) = \frac{1}{2}\cos(2x - (-\pi))$

The equation $y = \frac{1}{2}\cos(2x - (-\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = \frac{1}{2}$, and $B = 2$, and $C = -\pi$. Thus, the amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{-\pi}{2} = -\frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

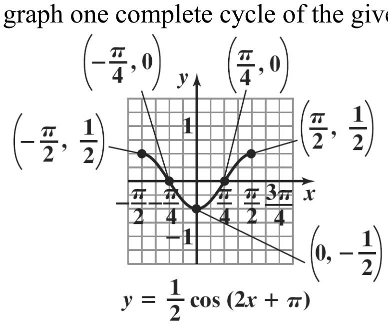
$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, \frac{1}{2}\right)$
$-\frac{\pi}{4}$	$\left(-\frac{\pi}{4}, 0\right)$
0	$\left(0, -\frac{1}{2}\right)$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, \frac{1}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



49. The equation $y = -3\cos\left(2x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -3$, and $B = 2$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$.

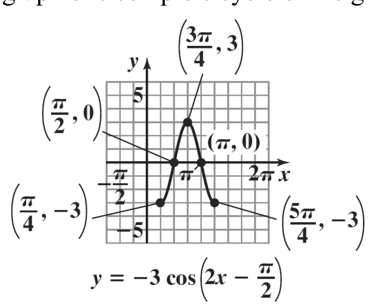
The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, -3\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 3\right)$
π	$(\pi, 0)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, -3\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



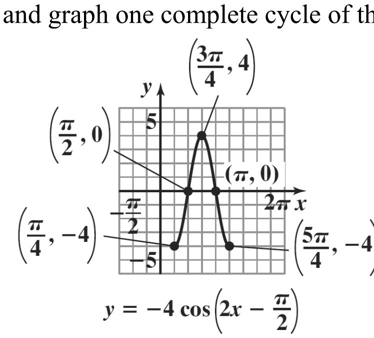
50. The equation $y = -4\cos\left(2x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -4$, and $B = 2$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |-4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, -4\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 4\right)$
π	$(\pi, 0)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, -4\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



51. $y = 2\cos(2\pi x + 8\pi) = 2\cos(2\pi x - (-8\pi))$

The equation $y = 2\cos(2\pi x - (-8\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = 2$, $B = 2\pi$, and $C = -8\pi$. Thus, the amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-8\pi}{2\pi} = -4$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -4$. Add quarter-periods to generate x-values for the key points.

$x = -4$

$x = -4 + \frac{1}{4} = -\frac{15}{4}$

$x = -\frac{15}{4} + \frac{1}{4} = -\frac{7}{2}$

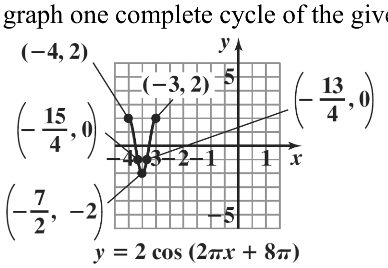
$x = -\frac{7}{2} + \frac{1}{4} = -\frac{13}{4}$

$x = -\frac{13}{4} + \frac{1}{4} = -3$

Evaluate the function at each value of x.

x	coordinates
–4	(–4, 2)
$-\frac{15}{4}$	$\left(-\frac{15}{4}, 0\right)$
$-\frac{7}{2}$	$\left(-\frac{7}{2}, -2\right)$
$-\frac{13}{4}$	$\left(-\frac{13}{4}, 0\right)$
–3	(–3, 2)

Connect the five points with a smooth curve and graph one complete cycle of the given function



52. $y = 3\cos(2\pi x + 4\pi) = 3\cos(2\pi x - (-4\pi))$

The equation $y = 3\cos(2\pi x - (-4\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = 3$, and $B = 2\pi$, and $C = -4\pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate x-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

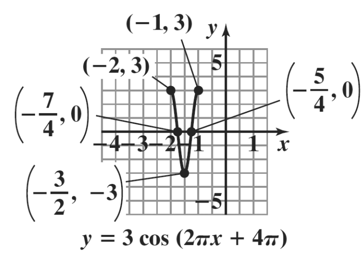
$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of x.

x	coordinates
–2	(–2, 3)
$-\frac{7}{4}$	$\left(-\frac{7}{4}, 0\right)$
$-\frac{3}{2}$	$\left(-\frac{3}{2}, -3\right)$
$-\frac{5}{4}$	$\left(-\frac{5\pi}{4}, 0\right)$
–1	(–1, 3)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



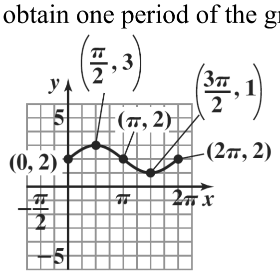
53. The graph of $y = \sin x + 2$ is the graph of $y = \sin x$ shifted up 2 units upward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$$x = 0$$
$$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$$
$$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$
$$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$
$$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$$

Evaluate the function at each value of x.

x	$y = \sin x + 2$	coordinates
0	$y = \sin 0 + 2$ $= 0 + 2 = 2$	(0, 2)
$\frac{\pi}{2}$	$y = \sin \frac{\pi}{2} + 2$ $= 1 + 2 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = \sin \pi + 2$ $= 0 + 2 = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = \sin \frac{3\pi}{2} + 2$ $= -1 + 2 = 1$	$\left(\frac{3\pi}{2}, 1\right)$
2π	$y = \sin 2\pi + 2$ $= 0 + 2 = 2$	$(2\pi, 2)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \sin x + 2$

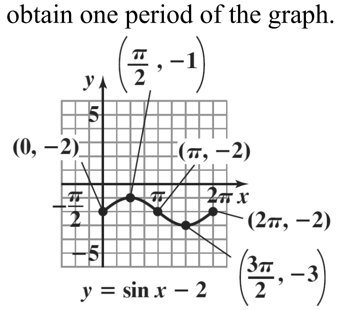
54. The graph of $y = \sin x - 2$ is the graph of $y = \sin x$ shifted 2 units downward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$$x = 0$$
$$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$$
$$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$
$$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$
$$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$$

Evaluate the function at each value of x.

x	$y = \sin x - 2$	coordinates
0	$y = \sin 0 - 2 = 0 - 2 = -2$	(0, –2)
$\frac{\pi}{2}$	$y = \sin \frac{\pi}{2} - 2 = 1 - 2 = -1$	$\left(\frac{\pi}{4}, -1\right)$
π	$y = \sin \pi - 2 = 0 - 2 = -2$	$(\pi, -2)$
$\frac{3\pi}{2}$	$y = \sin \frac{3\pi}{2} - 2 = -1 - 2 = -3$	$\left(\frac{3\pi}{2}, -3\right)$
2π	$y = \sin 2\pi - 2 = 0 - 2 = -2$	$(2\pi, -2)$

By connecting the points with a smooth curve we obtain one period of the graph.



55. The graph of $y = \cos x - 3$ is the graph of $y = \cos x$ shifted 3 units downward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

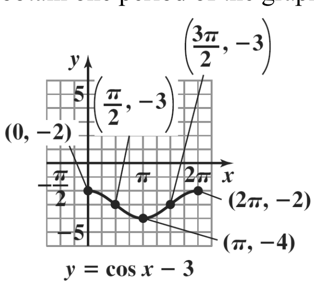
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \cos x - 3$	coordinates
0	$y = \cos 0 - 3$ $= 1 - 3 = -2$	(0, –2)
$\frac{\pi}{2}$	$y = \cos\frac{\pi}{2} - 3$ $= 0 - 3 = -3$	$\left(\frac{\pi}{2}, -3\right)$
π	$y = \cos\pi - 3$ $= -1 - 3 = -4$	$(\pi, -4)$
$\frac{3\pi}{2}$	$y = \cos\frac{3\pi}{2} - 3$ $= 0 - 3 = -3$	$\left(\frac{3\pi}{2}, -3\right)$
2π	$y = \cos 2\pi - 3$ $= 1 - 3 = -2$	$(2\pi, -2)$

By connecting the points with a smooth curve we obtain one period of the graph.



56. The graph of $y = \cos x + 3$ is the graph of $y = \cos x$ shifted 3 units upward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

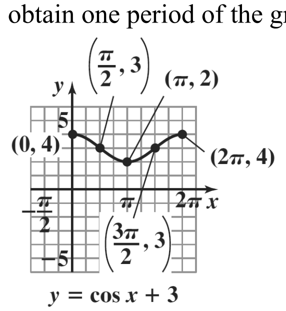
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \cos x + 3$	coordinates
0	$y = \cos 0 + 3 = 1 + 3 = 4$	(0, 4)
$\frac{\pi}{2}$	$y = \cos\frac{\pi}{2} + 3 = 0 + 3 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = \cos\pi + 3 = -1 + 3 = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = \cos\frac{3\pi}{2} + 3 = 0 + 3 = 3$	$\left(\frac{3\pi}{2}, 3\right)$
2π	$y = \cos 2\pi + 3 = 1 + 3 = 4$	$(2\pi, 4)$

By connecting the points with a smooth curve we obtain one period of the graph.



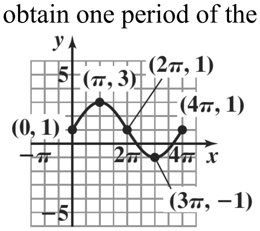
57. The graph of $y = 2\sin\frac{1}{2}x + 1$ is the graph of $y = 2\sin\frac{1}{2}x$ shifted one unit upward. The amplitude for both functions is $|2| = 2$. The period for both functions is $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \pi = \pi$
$x = \pi + \pi = 2\pi$
$x = 2\pi + \pi = 3\pi$
$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{2}x + 1$	coordinates
0	$y = 2\sin\left(\frac{1}{2}\cdot 0\right) + 1$ $= 2\sin 0 + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(0, 1)$
π	$y = 2\sin\left(\frac{1}{2}\cdot \pi\right) + 1$ $= 2\sin\frac{\pi}{2} + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(\pi, 3)$
2π	$y = 2\sin\left(\frac{1}{2}\cdot 2\pi\right) + 1$ $= 2\sin\pi + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(2\pi, 1)$
3π	$y = 2\sin\left(\frac{1}{2}\cdot 3\pi\right) + 1$ $= 2\sin\frac{3\pi}{2} + 1$ $= 2\cdot(-1) + 1$ $= -2 + 1 = -1$	$(3\pi, -1)$
4π	$y = 2\sin\left(\frac{1}{2}\cdot 4\pi\right) + 1$ $= 2\sin 2\pi + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(4\pi, 1)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = 2\sin\frac{1}{2}x + 1$

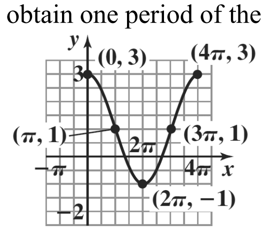
58. The graph of $y = 2\cos\frac{1}{2}x + 1$ is the graph of $y = 2\cos\frac{1}{2}x$ shifted one unit upward. The amplitude for both functions is $|2| = 2$. The period for both functions is $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \pi = \pi$
$x = \pi + \pi = 2\pi$
$x = 2\pi + \pi = 3\pi$
$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\cos\frac{1}{2}x + 1$	coordinates
0	$y = 2\cos\left(\frac{1}{2}\cdot 0\right) + 1$ $= 2\cos 0 + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(0, 3)$
π	$y = 2\cos\left(\frac{1}{2}\cdot \pi\right) + 1$ $= 2\cos\frac{\pi}{2} + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(\pi, 1)$
2π	$y = 2\cos\left(\frac{1}{2}\cdot 2\pi\right) + 1$ $= 2\cos\pi + 1$ $= 2\cdot(-1) + 1 = -2 + 1 = -1$	$(2\pi, -1)$
3π	$y = 2\cos\left(\frac{1}{2}\cdot 3\pi\right) + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(3\pi, 1)$
4π	$y = 2\cos\left(\frac{1}{2}\cdot 4\pi\right) + 1$ $= 2\cos 2\pi + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(4\pi, 3)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = 2\cos\frac{1}{2}x + 1$

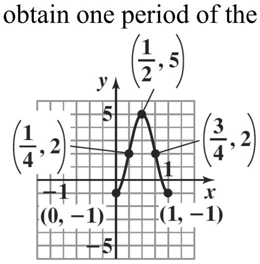
59. The graph of $y=-3\cos 2\pi x+2$ is the graph of $y=-3\cos 2\pi x$ shifted 2 units upward. The amplitude for both functions is $|-3|=3$. The period for both functions is $\frac{2\pi}{2\pi}=1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x=0$
$x=0+\frac{1}{4}=\frac{1}{4}$
$x=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$
$x=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$
$x=\frac{3}{4}+\frac{1}{4}=1$

Evaluate the function at each value of x.

x	$y=-3\cos 2\pi x+2$	coordinates
0	$y=-3\cos(2\pi\cdot 0)+2$ $=-3\cos 0+2$ $=-3\cdot 1+2$ $=-3+2=-1$	(0, –1)
$\frac{1}{4}$	$y=-3\cos\left(2\pi\cdot\frac{1}{4}\right)+2$ $=-3\cos\frac{\pi}{2}+2$ $=-3\cdot 0+2$ $=0+2=2$	$\left(\frac{1}{4}, 2\right)$
$\frac{1}{2}$	$y=-3\cos\left(2\pi\cdot\frac{1}{2}\right)+2$ $=-3\cos\pi+2$ $=-3\cdot(-1)+2$ $=3+2=5$	$\left(\frac{1}{2}, 5\right)$
$\frac{3}{4}$	$y=-3\cos\left(2\pi\cdot\frac{3}{4}\right)+2$ $=-3\cos\frac{3\pi}{2}+2$ $=-3\cdot 0+2$ $=0+2=2$	$\left(\frac{3}{4}, 2\right)$
1	$y=-3\cos(2\pi\cdot 1)+2$ $=-3\cos 2\pi+2$ $=-3\cdot 1+2$ $=-3+2=-1$	(1, –1)

By connecting the points with a smooth curve we obtain one period of the graph.



$y = -3\cos 2\pi x + 2$

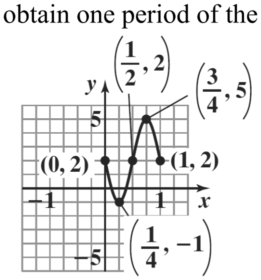
60. The graph of $y=-3\sin 2\pi x+2$ is the graph of $y=-3\sin 2\pi x$ shifted two units upward. The amplitude for both functions is $|A|=|-3|=3$. The period for both functions is $\frac{2\pi}{2\pi}=1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter–periods to generate x-values for the key points.

$x=0$
$x=0+\frac{1}{4}=\frac{1}{4}$
$x=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$
$x=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$
$x=\frac{3}{4}+\frac{1}{4}=1$

Evaluate the function at each value of x.

x	$y=-3\sin 2\pi x+2$	coordinates
0	$y=-3\sin(2\pi\cdot 0)+2$ $=-3\sin 0+2$ $=-3\cdot 0+2=0+2=2$	(0, 2)
$\frac{1}{4}$	$y=-3\sin\left(2\pi\cdot\frac{1}{4}\right)+2$ $=-3\sin\frac{\pi}{2}+2$ $=-3\cdot 1+2=-3+2=-1$	$\left(\frac{1}{4}, -1\right)$
$\frac{1}{2}$	$y=-3\sin\left(2\pi\cdot\frac{1}{2}\right)+2$ $=-3\sin\pi+2$ $=-3\cdot 0+2=0+2=2$	$\left(\frac{1}{2}, 2\right)$
$\frac{3}{4}$	$y=-3\sin\left(2\pi\cdot\frac{3}{4}\right)+2$ $=-3\sin\frac{3\pi}{2}+2$ $=-3\cdot(-1)+2=3+2=5$	$\left(\frac{3}{4}, 5\right)$
1	$y=-3\sin(2\pi\cdot 1)+2$ $=-3\sin 2\pi+2$ $=-3\cdot 0+2=0+2=2$	(1, 2)

By connecting the points with a smooth curve we obtain one period of the graph.



$y = -3\sin 2\pi x + 2$

61. Using $y = A\cos Bx$ the amplitude is 3 and $A = 3$,
The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4\pi} = \frac{1}{2}$$
$$y = A\cos Bx$$
$$y = 3\cos\frac{1}{2}x$$

62. Using $y = A\sin Bx$ the amplitude is 3 and $A = 3$, The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4\pi} = \frac{1}{2}$$
$$y = A\sin Bx$$
$$y = 3\sin\frac{1}{2}x$$

63. Using $y = A\sin Bx$ the amplitude is 2 and $A = -2$,
The period is π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$
$$y = A\sin Bx$$
$$y = -2\sin 2x$$

64. Using $y = A\cos Bx$ the amplitude is 2 and $A = -2$,
The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$
$$y = A\cos Bx$$
$$y = -2\cos 2x$$

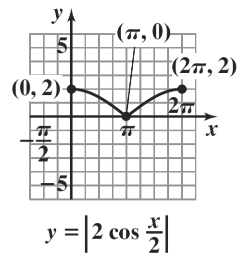
65. Using $y = A\sin Bx$ the amplitude is 2 and $A = 2$, The period is 4 and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$
$$y = A\sin Bx$$
$$y = 2\sin\left(\frac{\pi}{2}x\right)$$

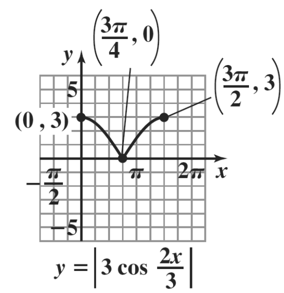
66. Using $y = A\cos Bx$ the amplitude is 2 and $A = 2$,
The period is 4 and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$
$$y = A\cos Bx$$
$$y = 2\cos\left(\frac{\pi}{2}x\right)$$

67.



68.



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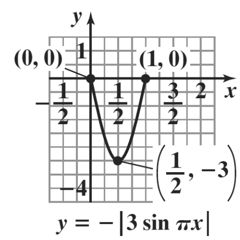

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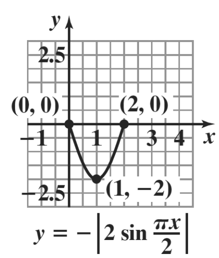

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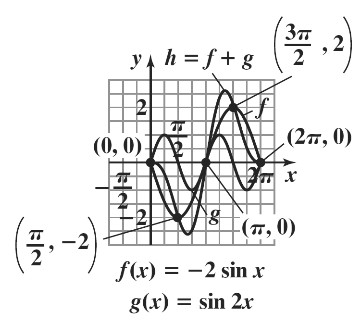
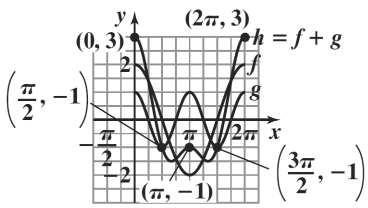
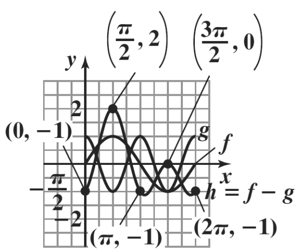


72.



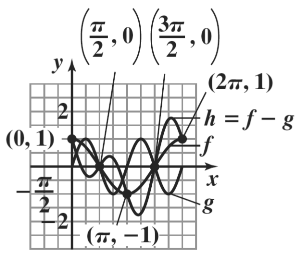
$f(x) = 2\cos x$
$g(x) = \cos 2x$

73.



$f(x) = \sin x$
$g(x) = \cos 2x$

74.



$f(x) = \cos x$
$g(x) = \sin 2x$

75. The period of the physical cycle is 33 days.

76. The period of the emotional cycle is 28 days.

77. The period of the intellectual cycle is 23 days.

78. In the month of February, the physical cycle is at a minimum on February 18. Thus, the author should not run in a marathon on February 18.

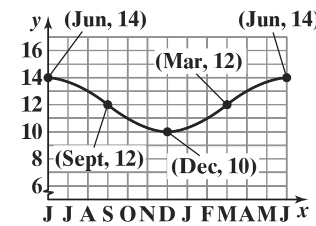
79. In the month of March, March 21 would be the best day to meet an on-line friend for the first time, because the emotional cycle is at a maximum.

80. In the month of February, the intellectual cycle is at a maximum on February 11. Thus, the author should begin writing the on February 11.

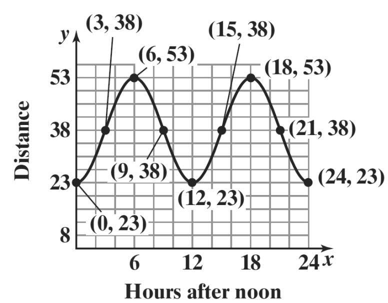
81. Answers may vary.

82. Answers may vary.

83. The information gives the five key point of the graph.
(0, 14) corresponds to June,
(3, 12) corresponds to September,
(6, 10) corresponds to December,
(9, 12) corresponds to March,
(12, 14) corresponds to June
By connecting the five key points with a smooth curve we graph the information from June of one year to June of the following year.



84. The information gives the five key points of the graph.
(0, 23) corresponds to Noon,
(3, 38) corresponds to 3 P.M.,
(6, 53) corresponds to 6 P.M.,
(9, 38) corresponds to 9 P.M.,
(12, 23) corresponds to Midnight.
By connecting the five key points with a smooth curve we graph information from noon to midnight. Extend the graph one cycle to the right to graph the information for $0 \le x \le 24$.



85. The function $y = 3\sin\frac{2\pi}{365}(x-79)+12$ is of the form $y = A\sin B\left(x - \frac{C}{B}\right) + D$ with $A = 3$ and $B = \frac{2\pi}{365}$.

a. The amplitude is $|A| = |3| = 3$.

b. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{2\pi}{365}} = 2\pi \cdot \frac{365}{2\pi} = 365$.

c. The longest day of the year will have the most hours of daylight. This occurs when the sine function equals 1.

$$y = 3\sin\frac{2\pi}{365}(x-79)+12$$
$$y = 3(1)+12$$
$$y = 15$$

There will be 15 hours of daylight.

d. The shortest day of the year will have the least hours of daylight. This occurs when the sine function equals –1.

$$y = 3\sin\frac{2\pi}{365}(x-79)+12$$
$$y = 3(-1)+12$$
$$y = 9$$

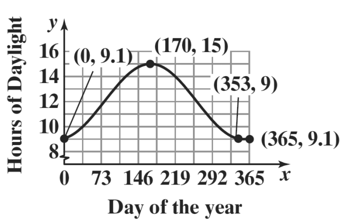
There will be 9 hours of daylight.

e. The amplitude is 3. The period is 365. The phase shift is $\frac{C}{B} = 79$. The quarter-period is $\frac{365}{4} = 91.25$. The cycle begins at $x = 79$. Add quarter-periods to find the x-values of the key points.

$$x = 79$$
$$x = 79+91.25 = 170.25$$
$$x = 170.25+91.25 = 261.5$$
$$x = 261.5+91.25 = 352.75$$
$$x = 352.75+91.25 = 444$$

Because we are graphing for $0 \le x \le 365$, we will evaluate the function for the first four x-values along with $x = 0$ and $x = 365$. Using a calculator we have the following points.
(0, 9.1) (79, 12) (170.25, 15)
(261.5, 12) (352.75, 9) (365, 9.1)

By connecting the points with a smooth curve we obtain one period of the graph, starting on January 1.

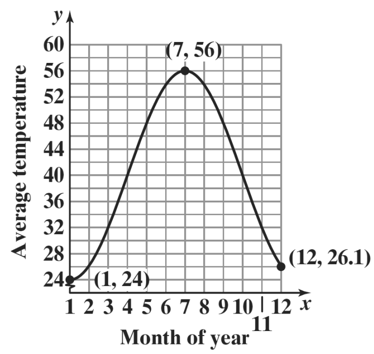


86. The function $y = 16\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right)+40$ is in the form $y = A\sin(Bx - C)+D$ with $A = 16$, $B = \frac{\pi}{6}$, and $C = \frac{2\pi}{3}$. The amplitude is $|A| = |16| = 16$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{6}} = 2\pi\cdot\frac{6}{\pi} = 12$. The phase shift is $\frac{C}{B} = \frac{\frac{2\pi}{3}}{\frac{\pi}{6}} = \frac{2\pi}{3}\cdot\frac{6}{\pi} = 4$. The quarter-period is $\frac{12}{4} = 3$.

The cycle begins at $x = 4$. Add quarter-periods to find the x-values for the key points.

$$x = 4$$
$$x = 4+3 = 7$$
$$x = 7+3 = 10$$
$$x = 10+3 = 13$$
$$x = 13+3 = 16$$

Because we are graphing for $1 \le x \le 12$, we will evaluate the function for the three x-values between 1 and 12, along with $x = 1$ and $x = 12$. Using a calculator we have the following points.
(1, 24) (4, 40) (7, 56) (10, 40) (12, 26.1)
By connecting the points with a smooth curve we obtain the graph for $1 \le x \le 12$.



The highest average monthly temperature is 56° in July.

87. Because the depth of the water ranges from a minimum of 6 feet to a maximum of 12 feet, the curve oscillates about the middle value, 9 feet. Thus, $D = 9$. The maximum depth of the water is 3 feet above 9 feet. Thus, $A = 3$. The graph shows that one complete cycle occurs in 12-0, or 12 hours. The period is 12.
Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

Substitute these values into $y = A\cos Bx + D$. The depth of the water is modeled by $y = 3\cos\frac{\pi x}{6}+9$.

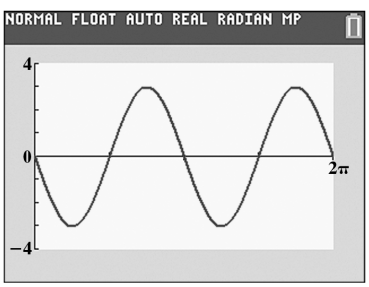
88. Because the depth of the water ranges from a minimum of 3 feet to a maximum of 5 feet, the curve oscillates about the middle value, 4 feet. Thus, $D = 4$. The maximum depth of the water is 1 foot above 4 feet. Thus, $A = 1$. The graph shows that one complete cycle occurs in 12–0, or 12 hours. The period is 12. Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

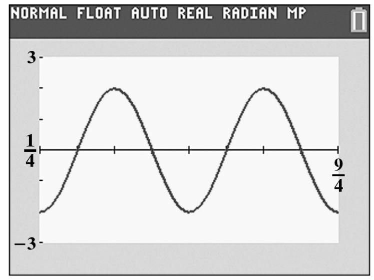
Substitute these values into $y = A\cos Bx + D$. The depth of the water is modeled by $y = \cos\frac{\pi x}{6} + 4$.

89. – 100. Answers may vary.

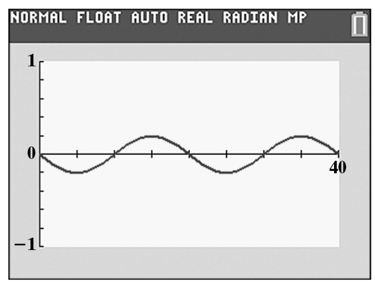
101. The function $y = 3\sin(2x+\pi) = 3\sin(2x-(-\pi))$ is of the form $y = A\sin(Bx-C)$ with $A = 3$, $B = 2$, and $C = -\pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The cycle begins at $x = \frac{C}{B} = \frac{-\pi}{2} = -\frac{\pi}{2}$. We choose $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, and $-4 \le y \le 4$ for our graph.



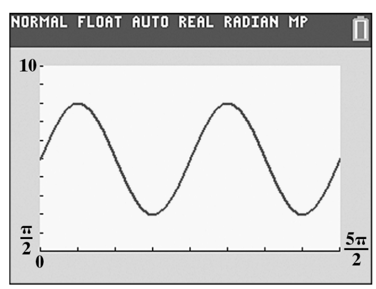
102. The function $y = -2\cos\left(2\pi x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx-C)$ with $A = -2$, $B = 2\pi$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The cycle begins at $x = \frac{C}{B} = \frac{\frac{\pi}{2}}{2\pi} = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}$. We choose $\frac{1}{4} \le x \le \frac{9}{4}$, and $-3 \le y \le 3$ for our graph.



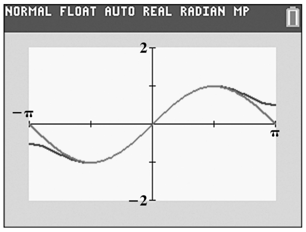
103. The function $y = 0.2\sin\left(\frac{\pi}{10}x+\pi\right) = 0.2\sin\left(\frac{\pi}{10}x-(-\pi)\right)$ is of the form $y = A\sin(Bx-C)$ with $A = 0.2$, $B = \frac{\pi}{10}$, and $C = -\pi$. The amplitude is $|A| = |0.2| = 0.2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{10}} = 2\pi\cdot\frac{10}{\pi} = 20$. The cycle begins at $x = \frac{C}{B} = \frac{-\pi}{\frac{\pi}{10}} = -\pi\cdot\frac{10}{\pi} = -10$. We choose $-10 \le x \le 30$, and $-1 \le y \le 1$ for our graph.



104. The function $y = 3\sin(2x-\pi)+5$ is of the form $y = A\cos(Bx-C)+D$ with $A = 3$, $B = 2$, $C = \pi$, and $D = 5$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The cycle begins at $x = \frac{C}{B} = \frac{\pi}{2}$. Because $D = 5$, the graph has a vertical shift 5 units upward. We choose $\frac{\pi}{2} \le x \le \frac{5\pi}{2}$, and $0 \le y \le 10$ for our graph.

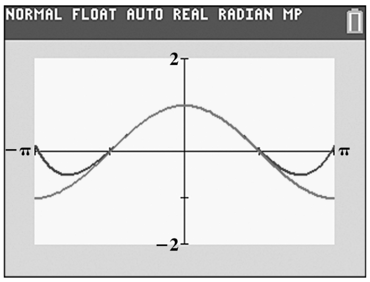


105.



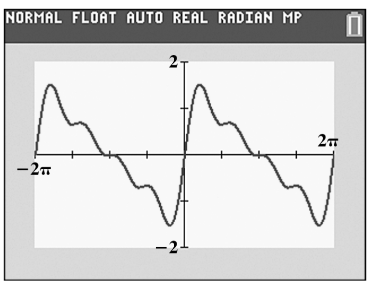
The graphs appear to be the same from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

106.



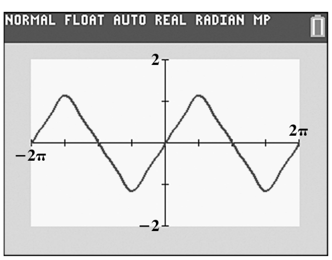
The graphs appear to be the same from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

107.



The graph is similar to $y = \sin x$, except the amplitude is greater and the curve is less smooth.

108.

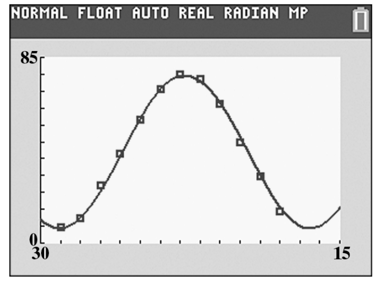


The graph is very similar to $y = \sin x$, except not smooth.

109. **a.** see part **c**.

b. $y = 22.61\sin(0.50x - 2.04) + 57.17$

c. graph for parts **a** and **c**:



110. Answers may vary.

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: It may be easier to start at the highest point.

113. makes sense

114. makes sense

115. **a.** Since $A = 3$ and $D = -2$, the maximum will occur at $3 - 2 = 1$ and the minimum will occur at $-3 - 2 = -5$. Thus the range is $[-5, 1]$

Viewing rectangle: $\left[-\frac{\pi}{6}, \frac{23\pi}{6}, \frac{\pi}{6}\right]$ by $[-5, 1, 1]$

b. Since $A = 1$ and $D = -2$, the maximum will occur at $1 - 2 = -1$ and the minimum will occur at $-1 - 2 = -3$. Thus the range is $[-3, -1]$

Viewing rectangle: $\left[-\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{6}\right]$ by $[-3, -1, 1]$

116. $A = \pi$

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{1} = 2\pi$$

$$\frac{C}{B} = \frac{C}{2\pi} = -2$$

$$C = -4\pi$$

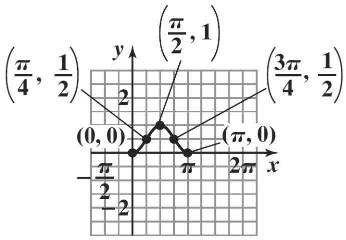
$$y = A\cos(Bx - C)$$

$$y = \pi\cos(2\pi x + 4\pi)$$

or

$$y = \pi\cos\left[2\pi(x + 2)\right]$$

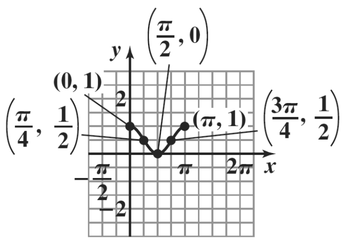
117. $y = \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$



$y = \sin^2 x$

or $y = \frac{1}{2} - \frac{1}{2}\cos 2x$

118. $y = \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$



$y = \cos^2 x$

or $y = \frac{1}{2} + \frac{1}{2}\cos 2x$

119. Answers may vary.

120.

2	2	–7	–1
		4	–6
	2	–3	–7

the equation of the slant asymptote is $y = 2x - 3$.

121.

$$8^{x+5} = 4^{x-1}$$
$$\left(2^3\right)^{x+5} = \left(2^2\right)^{x-1}$$
$$2^{3x+15} = 2^{2x-2}$$
$$3x + 15 = 2x - 2$$
$$x = -17$$

The solution set is $\{-17\}$.

122. $\log_2(2x+1) - \log_2(x-2) = 1$

$$\log_2 \frac{2x+1}{x-2} = 1$$
$$\frac{2x+1}{x-2} = 2^1$$
$$2x + 1 = 2x - 4$$
$$1 = -4$$

This is a false statement, thus the solution set is { }.

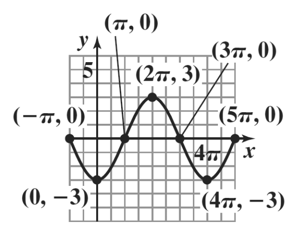
123.

$$-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$$
$$-\frac{\pi}{2} - \frac{\pi}{4} < x + \frac{\pi}{4} - \frac{\pi}{4} < \frac{\pi}{2} - \frac{\pi}{4}$$
$$-\frac{2\pi}{4} - \frac{\pi}{4} < x < \frac{2\pi}{4} - \frac{\pi}{4}$$
$$-\frac{3\pi}{4} < x < \frac{\pi}{4}$$

$\left\{x \middle| -\frac{3\pi}{4} < x < \frac{\pi}{4}\right\}$ or $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$

124. $\dfrac{-\frac{3\pi}{4} + \frac{\pi}{4}}{2} = \dfrac{-\frac{2\pi}{4}}{2} = \dfrac{-\frac{\pi}{2}}{2} = -\dfrac{\pi}{4}$

125. a.



$y = -3\cos\frac{x}{2}$

b. The reciprocal function is undefined.

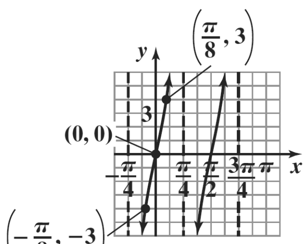
Section 4.6

Check Point Exercises

1. Solve the equations $2x = -\frac{\pi}{2}$ and $2x = \frac{\pi}{2}$

$x = -\frac{\pi}{4}$ $\qquad x = \frac{\pi}{4}$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$. Midway between these asymptotes is $x = 0$. An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –3 and 3. Use the two asymptotes, the x-intercept, and the points midway between to graph one period of $y = 3\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$. In order to graph for $-\frac{\pi}{4} < x < \frac{3\pi}{4}$, Continue the pattern and extend the graph another full period to the right.



$y = 3\tan 2x$

2. Solve the equations

$x - \frac{\pi}{2} = -\frac{\pi}{2}$ and $x - \frac{\pi}{2} = \frac{\pi}{2}$

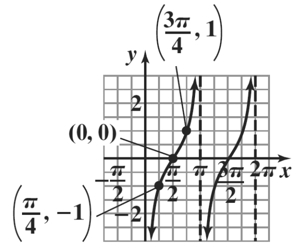
$x = \frac{\pi}{2} - \frac{\pi}{2}$ $\qquad x = \frac{\pi}{2} + \frac{\pi}{2}$

$x = 0$ $\qquad x = \pi$

Thus, two consecutive asymptotes occur at $x = 0$ and $x = \pi$.

x-intercept $= \frac{0+\pi}{2} = \frac{\pi}{2}$

An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = 0$ and $x = \pi$, to graph one full period of $y = \tan\left(x - \frac{\pi}{2}\right)$ from 0 to π. Continue the pattern and extend the graph another full period to the right.



$y = \tan\left(x - \frac{\pi}{2}\right)$

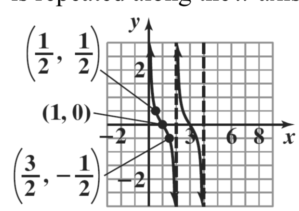
3. Solve the equations

$\frac{\pi}{2}x = 0$ and $\frac{\pi}{2}x = \pi$

$x = 0$ $\qquad x = \frac{\pi}{\frac{\pi}{2}}$

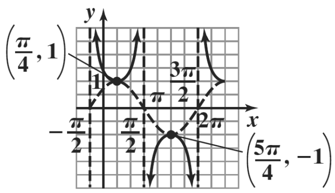
$x = 2$

Two consecutive asymptotes occur at $x = 0$ and $x = 2$. Midway between $x = 0$ and $x = 2$ is $x = 1$. An x-intercept is 1 and the graph passes through (1, 0). Because the coefficient of the cotangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $-\frac{1}{2}$ and $\frac{1}{2}$. Use the two consecutive asymptotes, $x = 0$ and $x = 2$, to graph one full period of $y = \frac{1}{2}\cot\frac{\pi}{2}x$. The curve is repeated along the x-axis one full period as shown.



$y = \frac{1}{2}\cot\frac{\pi}{2}x$

4. The x-intercepts of $y=\sin\left(x+\frac{\pi}{4}\right)$ correspond to vertical asymptotes of $y=\csc\left(x+\frac{\pi}{4}\right)$.

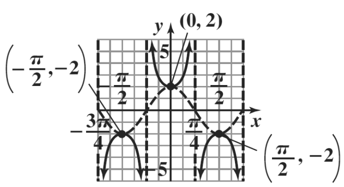


$y=\csc\left(x+\frac{\pi}{4}\right)$

5. Graph the reciprocal cosine function, $y=2\cos 2x$. The equation is of the form $y=A\cos Bx$ with $A = 2$ and $B = 2$.
amplitude: $|A|=|2|=2$

period: $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$

Use quarter-periods, $\frac{\pi}{4}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4}$, and π. Evaluating the function at each value of x, the key points are $(0,2)$, $\left(\frac{\pi}{4},0\right)$, $\left(\frac{\pi}{2},-2\right)$, $\left(\frac{3\pi}{4},0\right)$, $(\pi,2)$. In order to graph for $-\frac{3\pi}{4}\le x\le\frac{3\pi}{4}$, Use the first four points and extend the graph $-\frac{3\pi}{4}$ units to the left. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y=2\sec 2x$.



$y = 2\sec 2x$

Concept and Vocabulary Check 4.6

1. $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$; $-\frac{\pi}{4}$; $\frac{\pi}{4}$

2. $(0,\pi)$; 0; π

3. (0, 2); 0; 2

4. $\left(-\frac{\pi}{4},\frac{3\pi}{4}\right)$; $-\frac{\pi}{4}$; $\frac{3\pi}{4}$

5. $3\sin 2x$

6. $y=2\cos\pi x$

7. false

8. true

Exercise Set 4.6

1. The graph has an asymptote at $x=-\frac{\pi}{2}$.

The phase shift, $\frac{C}{B}$, from $\frac{\pi}{2}$ to $-\frac{\pi}{2}$ is $-\pi$ units.

Thus, $\frac{C}{B}=\frac{C}{1}=-\pi$
$C=-\pi$

The function with $C=-\pi$ is $y=\tan(x+\pi)$.

2. The graph has an asymptote at $x=0$.

The phase shift, $\frac{C}{B}$, from $\frac{\pi}{2}$ to 0 is $-\frac{\pi}{2}$ units. Thus,

$\frac{C}{B}=\frac{C}{1}=-\frac{\pi}{2}$
$C=-\frac{\pi}{2}$

The function with $C=-\frac{\pi}{2}$ is $y=\tan\left(x+\frac{\pi}{2}\right)$.

3. The graph has an asymptote at $x=\pi$.

$\pi=\frac{\pi}{2}+C$
$C=\frac{\pi}{2}$

The function is $y=-\tan\left(x-\frac{\pi}{2}\right)$.

4. The graph has an asymptote at $\frac{\pi}{2}$.

There is no phase shift. Thus, $\frac{C}{B}=\frac{C}{1}=0$
$C=0$

The function with $C = 0$ is $y=-\tan x$.

5. Solve the equations $\frac{x}{4} = -\frac{\pi}{2}$ and $\frac{x}{4} = \frac{\pi}{2}$

$$x = \left(-\frac{\pi}{2}\right)4 \qquad x = \left(\frac{\pi}{2}\right)4$$

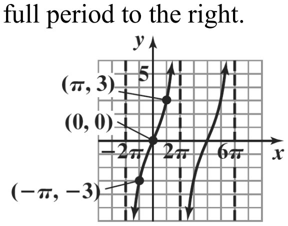
$$x = -2\pi \qquad x = 2\pi$$

Thus, two consecutive asymptotes occur at $x = -2\pi$ and $x = 2\pi$.

x-intercept $= \frac{-2\pi + 2\pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –3 and 3. Use the two consecutive asymptotes, $x = -2\pi$ and $x = 2\pi$, to graph one full period of $y = 3\tan\frac{x}{4}$ from -2π to 2π.

Continue the pattern and extend the graph another full period to the right.



$y = 3\tan\frac{x}{4}$

6. Solve the equations

$$\frac{x}{4} = -\frac{\pi}{2} \quad \text{and} \quad \frac{x}{4} = \frac{\pi}{2}$$

$$x = \left(-\frac{\pi}{2}\right)4 \qquad x = \left(\frac{\pi}{2}\right)4$$

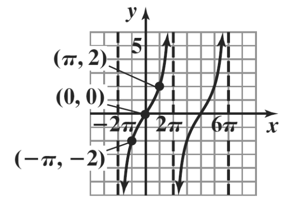
$$x = -2\pi \qquad x = 2\pi$$

Thus, two consecutive asymptotes occur at $x = -2\pi$ and $x = 2\pi$.

x-intercept $= \frac{-2\pi + 2\pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = -2\pi$ and $x = 2\pi$, to graph one full period of $y = 2\tan\frac{x}{4}$ from -2π to 2π.

Continue the pattern and extend the graph another full period to the right.



$y = 2\tan\frac{x}{4}$

7. Solve the equations $2x = -\frac{\pi}{2}$ and $2x = \frac{\pi}{2}$

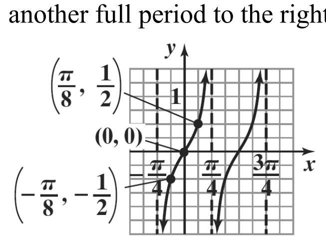
$$x = \frac{-\frac{\pi}{2}}{2} \qquad x = \frac{\frac{\pi}{2}}{2}$$

$$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

x-intercept $= \frac{-\frac{\pi}{4} + \frac{\pi}{4}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $-\frac{1}{2}$ and $\frac{1}{2}$. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = \frac{1}{2}\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$. Continue the pattern and extend the graph another full period to the right.



$y = \frac{1}{2}\tan 2x$

8. Solve the equations

$$2x = -\frac{\pi}{2} \quad \text{and} \quad 2x = \frac{\pi}{2}$$

$$x = \frac{-\frac{\pi}{2}}{2} \qquad x = \frac{\frac{\pi}{2}}{2}$$

$$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$$

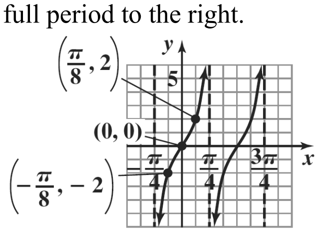
Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

x-intercept $= \dfrac{-\frac{\pi}{4} + \frac{\pi}{4}}{2} = \dfrac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2.

Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = 2\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$.

Continue the pattern and extend the graph another full period to the right.



$y = 2\tan 2x$

9. Solve the equations

$$\frac{1}{2}x = -\frac{\pi}{2} \quad \text{and} \quad \frac{1}{2}x = \frac{\pi}{2}$$

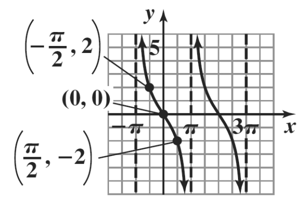
$$x = \left(-\frac{\pi}{2}\right)2 \qquad x = \left(\frac{\pi}{2}\right)2$$

$$x = -\pi \qquad x = \pi$$

Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

x-intercept $= \dfrac{-\pi + \pi}{2} = \dfrac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is –2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, to graph one full period of $y = -2\tan\frac{1}{2}x$ from $-\pi$ to π. Continue the pattern and extend the graph another full period to the right.



$y = -2\tan\frac{1}{2}x$

10. Solve the equations

$$\frac{1}{2}x = -\frac{\pi}{2} \quad \text{and} \quad \frac{1}{2}x = \frac{\pi}{2}$$

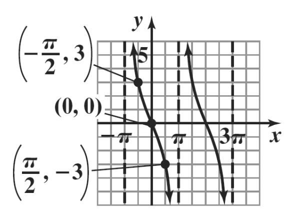
$$x = \left(-\frac{\pi}{2}\right)2 \qquad x = \left(\frac{\pi}{2}\right)2$$

$$x = -\pi \qquad x = \pi$$

Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

x-intercept $= \dfrac{-\pi + \pi}{2} = \dfrac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is –3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 3 and –3. Use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, to graph one full period of $y = -3\tan\frac{1}{2}x$ from $-\pi$ to π. Continue the pattern and extend the graph another full period to the right.



$y = -3\tan\frac{1}{2}x$

11. Solve the equations

$$x - \pi = -\frac{\pi}{2} \quad \text{and} \quad x - \pi = \frac{\pi}{2}$$

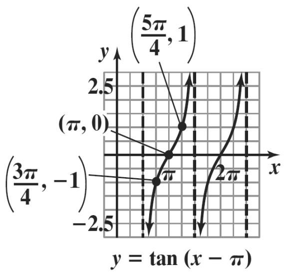
$$x = -\frac{\pi}{2} + \pi \qquad x = \frac{\pi}{2} + \pi$$

$$x = \frac{\pi}{2} \qquad x = \frac{3\pi}{2}$$

Thus, two consecutive asymptotes occur at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

x-intercept $= \dfrac{\frac{\pi}{2} + \frac{3\pi}{2}}{2} = \dfrac{\frac{4\pi}{2}}{2} = \dfrac{4\pi}{4} = \pi$

An x-intercept is π and the graph passes through $(\pi, 0)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$, to graph one full period of $y = \tan(x - \pi)$ from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$. Continue the pattern and extend the graph another full period to the right.



12. Solve the equations

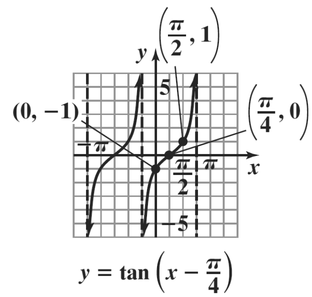
$$x - \frac{\pi}{4} = -\frac{\pi}{2} \qquad \text{and} \qquad x - \frac{\pi}{4} = \frac{\pi}{2}$$
$$x = -\frac{2\pi}{4} + \frac{\pi}{4} \qquad\qquad x = \frac{2\pi}{4} + \frac{\pi}{4}$$
$$x = -\frac{\pi}{4} \qquad\qquad x = \frac{3\pi}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{4} + \frac{3\pi}{4}}{2} = \frac{\frac{2\pi}{4}}{2} = \frac{\pi}{4}$$

An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, to graph one full period of $y = \tan\left(x - \frac{\pi}{4}\right)$ from 0 to π.

Continue the pattern and extend the graph another full period to the right.



13. There is no phase shift. Thus,

$$\frac{C}{B} = \frac{C}{1} = 0$$
$$C = 0$$

Because the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1, $A = –1$. The function with $C = 0$ and $A = –1$ is $y = -\cot x$.

14. The graph has an asymptote at $\frac{\pi}{2}$. The phase shift, $\frac{C}{B}$, from 0 to $\frac{\pi}{2}$ is $\frac{\pi}{2}$ units.

Thus, $\frac{C}{B} = \frac{C}{1} = \frac{\pi}{2}$

$$C = \frac{\pi}{2}$$

The function with $C = \frac{\pi}{2}$ is $y = -\cot\left(x - \frac{\pi}{2}\right)$.

15. The graph has an asymptote at $-\frac{\pi}{2}$. The phase shift, $\frac{C}{B}$, from 0 to $-\frac{\pi}{2}$ is $-\frac{\pi}{2}$ units. Thus, $\frac{C}{B} = \frac{C}{1} = -\frac{\pi}{2}$

$$C = -\frac{\pi}{2}$$

The function with $C = -\frac{\pi}{2}$ is $y = \cot\left(x + \frac{\pi}{2}\right)$.

16. The graph has an asymptote at $-\pi$. The phase shift, $\frac{C}{B}$, from 0 to $-\pi$ is $-\pi$ units.

Thus, $\frac{C}{B} = \frac{C}{1} = -\pi$

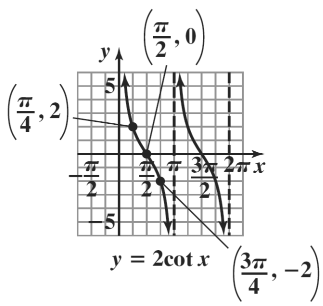
$$C = -\pi$$

The function with $C = -\pi$ is $y = \cot(x + \pi)$.

17. Solve the equations $x = 0$ and $x = \pi$. Two consecutive asymptotes occur at $x = 0$ and $x = \pi$.

x-intercept $= \frac{0+\pi}{2} = \frac{\pi}{2}$

An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = 0$ and $x = \pi$, to graph one full period of $y = 2\cot x$. The curve is repeated along the x-axis one full period as shown.

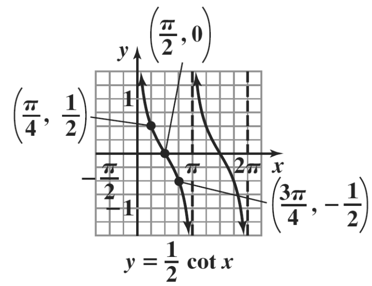


18. Solve the equations
$x = 0$ and $x = \pi$
Two consecutive asymptotes occur at $x = 0$ and $x = \pi$.

x-intercept $= \frac{0+\pi}{2} = \frac{\pi}{2}$

An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the cotangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $\frac{1}{2}$ and $-\frac{1}{2}$. Use the two consecutive asymptotes, $x = 0$ and $x = \pi$, to graph one full period of $y = \frac{1}{2}\cot x$.

The curve is repeated along the x-axis one full period as shown.



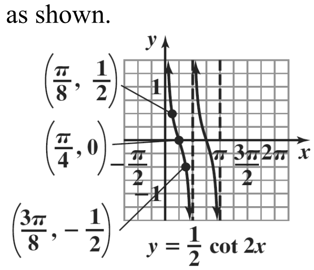
19. Solve the equations $2x = 0$ and $2x = \pi$

$x = 0 \qquad x = \frac{\pi}{2}$

Two consecutive asymptotes occur at $x = 0$ and $x = \frac{\pi}{2}$.

x-intercept $= \frac{0+\frac{\pi}{2}}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$. Because the coefficient of the cotangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $\frac{1}{2}$ and $-\frac{1}{2}$. Use the two consecutive asymptotes, $x = 0$ and $x = \frac{\pi}{2}$, to graph one full period of $y = \frac{1}{2}\cot 2x$. The curve is repeated along the x-axis one full period as shown.



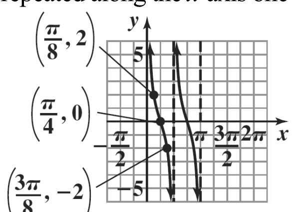
20. Solve the equations $2x = 0$ and $2x = \pi$

$x = 0 \qquad x = \dfrac{\pi}{2}$

Two consecutive asymptotes occur at $x = 0$ and $x = \dfrac{\pi}{2}$.

$x\text{-intercept} = \dfrac{0 + \frac{\pi}{2}}{2} = \dfrac{\frac{\pi}{2}}{2} = \dfrac{\pi}{4}$

An x-intercept is $\dfrac{\pi}{4}$ and the graph passes through $\left(\dfrac{\pi}{4}, 0\right)$. Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = 0$ and $x = \dfrac{\pi}{2}$, to graph one full period of $y = 2\cot 2x$. The curve is repeated along the x-axis one full period as shown.



$y = 2\cot 2x$

21. Solve the equations $\dfrac{\pi}{2}x = 0$ and $\dfrac{\pi}{2}x = \pi$

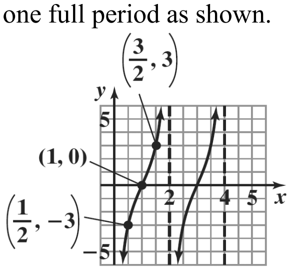
$x = 0 \qquad x = \dfrac{\pi}{\frac{\pi}{2}}$

$x = 2$

Two consecutive asymptotes occur at $x = 0$ and $x = 2$.

$x\text{-intercept} = \dfrac{0 + 2}{2} = \dfrac{2}{2} = 1$

An x-intercept is 1 and the graph passes through (1, 0). Because the coefficient of the cotangent is –3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –3 and 3. Use the two consecutive asymptotes, $x = 0$ and $x = 2$, to graph one full period of $y = -3\cot\dfrac{\pi}{2}x$. The curve is repeated along the x-axis one full period as shown.



$y = -3\cot\frac{\pi}{2}x$

22. Solve the equations $\dfrac{\pi}{4}x = 0$ and $\dfrac{\pi}{4}x = \pi$

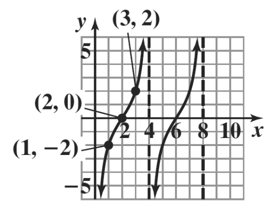
$x = 0 \qquad x = \dfrac{\pi}{\frac{\pi}{4}}$

$x = 4$

Two consecutive asymptotes occur at $x = 0$ and $x = 4$.

$x\text{-intercept} = \dfrac{0 + 4}{2} = \dfrac{4}{2} = 2$

An x-intercept is 2 and the graph passes through $(2, 0)$. Because the coefficient of the cotangent is –2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = 0$ and $x = 4$, to graph one full period of $y = -2\cot\dfrac{\pi}{4}x$. The curve is repeated along the x-axis one full period as shown.



$y = -2\cot\frac{\pi}{4}x$

23. Solve the equations

$x + \dfrac{\pi}{2} = 0$ and $x + \dfrac{\pi}{2} = \pi$

$x = 0 - \dfrac{\pi}{2} \qquad x = \pi - \dfrac{\pi}{2}$

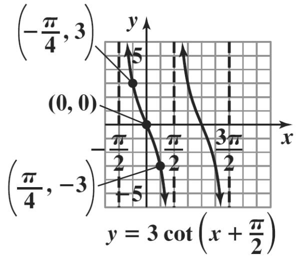
$x = -\dfrac{\pi}{2} \qquad x = \dfrac{\pi}{2}$

Two consecutive asymptotes occur at $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$.

$x\text{-intercept} = \dfrac{-\frac{\pi}{2} + \frac{\pi}{2}}{2} = \dfrac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the cotangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 3 and –3. Use the two consecutive asymptotes, $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$, to graph one full period of $y = 3\cot\left(x + \dfrac{\pi}{2}\right)$.

The curve is repeated along the x-axis one full period as shown.



24. Solve the equations

$$x + \frac{\pi}{4} = 0 \qquad \text{and} \qquad x + \frac{\pi}{4} = \pi$$

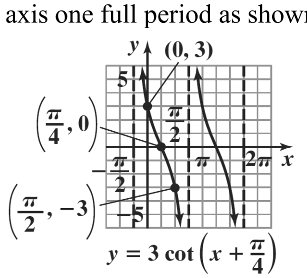
$$x = 0 - \frac{\pi}{4} \qquad\qquad x = \pi - \frac{\pi}{4}$$

$$x = -\frac{\pi}{4} \qquad\qquad x = \frac{3\pi}{4}$$

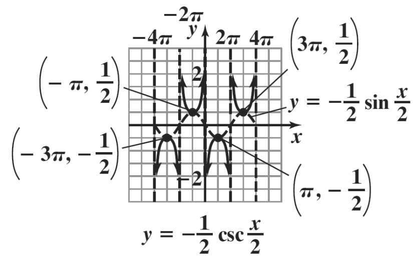
Two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{4} + \frac{3\pi}{4}}{2} = \frac{\frac{2\pi}{4}}{2} = \frac{\pi}{4}$$

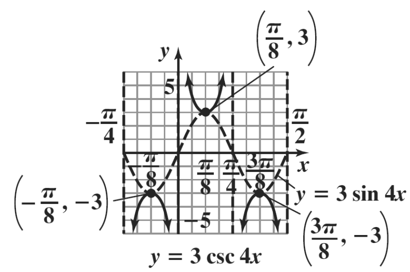
An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$. Because the coefficient of the cotangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 3 and –3. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, to graph one full period of $y = 3\cot\left(x + \frac{\pi}{4}\right)$. The curve is repeated along the x-axis one full period as shown.



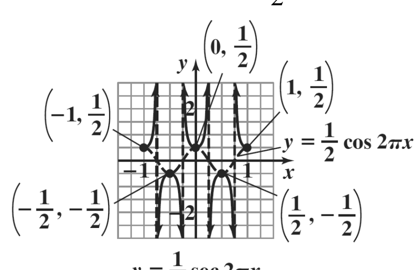
25. The x-intercepts of $y = -\frac{1}{2}\sin\frac{x}{2}$ corresponds to vertical asymptotes of $y = -\frac{1}{2}\csc\frac{x}{2}$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y = -\frac{1}{2}\csc\frac{x}{2}$.



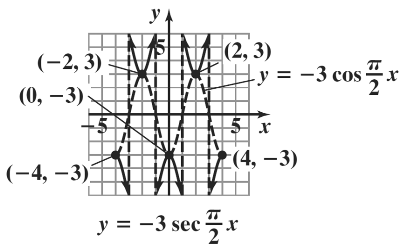
26. The x-intercepts of $y = 3\sin 4x$ correspond to vertical asymptotes of $y = 3\csc 4x$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y = 3\csc 4x$.



27. The x-intercepts of $y = \frac{1}{2}\cos 2\pi x$ corresponds to vertical asymptotes of $y = \frac{1}{2}\sec 2\pi x$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y = \frac{1}{2}\sec 2\pi x$.



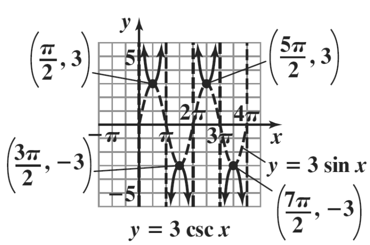
28. The x-intercepts of $y = -3\cos\frac{\pi}{2}x$ correspond to vertical asymptotes of $y = -3\sec\frac{\pi}{2}x$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y = -3\sec\frac{\pi}{2}x$.



29. Graph the reciprocal sine function, $y = 3\sin x$. The equation is of the form $y = A\sin Bx$ with $A = 3$ and $B = 1$.
amplitude: $|A| = |3| = 3$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

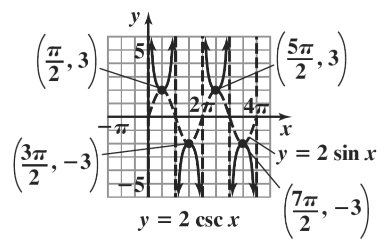
Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π. Evaluating the function at each value of x, the key points are $(0, 0)$, $\left(\frac{\pi}{2}, 3\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -3\right)$, and $(2\pi, 0)$. Use these key points to graph $y = 3\sin x$ from 0 to 2π. Extend the graph one cycle to the right.
Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\csc x$.



30. Graph the reciprocal sine function, $y = 2\sin x$. The equation is of the form $y = A\sin Bx$ with $A = 2$ and $B = 1$.
amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π. Evaluating the function at each value of x, the key points are $(0, 0)$, $\left(\frac{\pi}{2}, 2\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -2\right)$, and $(2\pi, 0)$.
Use these key points to graph $y = 2\sin x$ from 0 to 2π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\csc x$.



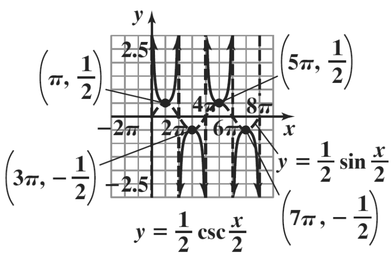
31. Graph the reciprocal sine function, $y = \frac{1}{2}\sin\frac{x}{2}$. The equation is of the form $y = A\sin Bx$ with $A = \frac{1}{2}$ and $B = \frac{1}{2}$.

amplitude: $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Use quarter-periods, π, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\pi, 2\pi, 3\pi$, and 4π. Evaluating the function at each value of x, the key points are $(0, 0)$, $\left(\pi, \frac{1}{2}\right)$, $(2\pi, 0)$, $\left(3\pi, -\frac{1}{2}\right)$, and $(4\pi, 0)$. Use these key points to graph $y = \frac{1}{2}\sin\frac{x}{2}$ from 0 to 4π.

Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \frac{1}{2}\csc\frac{x}{2}$.



32. Graph the reciprocal sine function, $y = \frac{3}{2}\sin\frac{x}{4}$. The equation is of the form $y = A\sin Bx$ with $A = \frac{3}{2}$ and $B = \frac{1}{4}$.

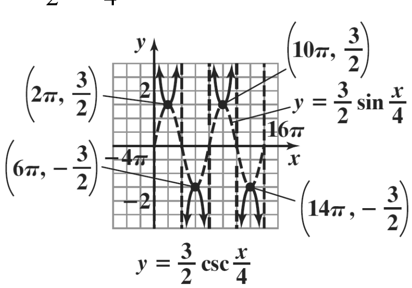
amplitude: $|A| = \left|\frac{3}{2}\right| = \frac{3}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$

Use quarter-periods, 2π, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, 2\pi, 4\pi, 6\pi,$ and 8π. Evaluating the function at each value of x, the key points are $(0, 0), \left(2\pi, \frac{3}{2}\right), (4\pi, 0), \left(6\pi, -\frac{3}{2}\right),$ and $(8\pi, 0)$.

Use these key points to graph $y = \frac{3}{2}\sin\frac{x}{4}$ from 0 to 8π. Extend the graph one cycle to the right.

Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \frac{3}{2}\csc\frac{x}{4}$.

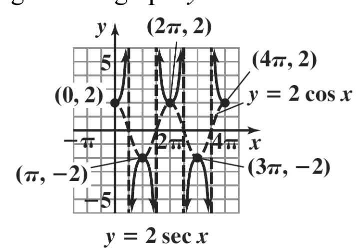


33. Graph the reciprocal cosine function, $y = 2\cos x$. The equation is of the form $y = A\cos Bx$ with $A = 2$ and $B = 1$.

amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$. Evaluating the function at each value of x, the key points are (0, 2), $\left(\frac{\pi}{2}, 0\right)$, $(\pi, -2)$, $\left(\frac{3\pi}{2}, 0\right)$, and $(2\pi, 2)$. Use these key points to graph $y = 2\cos x$ from 0 to 2π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec x$.



34. Graph the reciprocal cosine function, $y = 3\cos x$. The equation is of the form $y = A\cos Bx$ with $A = 3$ and $B = 1$.

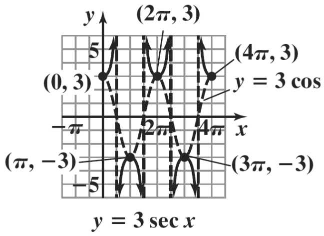
amplitude: $|A| = |3| = 3$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2},$ and 2π. Evaluating the function at each value of x, the key points are $(0, 3), \left(\frac{\pi}{2}, 0\right), (\pi, -3), \left(\frac{3\pi}{2}, 0\right), (2\pi, 3)$.

Use these key points to graph $y = 3\cos x$ from 0 to 2π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function.

Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\sec x$.

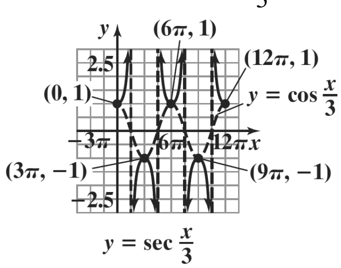


35. Graph the reciprocal cosine function, $y = \cos\frac{x}{3}$. The equation is of the form $y = A\cos Bx$ with $A = 1$ and $B = \frac{1}{3}$.

amplitude: $|A| = |1| = 1$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$

Use quarter-periods, $\frac{6\pi}{4} = \frac{3\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{3\pi}{2}, 3\pi, \frac{9\pi}{2}$, and 6π. Evaluating the function at each value of x, the key points are $(0, 1)$, $\left(\frac{3\pi}{2}, 0\right)$, $(3\pi, -1)$, $\left(\frac{9\pi}{2}, 0\right)$, and $(6\pi, 1)$. Use these key points to graph $y = \cos\frac{x}{3}$ from 0 to 6π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \sec\frac{x}{3}$.



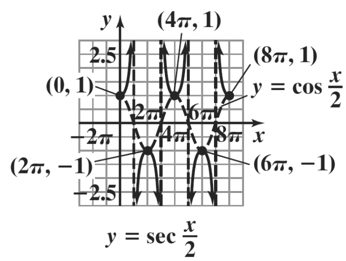
36. Graph the reciprocal cosine function, $y = \cos\frac{x}{2}$. The equation is of the form $y = A\cos Bx$ with $A = 1$ and $B = \frac{1}{2}$.

amplitude: $|A| = |1| = 1$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Use quarter-periods, π, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \pi, 2\pi, 3\pi$, and 4π. Evaluating the function at each value of x, the key points are $(0, 1), (\pi, 0), (2\pi, -1), (3\pi, 0)$, and $(4\pi, 1)$.

Use these key points to graph $y = \cos\frac{x}{2}$ from 0 to 4π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \sec\frac{x}{2}$.



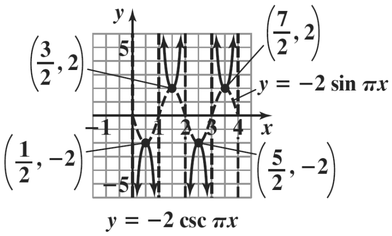
37. Graph the reciprocal sine function, $y = -2\sin\pi x$. The equation is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$.

amplitude: $|A| = |-2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2}$, and 2. Evaluating the function at each value of x, the key points are $(0, 0)$, $\left(\frac{1}{2}, -2\right)$, $(1, 0)$, $\left(\frac{3}{2}, 2\right)$, and $(2, 0)$. Use these key points to graph $y = -2\sin\pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function.

Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -2\csc \pi x$.



38. Graph the reciprocal sine function, $y = -\frac{1}{2}\sin \pi x$.

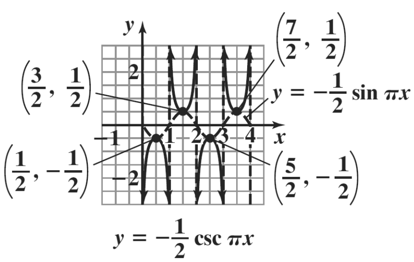
The equation is of the form $y = A\sin Bx$ with $A = -\frac{1}{2}$ and $B = \pi$.

amplitude: $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2}$, and 2. Evaluating the function at each value of x, the key points are $(0, 0), \left(\frac{1}{2}, -\frac{1}{2}\right), (1, 0), \left(\frac{3}{2}, \frac{1}{2}\right)$, and $(2, 0)$.

Use these key points to graph $y = -\frac{1}{2}\sin \pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{1}{2}\csc \pi x$.



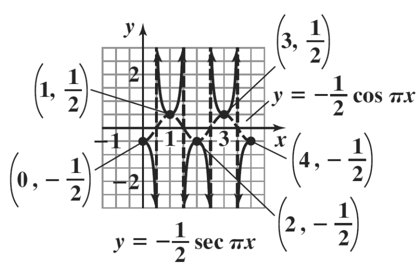
39. Graph the reciprocal cosine function, $y = -\frac{1}{2}\cos \pi x$.

The equation is of the form $y = A\cos Bx$ with $A = -\frac{1}{2}$ and $B = \pi$.

amplitude: $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2}$, and 2. Evaluating the function at each value of x, the key points are $\left(0, -\frac{1}{2}\right)$, $\left(\frac{1}{2}, 0\right), \left(1, \frac{1}{2}\right), \left(\frac{3}{2}, 0\right), \left(2, -\frac{1}{2}\right)$. Use these key points to graph $y = -\frac{1}{2}\cos \pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{1}{2}\sec \pi x$.



40. Graph the reciprocal cosine function, $y = -\frac{3}{2}\cos \pi x$.

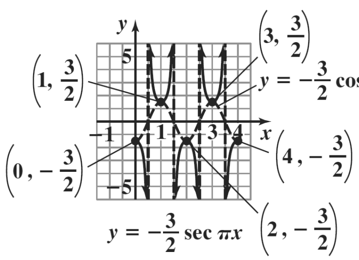
The equation is of the form $y = A\cos Bx$ with $A = -\frac{3}{2}$ and $B = \pi$.

amplitude: $|A| = \left|-\frac{3}{2}\right| = \frac{3}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4}=\frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2},$ and 2. Evaluating the function at each value of x, the key points are $\left(0, -\frac{3}{2}\right), \left(\frac{1}{2}, 0\right), \left(1, \frac{3}{2}\right), \left(\frac{3}{2}, 0\right), \left(2, -\frac{3}{2}\right)$.

Use these key points to graph $y=-\frac{3}{2}\cos \pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y=-\frac{3}{2}\sec \pi x$.

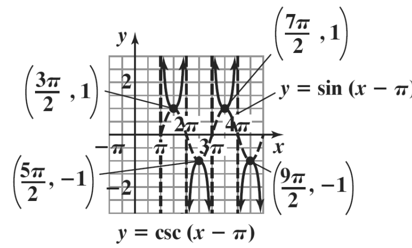


41. Graph the reciprocal sine function, $y=\sin(x-\pi)$. The equation is of the form $y=A\sin(Bx-C)$ with $A = 1$, and $B = 1$, and $C=\pi$.
amplitude: $|A|=|1|=1$

period: $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$

phase shift: $\frac{C}{B}=\frac{\pi}{1}=\pi$

Use quarter-periods, $\frac{2\pi}{4}=\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x=\pi$, the x-values are $\pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2},$ and 3π. Evaluating the function at each value of x, the key points are $(\pi, 0), \left(\frac{3\pi}{2}, 1\right), (2\pi, 0), \left(\frac{5\pi}{2}, -1\right), (3\pi, 0)$. Use these key points to graph $y=\sin(x-\pi)$ from π to 3π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y=\csc(x-\pi)$.



42. Graph the reciprocal sine function, $y=\sin\left(x-\frac{\pi}{2}\right)$.

The equation is of the form $y=A\sin(Bx-C)$ with $A=1$, $B=1$, and $C=\frac{\pi}{2}$.

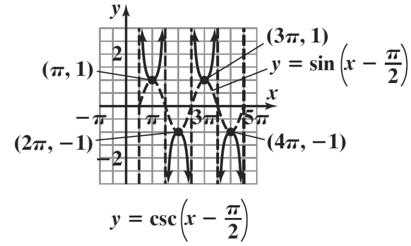
amplitude: $|A|=|1|=1$

period: $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$

phase shift: $\frac{C}{B}=\frac{\frac{\pi}{2}}{1}=\frac{\pi}{2}$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x=\frac{\pi}{2}$, the x-values are $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi,$ and $\frac{5\pi}{2}$. Evaluating the function at each value of x, the key points are $\left(\frac{\pi}{2}, 0\right), (\pi, 1), \left(\frac{3\pi}{2}, 0\right), (2\pi, -1),$ and $\left(\frac{5\pi}{2}, 0\right)$.

Use these key points to graph $y=\sin\left(x-\frac{\pi}{2}\right)$ from $\frac{\pi}{2}$ to $\frac{5\pi}{2}$. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y=\csc\left(x-\frac{\pi}{2}\right)$.



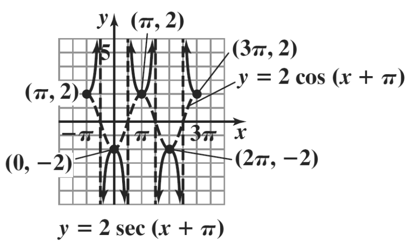
43. Graph the reciprocal cosine function, $y = 2\cos(x+\pi)$. The equation is of the form $y = A\cos(Bx+C)$ with $A = 2$, $B = 1$, and $C = -\pi$.

amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

phase shift: $\frac{C}{B} = \frac{-\pi}{1} = -\pi$

Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = -\pi$, the x-values are $-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}$, and π. Evaluating the function at each value of x, the key points are $(-\pi, 2)$, $\left(-\frac{\pi}{2}, 0\right)$, $(0, -2)$, $\left(\frac{\pi}{2}, 0\right)$, and $(\pi, 2)$. Use these key points to graph $y = 2\cos(x+\pi)$ from $-\pi$ to π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec(x+\pi)$.



44. Graph the reciprocal cosine function, $y = 2\cos\left(x+\frac{\pi}{2}\right)$. The equation is of the form $y = A\cos(Bx+C)$ with $A = 2$ and $B = 1$, and $C = -\frac{\pi}{2}$.

amplitude: $|A| = |2| = 2$

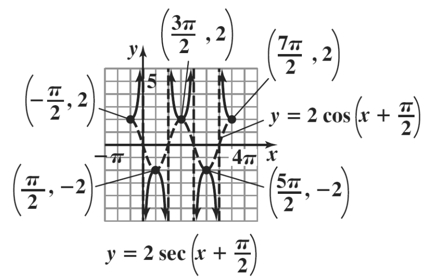
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

phase shift: $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$

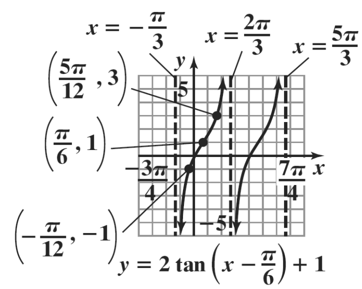
Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = -\frac{\pi}{2}$, the x-values are $-\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$.

Evaluating the function at each value of x, the key points are $\left(-\frac{\pi}{2}, 2\right)$, $(0, 0)$, $\left(\frac{\pi}{2}, -2\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, 2\right)$.

Use these key points to graph $y = 2\cos\left(x+\frac{\pi}{2}\right)$ from $-\frac{\pi}{2}$ to $\frac{3\pi}{2}$. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec\left(x+\frac{\pi}{2}\right)$.



45.



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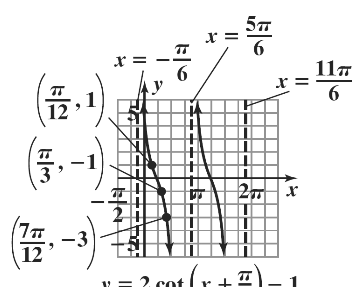

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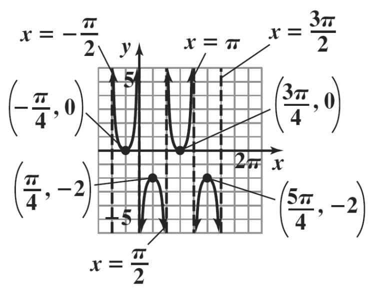
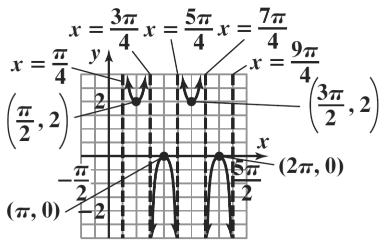


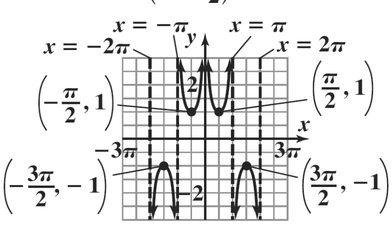
$y = \sec\left(2x + \frac{\pi}{2}\right) - 1$

48.



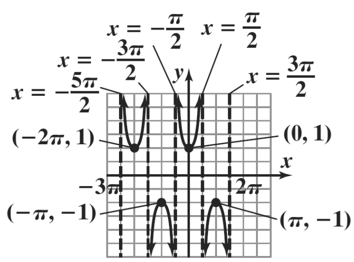
$y = \csc\left(2x - \frac{\pi}{2}\right) + 1$

49.



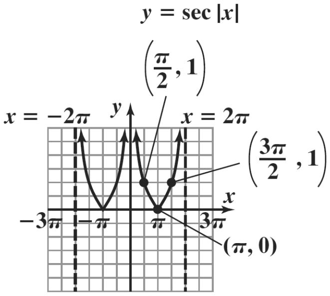
$y = \csc|x|$

50.



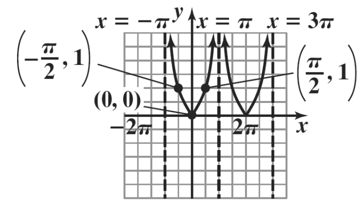
$y = \sec|x|$

51.



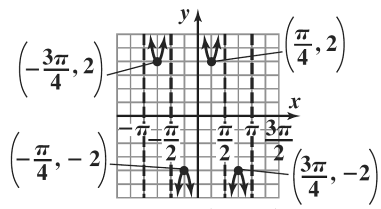
$y = \left|\cot \frac{1}{2}x\right|$

52.



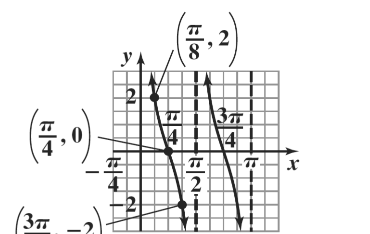
$y = \left|\tan \frac{1}{2}x\right|$

53. $y = (f \circ h)(x) = f(h(x)) = 2\sec\left(2x - \frac{\pi}{2}\right)$



$y = 2\sec\left(2x - \frac{\pi}{2}\right)$

54. $y = (g \circ h)(x) = g(h(x)) = -2\tan\left(2x - \frac{\pi}{2}\right)$



$y = -2\tan\left(2x - \frac{\pi}{2}\right)$

55. Use a graphing utility with $y_1 = \tan x$ and $y_2 = -1$. For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.

$x = -\frac{5\pi}{4},\ -\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{7\pi}{4}$
$x \approx -3.93,\ -0.79,\ 2.36,\ 5.50$

56. Use a graphing utility with $y_1 = 1/\tan x$ and $y_2 = -1$. For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.

$x = -\frac{5\pi}{4},\ -\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{7\pi}{4}$
$x \approx -3.93,\ -0.79,\ 2.36,\ 5.50$

57. Use a graphing utility with $y_1 = 1/\sin x$ and $y_2 = 1$. For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.

$x = -\frac{3\pi}{2},\ \frac{\pi}{2}$
$x \approx -4.71,\ 1.57$

58. Use a graphing utility with $y_1 = 1/\cos x$ and $y_2 = 1$. For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.
$x = -2\pi,\ 0,\ 2\pi$
$x \approx -6.28,\ 0,\ 6.28$

59. $d = 12\tan 2\pi t$

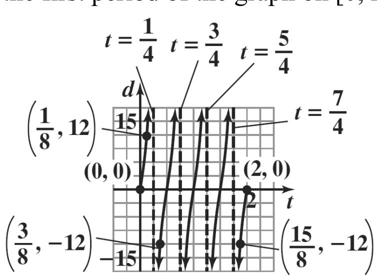
a. Solve the equations

$$2\pi t = -\frac{\pi}{2} \quad \text{and} \quad 2\pi t = \frac{\pi}{2}$$
$$t = \frac{-\frac{\pi}{2}}{2\pi} \qquad\qquad t = \frac{\frac{\pi}{2}}{2\pi}$$
$$t = -\frac{1}{4} \qquad\qquad t = \frac{1}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{1}{4}$ and $x = \frac{1}{4}$.

x-intercept $= \dfrac{-\frac{1}{4} + \frac{1}{4}}{2} = \dfrac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 12, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –12 and 12. Use the two consecutive asymptotes, $x = -\frac{1}{4}$ and $x = \frac{1}{4}$, to graph one full period of $d = 12\tan 2\pi t$. To graph on [0, 2], continue the pattern and extend the graph to 2. (Do not use the left hand side of the first period of the graph on [0, 2].)



$d = 12\tan 2\pi t$

b. The function is undefined for t = 0.25, 0.75, 1.25, and 1.75.
The beam is shining parallel to the wall at these times.

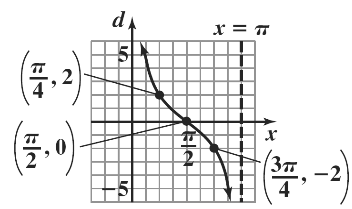
60. In a right triangle the angle of elevation is one of the acute angles, the adjacent leg is the distance d, and the opposite leg is 2 mi. Use the cotangent function.

$$\cot x = \frac{d}{2}$$
$$d = 2\cot x$$

Use the equations $x = 0$ and $x = \pi$.

Two consecutive asymptotes occur at $x = 0$ and $x = \pi$. Midway between $x = 0$ and $x = \pi$ is $x = \frac{\pi}{2}$.

An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = 0$ and $x = \pi$, to graph $y = 2\cot x$ for $0 < x < \pi$.



$d = 2\cot x$

61. Use the function that relates the acute angle with the hypotenuse and the adjacent leg, the secant function.

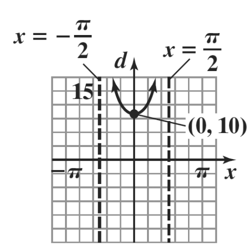
$$\sec x = \frac{d}{10}$$
$$d = 10\sec x$$

Graph the reciprocal cosine function, $y = 10\cos x$.
The equation is of the form $y = A\cos Bx$ with $A = 10$ and $B = 1$.
amplitude: $|A| = |10| = 10$

period: $\dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$

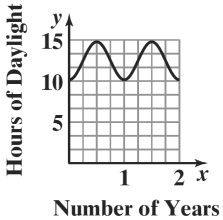
For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, use the x-values $-\frac{\pi}{2}$, 0, and $\frac{\pi}{2}$ to find the key points $\left(-\frac{\pi}{2}, 0\right)$, (0, 10), and $\left(\frac{\pi}{2}, 0\right)$.

Connect these points with a smooth curve, then draw vertical asymptotes through the x-intercepts, and use them as guides to graph $d = 10\sec x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

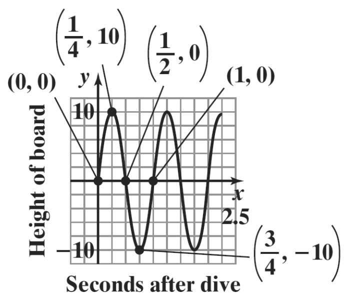


$d = 10\sec x$

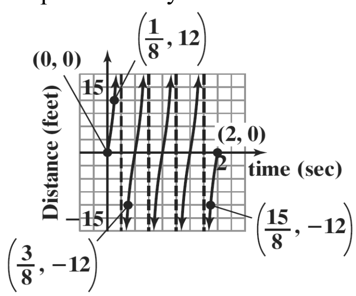
62. Graphs will vary.



63.



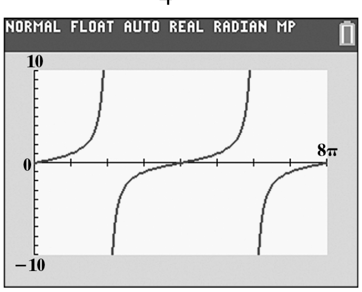
64. Graphs will vary.



65. – 76. Answers may vary.

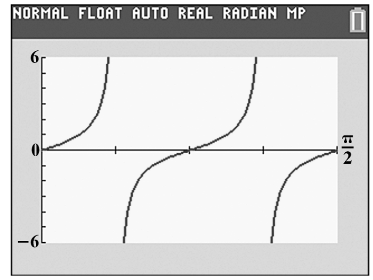
77. period: $\frac{\pi}{B} = \frac{\pi}{\frac{1}{4}} = \pi \cdot 4 = 4\pi$

Graph $y = \tan\frac{x}{4}$ for $0 \le x \le 8\pi$.



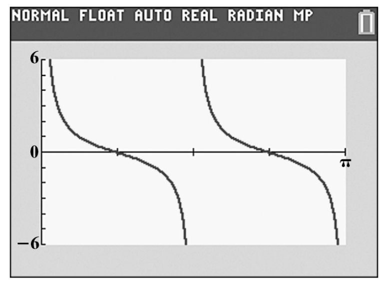
78. period: $\frac{\pi}{B} = \frac{\pi}{4}$

Graph $y = \tan 4x$ for $0 \le x \le \frac{\pi}{2}$.



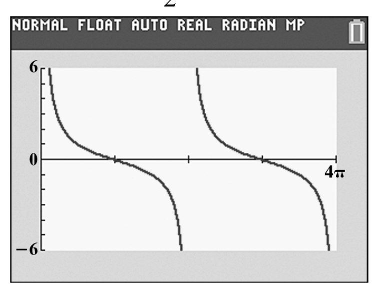
79. period: $\frac{\pi}{B} = \frac{\pi}{2}$

Graph $y = \cot 2x$ for $0 \le x \le \pi$.



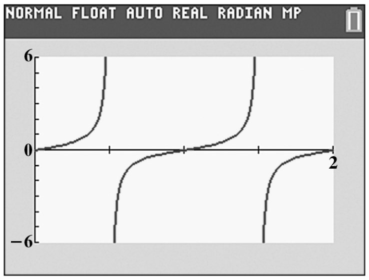
80. period: $\frac{\pi}{B} = \frac{\pi}{\frac{1}{2}} = \pi \cdot 2 = 2\pi$

Graph $y = \cot\frac{x}{2}$ for $0 \le x \le 4\pi$.



81. period: $\frac{\pi}{B} = \frac{\pi}{\pi} = 1$

Graph $y = \frac{1}{2}\tan \pi x$ for $0 \le x \le 2$.



82. Solve the equations

$$\pi x + 1 = -\frac{\pi}{2} \quad \text{and} \quad \pi x + 1 = \frac{\pi}{2}$$

$$\pi x = -\frac{\pi}{2} - 1 \qquad \pi x = \frac{\pi}{2} - 1$$

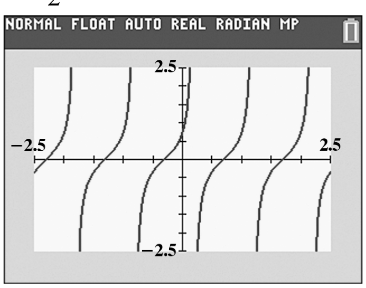
$$x = \frac{\frac{-\pi}{2} - 1}{\pi} \qquad x = \frac{\frac{\pi}{2} - 1}{\pi}$$

$$x = \frac{-\pi - 2}{2\pi} \qquad x = \frac{\pi - 2}{2\pi}$$

$$x \approx -0.82 \qquad x \approx 0.18$$

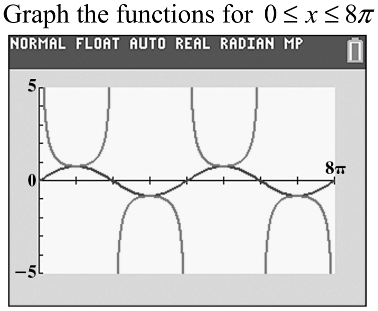
period: $\frac{\pi}{B} = \frac{\pi}{\pi} = 1$

Thus, we include $-0.82 \le x \le 1.18$ in our graph of $y = \frac{1}{2}\tan(\pi x + 1)$, and graph for $-0.85 \le x \le 1.2$.



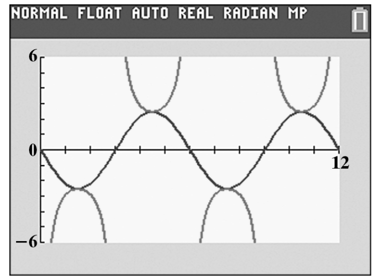
83. period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Graph the functions for $0 \le x \le 8\pi$.



84. period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6$

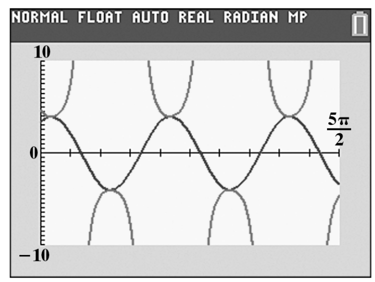
Graph the functions for $0 \le x \le 12$.



85. period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

phase shift: $\frac{C}{B} = \frac{\frac{\pi}{6}}{2} = \frac{\pi}{12}$

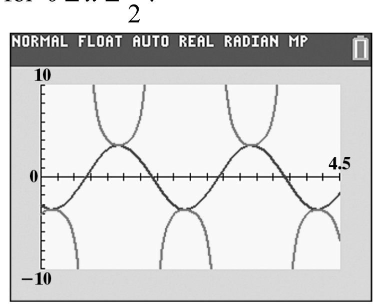
Thus, we include $\frac{\pi}{12} \le x \le \frac{25\pi}{12}$ in our graph, and graph for $0 \le x \le \frac{5\pi}{2}$.

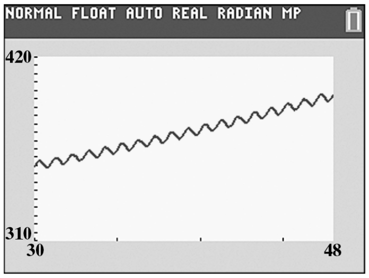


86. period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

phase shift: $\frac{C}{B} = \frac{\frac{\pi}{6}}{\pi} = \frac{\pi}{6} \cdot \frac{1}{\pi} = \frac{1}{6}$

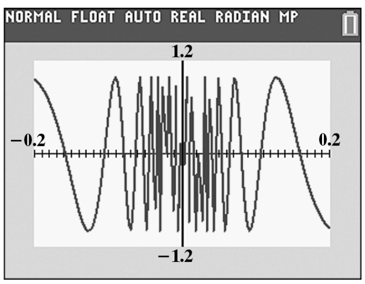
Thus, we include $\frac{1}{6} \le x \le \frac{25}{6}$ in our graph, and graph for $0 \le x \le \frac{9}{2}$.



87. 

The graph shows that carbon dioxide concentration rises and falls each year, but over all the concentration increased from 1990 to 2008.

88. $y = \sin\frac{1}{x}$



The graph is oscillating between 1 and –1.
The oscillation is faster as x gets closer to 0.
Explanations may vary.

89. makes sense

90. makes sense

91. does not make sense; Explanations will vary. Sample explanation: To obtain a cosecant graph, you can use a sine graph.

92. does not make sense; Explanations will vary. Sample explanation: To model a cyclical temperature, use sine or cosine.

93. The graph has the shape of a cotangent function with consecutive asymptotes at

$x = 0$ and $x = \frac{2\pi}{3}$. The period is $\frac{2\pi}{3} - 0 = \frac{2\pi}{3}$. Thus,

$$\frac{\pi}{B} = \frac{2\pi}{3}$$
$$2\pi B = 3\pi$$
$$B = \frac{3\pi}{2\pi} = \frac{3}{2}$$

The points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 1 and –1. Thus, $A = 1$. There is no phase shift. Thus, $C = 0$. An equation for this graph is $y = \cot\frac{3}{2}x$.

94. The graph has the shape of a secant function.
The reciprocal function has amplitude $|A| = 1$. The period is $\frac{8\pi}{3}$. Thus, $\frac{2\pi}{B} = \frac{8\pi}{3}$

$$8\pi B = 6\pi$$
$$B = \frac{6\pi}{8\pi} = \frac{3}{4}$$

There is no phase shift. Thus, $C = 0$. An equation for the reciprocal function is $y = \cos\frac{3}{4}x$. Thus, an equation for this graph is $y = \sec\frac{3}{4}x$.

95. The range shows that $A = 2$.
Since the period is 3π, the coefficient of x is given by B where $\frac{2\pi}{B} = 3\pi$

$$\frac{2\pi}{B} = 3\pi$$
$$3B\pi = 2\pi$$
$$B = \frac{2}{3}$$

Thus, $y = 2\csc\frac{2x}{3}$

96. The range shows that $A = \pi$.
Since the period is 2, the coefficient of x is given by B where $\frac{2\pi}{B} = 2$

$$\frac{2\pi}{B} = 2$$
$$2B = 2\pi$$
$$B = \pi$$

Thus, $y = \pi\csc\pi x$

97. **a.** Since A=1, the range is $(-\infty, -1] \cup [1, \infty)$

Viewing rectangle: $\left[-\frac{\pi}{6}, \pi, \frac{7\pi}{6}\right]$ by $[-3, 3, 1]$

b. Since A=3, the range is $(-\infty, -3] \cup [3, \infty)$

Viewing rectangle: $\left[-\frac{1}{2}, \frac{7}{2}, 1\right]$ by $[-6, 6, 1]$

98. $y = 2^{-x}\sin x$

2^{-x} decreases the amplitude as x gets larger.
Examples may vary.

99. $\pm\frac{1}{2},\ \pm 1,\ \pm 2$ are possible rational zeros

1	2	–5	1	2
		2	–3	–2
	2	–3	–2	0

1 is a zero.

$2x^2 - 3x + 2 = 0$

$(2x+1)(x-2) = 0$

$x = -\frac{1}{2}$ or $x = 2$

The zeros are $-\frac{1}{2}, 1,$ and $2.$

100. Let x = the number of bridge crossings at which the costs of the two plans are the same.

$$\underbrace{8x}_{\text{No Pass}} = \underbrace{36+5x}_{\text{Discount Pass}}$$

$8x - 5x = 36$

$3x = 36$

$x = 12$

The two plans cost the same for 12 bridge crossings. The cost will be $8(12)$ or \$96.

101. $m = \frac{4-(-2)}{-3-(-1)} = \frac{6}{-2} = -3$,

so the slope is –3.

Using the point (–1, –2), we get the following point-slope equation:

$y - y_1 = m(x - x_1)$

$y-(-2) = -3[x-(-1)]$

$y + 2 = -3(x+1)$

Using the point (–3, 4), we get the following point-slope equation:

$y - y_1 = m(x - x_1)$

$y - 4 = -3[x-(-3)]$

$y - 4 = -3(x+3)$

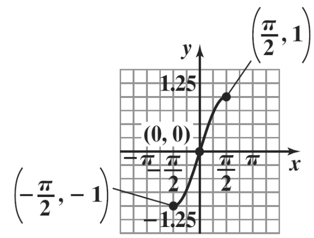
Solve the equation for y:

$y - 4 = -3(x+3)$

$y - 4 = -3x - 9$

$y = -3x - 5.$

102. a.



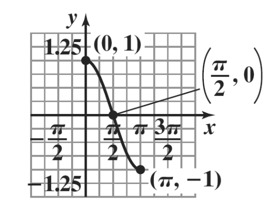
$y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

b. yes; Explanations will vary.

c. The angle is $-\frac{\pi}{6}$.

This is represented by the point $\left(-\frac{\pi}{6}, -\frac{1}{2}\right)$.

103. a.



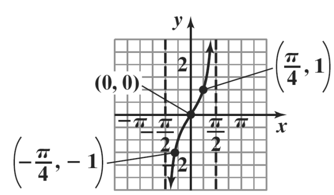
$y = \cos x,\ 0 \le x \le \pi$

b. yes; Explanations will vary.

c. The angle is $\frac{5\pi}{6}$.

This is represented by the point $\left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right)$.

104. a.



$y = \tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$

b. yes; Explanations will vary.

c. The angle is $-\frac{\pi}{3}$.

This is represented by the point $\left(-\frac{\pi}{3}, -\sqrt{3}\right)$.

Section 4.7

Check Point Exercises

1. Let $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$, then $\sin\theta = \frac{\sqrt{3}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$. Thus, $\theta = \frac{\pi}{3}$, or $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

2. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{2}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\cos\theta = -\frac{\sqrt{2}}{2}$ is $-\frac{\pi}{4}$. Thus $\theta = -\frac{\pi}{4}$, or $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$.

3. Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$, then $\cos\theta = -\frac{1}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{1}{2}$ is $\frac{2\pi}{3}$.

Thus, $\theta = \frac{2\pi}{3}$, or $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

4. Let $\theta = \tan^{-1}(-1)$, then $\tan\theta = -1$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -1$ is $-\frac{\pi}{4}$.

Thus $\theta = -\frac{\pi}{4}$ or $\tan^{-1}\theta = -\frac{\pi}{4}$.

5.

	Scientific Calculator Solution			
	Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
a.	$\cos^{-1}\left(\frac{1}{3}\right)$	Radian	1 [÷] 3 [=] [COS^{-1}]	1.2310
b.	$\tan^{-1}(-35.85)$	Radian	35.85 [+/−] [TAN^{-1}]	–1.5429

	Graphing Calculator Solution			
	Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
a.	$\cos^{-1}\left(\frac{1}{3}\right)$	Radian	[COS^{-1}] [(] 1 [÷] 3 [)] [ENTER]	1.2310
b.	$\tan^{-1}(-35.85)$	Radian	[TAN^{-1}] [–] 35.85 [ENTER]	–1.5429

6. **a.** $\cos\left(\cos^{-1} 0.7\right)$

$x = 0.7$, x is in $[-1,1]$ so $\cos(\cos^{-1} 0.7) = 0.7$

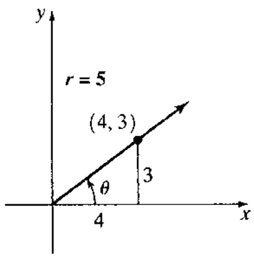
b. $\sin^{-1}(\sin \pi)$

$x = \pi$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. x is in the domain of $\sin x$, so $\sin^{-1}(\sin \pi) = \sin^{-1}(0) = 0$

c. $\cos\left(\cos^{-1} \pi\right)$

$x = \pi$, x is not in $[-1,1]$ so $\cos\left(\cos^{-1} \pi\right)$ is not defined.

7. Let $\theta = \tan^{-1}\left(\frac{3}{4}\right)$, then $\tan\theta = \frac{3}{4}$. Because $\tan\theta$ is positive, θ is in the first quadrant.



Use the Pythagorean Theorem to find r.

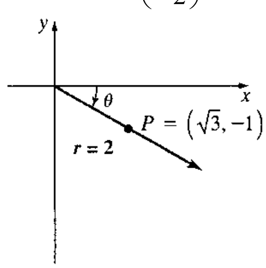
$r^2 = 3^2 + 4^2 = 9 + 16 = 25$

$r = \sqrt{25} = 5$

Use the right triangle to find the exact value.

$\sin\left(\tan^{-1}\frac{3}{4}\right) = \sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{3}{5}$

8. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. Because $\sin\theta$ is negative, θ is in quadrant IV.



Use the Pythagorean Theorem to find x.

$x^2 + (-1)^2 = 2^2$

$x^2 + 1 = 4$

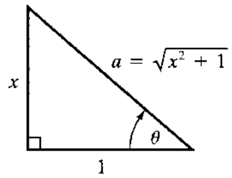
$x^2 = 3$

$x = \sqrt{3}$

Use values for x and r to find the exact value.

$\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$

9. Let $\theta = \tan^{-1} x$, then $\tan\theta = x = \frac{x}{1}$.



Use the Pythagorean Theorem to find the third side, a.

$a^2 = x^2 + 1^2$

$a = \sqrt{x^2+1}$

Use the right triangle to write the algebraic expression.

$\sec\left(\tan^{-1} x\right) = \sec\theta = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$

Concept and Vocabulary Check 4.7

1. $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$; $\sin^{-1} x$

2. $0 \le x \le \pi$; $\cos^{-1} x$

3. $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$; $\tan^{-1} x$

4. $[-1,1]$; $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

5. $[-1,1]$; $[0,\pi]$

6. $(-\infty,\infty)$; $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

7. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

8. $[0,\pi]$

9. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

10. false

Exercise Set 4.7

1. Let $\theta = \sin^{-1}\frac{1}{2}$, then $\sin\theta = \frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{1}{2}$ is $\frac{\pi}{6}$. Thus, $\theta = \frac{\pi}{6}$, or $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

2. Let $\theta = \sin^{-1} 0$, then $\sin\theta = 0$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = 0$ is 0. Thus $\theta = 0$, or $\sin^{-1} 0 = 0$.

3. Let $\theta = \sin^{-1}\frac{\sqrt{2}}{2}$, then $\sin\theta = \frac{\sqrt{2}}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

4. Let $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$, then $\sin\theta = \frac{\sqrt{3}}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$. Thus $\theta = \frac{\pi}{3}$, or $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

5. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{1}{2}$ is $-\frac{\pi}{6}$. Thus $\theta = -\frac{\pi}{6}$, or $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.

6. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{3}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{\sqrt{3}}{2}$ is $-\frac{\pi}{3}$. Thus $\theta = -\frac{\pi}{3}$, or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

7. Let $\theta = \cos^{-1}\frac{\sqrt{3}}{2}$, then $\cos\theta = \frac{\sqrt{3}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$. Thus $\theta = \frac{\pi}{6}$, or $\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

8. Let $\theta = \cos^{-1}\frac{\sqrt{2}}{2}$, then $\cos\theta = \frac{\sqrt{2}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = \frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

9. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{2}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{\sqrt{2}}{2}$ is $\frac{3\pi}{4}$. Thus $\theta = \frac{3\pi}{4}$, or $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$.

10. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{3}}{2}$.
The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{\sqrt{3}}{2}$ is $\frac{5\pi}{6}$. Thus $\theta = \frac{5\pi}{6}$, or $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$.

11. Let $\theta = \cos^{-1} 0$, then $\cos\theta = 0$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 0$ is $\frac{\pi}{2}$. Thus $\theta = \frac{\pi}{2}$, or $\cos^{-1} 0 = \frac{\pi}{2}$.

12. Let $\theta = \cos^{-1} 1$, then $\cos\theta = 1$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 1$ is 0. Thus $\theta = 0$, or $\cos^{-1} 1 = 0$.

13. Let $\theta = \tan^{-1}\frac{\sqrt{3}}{3}$, then $\tan\theta = \frac{\sqrt{3}}{3}$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = \frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$. Thus $\theta = \frac{\pi}{6}$, or $\tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$.

14. Let $\theta = \tan^{-1} 1$, then $\tan\theta = 1$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = 1$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\tan^{-1} 1 = \frac{\pi}{4}$.

15. Let $\theta = \tan^{-1} 0$, then $\tan\theta = 0$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = 0$ is 0. Thus $\theta = 0$, or $\tan^{-1} 0 = 0$.

16. Let $\theta = \tan^{-1}(-1)$, then $\tan\theta = -1$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -1$ is $-\frac{\pi}{4}$. Thus $\theta = -\frac{\pi}{4}$, or $\tan^{-1}(-1) = -\frac{\pi}{4}$.

17. Let $\theta = \tan^{-1}\left(-\sqrt{3}\right)$, then $\tan\theta = -\sqrt{3}$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -\sqrt{3}$ is $-\frac{\pi}{3}$. Thus $\theta = -\frac{\pi}{3}$, or $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$.

18. Let $\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$, then $\tan\theta = -\frac{\sqrt{3}}{3}$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -\frac{\sqrt{3}}{3}$ is $-\frac{\pi}{6}$. Thus $\theta = -\frac{\pi}{6}$, or $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

19.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.3$	Radian	0.3 [SIN^{-1}]	0.30

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.3$	Radian	[SIN^{-1}] 0.3 [ENTER]	0.30

20.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.47$	Radian	0.47 [SIN^{-1}]	0.49

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.47$	Radian	[SIN^{-1}] 0.47 [ENTER]	0.49

21.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.32)$	Radian	0.32 [+/–] [SIN^{-1}]	–0.33

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.32)$	Radian	[SIN^{-1}] [–] 0.32 [ENTER]	–0.33

22.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.625)$	Radian	0.625 [+/–] [SIN^{-1}]	–0.68

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.625)$	Radian	[SIN^{-1}] [–] 0.625 [ENTER]	–0.68

23.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{3}{8}\right)$	Radian	3 [÷] 8 [=] [COS^{-1}]	1.19

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{3}{8}\right)$	Radian	[COS^{-1}] [(] 3 [÷] 8 [)] [ENTER]	1.19

24.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{4}{9}\right)$	Radian	4 [÷] 9 [=] [COS^{-1}]	1.11

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{4}{9}\right)$	Radian	[COS^{-1}] [(] 4 [÷] 9 [)] [ENTER]	1.11

25.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{5}}{7}$	Radian	5 [$\sqrt{\ }$] [÷] 7 [=] [COS^{-1}]	1.25

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{5}}{7}$	Radian	[COS^{-1}] [(] [$\sqrt{\ }$] 5 [÷] 7 [)] [ENTER]	1.25

26.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{7}}{10}$	Radian	7 [$\sqrt{\ }$] [÷] 10 [=] [COS^{-1}]	1.30

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{7}}{10}$	Radian	[COS^{-1}] [(] [$\sqrt{\ }$] 7 [÷] 10 [)] [ENTER]	1.30

27.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-20)$	Radian	20 [+/−] [TAN^{-1}]	–1.52

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-20)$	Radian	[TAN^{-1}] [−] 20 [ENTER]	–1.52

28.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-30)$	Radian	30 [+/−] [TAN^{-1}]	–1.54

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-30)$	Radian	[TAN^{-1}] [−] 30 [ENTER]	–1.54

29.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{473}\right)$	Radian	473 [$\sqrt{\ }$] [+/−] [TAN^{-1}]	–1.52

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{473}\right)$	Radian	[TAN^{-1}] [(] [−] [$\sqrt{\ }$] 473 [)] [ENTER]	–1.52

30.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{5061}\right)$	Radian	5061 [$\sqrt{\ }$] [+/−] [TAN^{-1}]	–1.56

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{5061}\right)$	Radian	[TAN^{-1}] [(] [−] [$\sqrt{\ }$] 5061 [)] [ENTER]	–1.56

31. $\sin\left(\sin^{-1} 0.9\right)$

$x = 0.9$, x is in $[-1, 1]$, so $\sin(\sin^{-1} 0.9) = 0.9$

32. $\cos(\cos^{-1} 0.57)$
$x = 0.57$, x is in $[-1, 1]$,
so $\cos(\cos^{-1} 0.57) = 0.57$

33. $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$x = \frac{\pi}{3}$, x is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

34. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$
$x = \frac{2\pi}{3}$, x is in $[0, \pi]$,
so $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

35. $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

$x = \frac{5\pi}{6}$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, x is in the domain of $\sin x$, so $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

36. $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$
$x = \frac{4\pi}{3}$, x is not in $[0, \pi]$,
x is in the domain of $\cos x$,
so $\cos^{-1}\left(\cos\frac{4\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

37. $\tan\left(\tan^{-1} 125\right)$

$x = 125$, x is a real number, so $\tan\left(\tan^{-1} 125\right) = 125$

38. $\tan(\tan^{-1} 380)$
$x = 380$, x is a real number,
so $\tan(\tan^{-1} 380) = 380$

39. $\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]$

$x = -\frac{\pi}{6}$, x is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so

$\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$

40. $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right]$
$x = -\frac{\pi}{3}$, x is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,
so $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$

41. $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$x = \frac{2\pi}{3}$, x is not in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, x is in the domain of $\tan x$, so $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

42. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$
$x = \frac{3\pi}{4}$, x is not in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,
x is in the domain of $\tan x$
so $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

43. $\sin^{-1}(\sin\pi)$
$x = \pi$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,
x is in the domain of $\sin x$, so
$\sin^{-1}(\sin\pi) = \sin^{-1} 0 = 0$

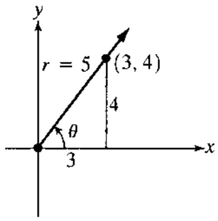
44. $\cos^{-1}(\cos 2\pi)$
$x = 2\pi$, x is not in $[0, \pi]$,
x is in the domain of $\cos x$,
so $\cos^{-1}(\cos 2\pi) = \cos^{-1} 1 = 0$

45. $\sin\left(\sin^{-1}\pi\right)$

$x = \pi$, x is not in $[-1, 1]$, so $\sin\left(\sin^{-1}\pi\right)$ is not defined.

46. $\cos(\cos^{-1} 3\pi)$
$x = 3\pi$, x is not in $[-1, 1]$
so $\cos(\cos^{-1} 3\pi)$ is not defined.

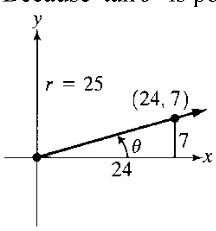
47. Let $\theta = \sin^{-1}\frac{4}{5}$, then $\sin\theta = \frac{4}{5}$. Because $\sin\theta$ is positive, θ is in the first quadrant.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + 4^2 &= 5^2 \\ x^2 &= 25 - 16 = 9 \\ x &= 3 \end{aligned}$$

$$\cos\left(\sin^{-1}\frac{4}{5}\right) = \cos\theta = \frac{x}{r} = \frac{3}{5}$$

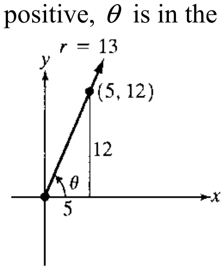
48. Let $\theta = \tan^{-1}\frac{7}{24}$, then $\tan\theta = \frac{7}{24}$.
Because $\tan\theta$ is positive, θ is in the first quadrant.



$$\begin{aligned} r^2 &= x^2 + y^2 \\ r^2 &= 7^2 + 24^2 \\ r^2 &= 625 \\ r &= 25 \end{aligned}$$

$$\sin\left(\tan^{-1}\frac{7}{24}\right) = \sin\theta = \frac{y}{r} = \frac{7}{25}$$

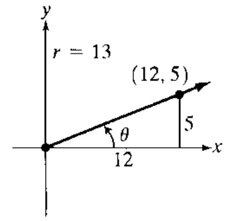
49. Let $\theta = \cos^{-1}\frac{5}{13}$, then $\cos\theta = \frac{5}{13}$. Because $\cos\theta$ is positive, θ is in the first quadrant.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ 5^2 + y^2 &= 13^2 \\ y^2 &= 169 - 25 \\ y^2 &= 144 \\ y &= 12 \end{aligned}$$

$$\tan\left(\cos^{-1}\frac{5}{13}\right) = \tan\theta = \frac{y}{x} = \frac{12}{5}$$

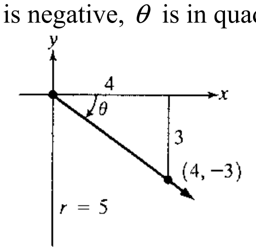
50. Let $\theta = \sin^{-1}\frac{5}{13}$ then $\sin\theta = \frac{5}{13}$.
because $\sin\theta$ is positive, θ is in the first quadrant.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + 5^2 &= 13^2 \\ x^2 &= 144 \\ x &= 12 \end{aligned}$$

$$\cot\left(\sin^{-1}\frac{5}{13}\right) = \cot\theta = \frac{x}{y} = \frac{12}{5}$$

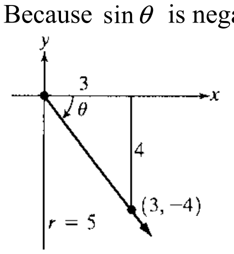
51. Let $\theta = \sin^{-1}\left(-\frac{3}{5}\right)$, then $\sin\theta = -\frac{3}{5}$. Because $\sin\theta$ is negative, θ is in quadrant IV.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-3)^2 &= 5^2 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

$$\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right] = \tan\theta = \frac{y}{x} = -\frac{3}{4}$$

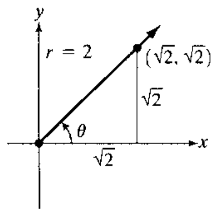
52. Let $\theta = \sin^{-1}\left(-\frac{4}{5}\right)$, then $\sin\theta = -\frac{4}{5}$.
Because $\sin\theta$ is negative, θ is in quadrant IV.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-4)^2 &= 5^2 \\ x^2 &= 9 \\ x &= 3 \end{aligned}$$

$$\cos\left[\sin^{-1}\left(-\frac{4}{5}\right)\right] = \cos\theta = \frac{x}{r} = \frac{3}{5}$$

53. Let, $\theta = \cos^{-1}\frac{\sqrt{2}}{2}$, then $\cos\theta = \frac{\sqrt{2}}{2}$. Because $\cos\theta$ is positive, θ is in the first quadrant.

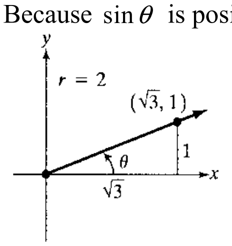


$$\begin{aligned} x^2 + y^2 &= r^2 \\ \left(\sqrt{2}\right)^2 + y^2 &= 2^2 \\ y^2 &= 2 \\ y &= \sqrt{2} \end{aligned}$$

$$\sin\left(\cos^{-1}\frac{\sqrt{2}}{2}\right) = \sin\theta = \frac{y}{r} = \frac{\sqrt{2}}{2}$$

54. Let $\theta = \sin^{-1}\frac{1}{2}$, then $\sin\theta = \frac{1}{2}$.

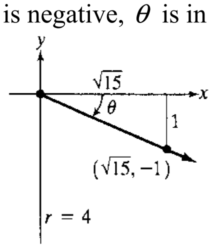
Because $\sin\theta$ is positive, θ is in the first quadrant.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + 1^2 &= 2^2 \\ x^2 &= 3 \\ x &= \sqrt{3} \end{aligned}$$

$$\cos\left(\sin^{-1}\frac{1}{2}\right) = \cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$$

55. Let $\theta = \sin^{-1}\left(-\frac{1}{4}\right)$, then $\sin\theta = -\frac{1}{4}$. Because $\sin\theta$ is negative, θ is in quadrant IV.

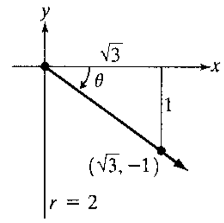


$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-1)^2 &= 4^2 \\ x^2 &= 15 \\ x &= \sqrt{15} \end{aligned}$$

$$\sec\left[\sin^{-1}\left(-\frac{1}{4}\right)\right] = \sec\theta = \frac{r}{x} = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

56. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$.

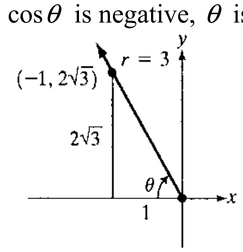
Because $\sin\theta$ is negative, θ is in quadrant IV.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-1)^2 &= 2^2 \\ x^2 &= 3 \\ x &= \sqrt{3} \end{aligned}$$

$$\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \sec\theta = \frac{r}{x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

57. Let $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$, then $\cos\theta = -\frac{1}{3}$. Because $\cos\theta$ is negative, θ is in quadrant II.



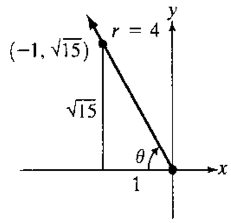
$$\begin{aligned} x^2 + y^2 &= r^2 \\ (-1)^2 + y^2 &= 3^2 \\ y^2 &= 8 \\ y &= \sqrt{8} \\ y &= 2\sqrt{2} \end{aligned}$$

Use the right triangle to find the exact value.

$$\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right] = \tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$$

58. Let $\theta = \cos^{-1}\left(-\frac{1}{4}\right)$, then $\cos\theta = -\frac{1}{4}$.

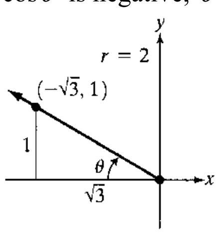
Because $\cos\theta$ is negative, θ is in quadrant II.



$$x^2 + y^2 = r^2$$
$$(-1)^2 + y^2 = 4^2$$
$$y^2 = 15$$
$$y = \sqrt{15}$$

$$\tan\left[\cos^{-1}\left(-\frac{1}{4}\right)\right] = \tan\theta = \frac{y}{x} = \frac{\sqrt{15}}{-1} = -\sqrt{15}$$

59. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{3}}{2}$. Because $\cos\theta$ is negative, θ is in quadrant II.

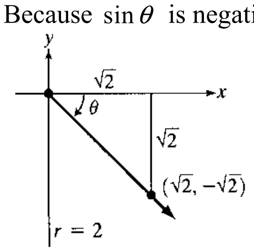


$$x^2 + y^2 = r^2$$
$$\left(-\sqrt{3}\right)^2 + y^2 = 2^2$$
$$y^2 = 1$$
$$y = 1$$

$$\csc\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \csc\theta = \frac{r}{y} = \frac{2}{1} = 2$$

60. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{2}}{2}$.

Because $\sin\theta$ is negative, θ is in quadrant IV.

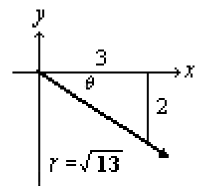


$$x^2 + y^2 = r^2$$
$$x^2 + \left(-\sqrt{2}\right)^2 = 2^2$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

$$\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] = \sec\theta = \frac{r}{x} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

61. Let $\theta = \tan^{-1}\left(-\frac{2}{3}\right)$, then $\tan\theta = -\frac{2}{3}$.

Because $\tan\theta$ is negative, θ is in quadrant IV.

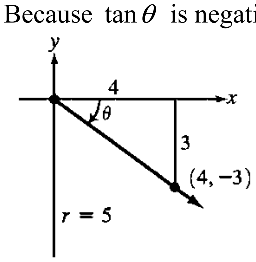


$$r^2 = x^2 + y^2$$
$$r^2 = 3^2 + (-2)^2$$
$$r^2 = 9 + 4$$
$$r^2 = 13$$
$$r = \sqrt{13}$$

$$\cos\left[\tan^{-1}\left(-\frac{2}{3}\right)\right] = \cos\theta = \frac{x}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

62. Let $\theta = \tan^{-1}\left(-\frac{3}{4}\right)$, then $\tan\theta = -\frac{3}{4}$.

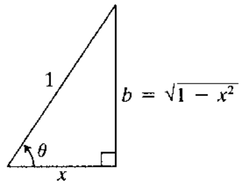
Because $\tan\theta$ is negative, θ is in quadrant IV.



$$r^2 = x^2 + y^2$$
$$r^2 = 4^2 + (-3)^2$$
$$r^2 = 16 + 9$$
$$r^2 = 25$$
$$r = 5$$

$$\sin\left[\tan^{-1}\left(-\frac{3}{4}\right)\right] = \sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$

63. Let $\theta = \cos^{-1} x$, then $\cos\theta = x = \frac{x}{1}$.



Use the Pythagorean Theorem to find the third side, b.

$x^2 + b^2 = 1^2$

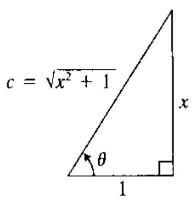
$b^2 = 1 - x^2$

$b = \sqrt{1 - x^2}$

Use the right triangle to write the algebraic expression.

$$\tan\left(\cos^{-1} x\right) = \tan\theta = \frac{\sqrt{1-x^2}}{x}$$

64. Let $\theta = \tan^{-1} x$, then $\tan\theta = x = \frac{x}{1}$.



Use the Pythagorean Theorem to find the third side, c.

$c^2 = x^2 + 1^2$

$c = \sqrt{x^2 + 1}$

Use the right triangle to write the algebraic expression.

$$\sin(\tan^{-1}) = \sin\theta = \frac{x}{\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = \frac{x\sqrt{x^2+1}}{x^2+1}$$

65. Let $\theta = \sin^{-1} 2x$, then $\sin\theta = 2x$

$y = 2x$, $r = 1$

Use the Pythagorean Theorem to find x.

$x^2 + (2x)^2 = 1^2$

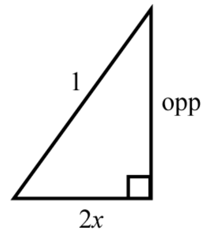
$x^2 = 1 - 4x^2$

$x = \sqrt{1 - 4x^2}$

$\cos(\sin^{-1} 2x) = \sqrt{1 - 4x^2}$

66. Let $\theta = \cos^{-1} 2x$.

Use the Pythagorean Theorem to find the third side, b.



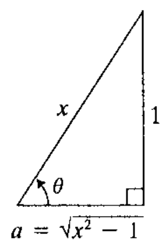
$(2x)^2 + b^2 = 1^2$

$b^2 = 1 - 4x^2$

$b = \sqrt{1 - 4x^2}$

$$\sin\left(\cos^{-1} 2x\right) = \frac{\sqrt{1-4x^2}}{1} = \sqrt{1-4x^2}$$

67. Let $\theta = \sin^{-1}\frac{1}{x}$, then $\sin\theta = \frac{1}{x}$.



Use the Pythagorean Theorem to find the third side, a.

$a^2 + 1^2 = x^2$

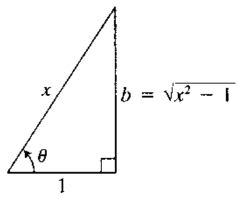
$a^2 = x^2 - 1$

$a = \sqrt{x^2 - 1}$

Use the right triangle to write the algebraic expression.

$$\cos\left(\sin^{-1}\frac{1}{x}\right) = \cos\theta = \frac{\sqrt{x^2-1}}{x}$$

68. Let $\theta = \cos^{-1}\frac{1}{x}$, then $\cos\theta = \frac{1}{x}$.



Use the Pythagorean Theorem to find the third side, b.

$$1^2 + b^2 = x^2$$
$$b^2 = x^2 - 1$$
$$b = \sqrt{x^2 - 1}$$

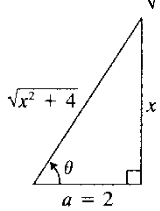
Use the right triangle to write the algebraic expression.

$$\sec\left(\cos^{-1}\frac{1}{x}\right) = \sec\theta = \frac{x}{1} = x$$

69. $\cot\left(\tan^{-1}\frac{x}{\sqrt{3}}\right) = \frac{\sqrt{3}}{x}$

70. $\cot\left(\tan^{-1}\frac{x}{\sqrt{2}}\right) = \frac{\sqrt{2}}{x}$

71. Let $\theta = \sin^{-1}\frac{x}{\sqrt{x^2+4}}$, then $\sin\theta = \frac{x}{\sqrt{x^2+4}}$.



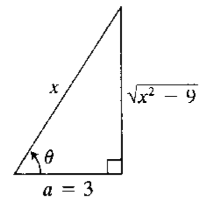
Use the Pythagorean Theorem to find the third side, a.

$$a^2 + x^2 = \left(\sqrt{x^2+4}\right)^2$$
$$a^2 = x^2 + 4 - x^2 = 4$$
$$a = 2$$

Use the right triangle to write the algebraic expression.

$$\sec\left(\sin^{-1}\frac{x}{\sqrt{x^2+4}}\right) = \sec\theta = \frac{\sqrt{x^2+4}}{2}$$

72. Let $\theta = \sin^{-1}\frac{\sqrt{x^2-9}}{x}$, then $\sin\theta = \frac{\sqrt{x^2-9}}{x}$.



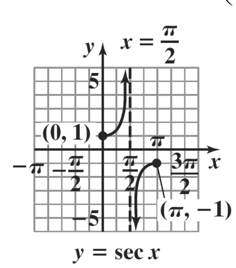
Use the Pythagorean Theorem to find the third side, a.

$$a^2 + \left(\sqrt{x^2-9}\right)^2 = x^2$$
$$a^2 = x^2 - x^2 + 9 = 9$$
$$a = 3$$

Use the right triangle to write the algebraic expression.

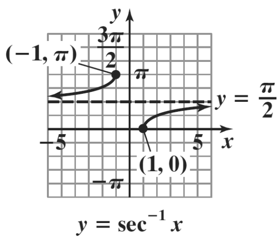
$$\cot\left(\sin^{-1}\frac{\sqrt{x^2-9}}{x}\right) = \frac{3}{\sqrt{x^2-9}}$$
$$= \frac{3}{\sqrt{x^2-9}} \cdot \frac{\sqrt{x^2-9}}{\sqrt{x^2-9}} = \frac{3\sqrt{x^2-9}}{x^2-9}$$

73. **a.** $y = \sec x$ is the reciprocal of $y = \cos x$. The x-values for the key points in the interval $[0, \pi]$ are $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4},$ and π. The key points are $(0, 1)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}\right)$, and $(\pi, -1)$, Draw a vertical asymptote at $x = \frac{\pi}{2}$. Now draw our graph from (0, 1) through $\left(\frac{\pi}{4}, \sqrt{2}\right)$ to ∞ on the left side of the asymptote. From $-\infty$ on the right side of the asymptote through $\left(\frac{3\pi}{4}, -\sqrt{2}\right)$ to $(\pi, -1)$.

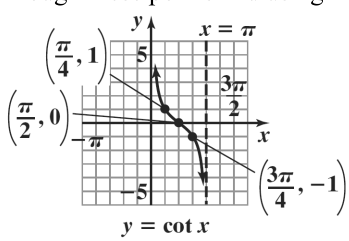


b. With this restricted domain, no horizontal line intersects the graph of $y = \sec x$ more than once, so the function is one-to-one and has an inverse function.

c. Reflecting the graph of the restricted secant function about the line $y = x$, we get the graph of $y = \sec^{-1} x$.

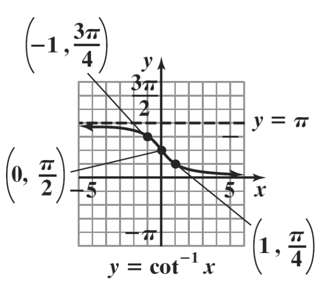


74. a. Two consecutive asymptotes occur at $x = 0$ and $x = \pi$. Midway between $x = 0$ and $x = \pi$ is $x = 0$ and $x = \pi$. An x-intercept for the graph is $\left(\frac{\pi}{2}, 0\right)$. The graph goes through the points $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{3\pi}{4}, -1\right)$. Now graph the function through these points and using the asymptotes.

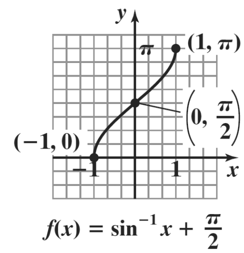


b. With this restricted domain no horizontal line intersects the graph of $y = \cot x$ more than once, so the function is one-to-one and has an inverse function. Reflecting the graph of the restricted cotangent function about the line $y = x$, we get the graph of $y = \cot^{-1} x$.

c.

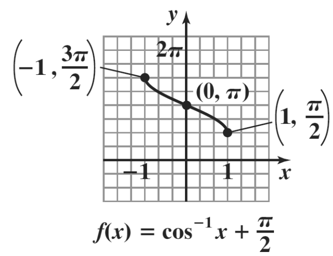


75.



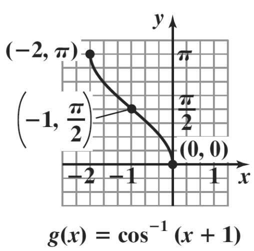
domain: $[-1, 1]$;
range: $[0, \pi]$

76.



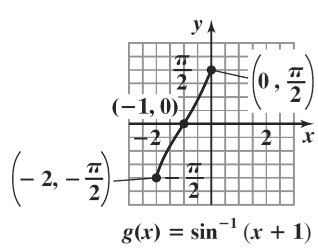
domain: $[-1, 1]$;
range: $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

77.



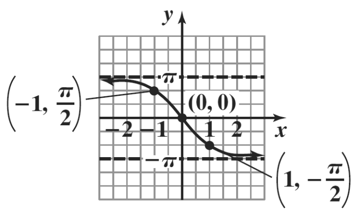
domain: $[-2, 0]$;
range: $[0, \pi]$

78.



domain: $[-2, 0]$;
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

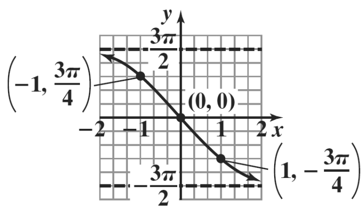
79.



$f(x) = -2\tan^{-1} x$

domain: $(-\infty, \infty)$;
range: $(-\pi, \pi)$

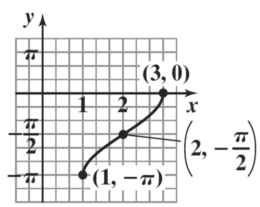
80.



$h(x) = -3\tan^{-1} x$

domain: $(-\infty, \infty)$;
range: $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

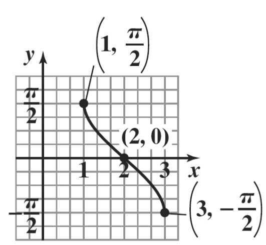
81.



$f(x) = \sin^{-1}(x-2) - \frac{\pi}{2}$

domain: (1, 3];
range: $[-\pi, 0]$

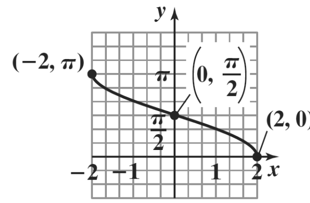
82.



$f(x) = \cos^{-1}(x-2) - \frac{\pi}{2}$

domain: [1, 3];
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

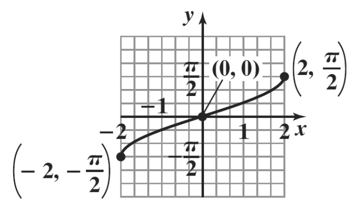
83.



$g(x) = \cos^{-1}\frac{x}{2}$

domain: $[-2, 2]$;
range: $[0, \pi]$

84.



$g(x) = \sin^{-1}\frac{x}{2}$

domain: $[-2, 2]$;
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

85. The inner function, $\sin^{-1} x$, accepts values on the interval $[-1,1]$. Since the inner and outer functions are inverses of each other, the domain and range are as follows.
domain: $[-1,1]$; range: $[-1,1]$

86. The inner function, $\cos^{-1} x$, accepts values on the interval $[-1,1]$. Since the inner and outer functions are inverses of each other, the domain and range are as follows.
domain: $[-1,1]$; range: $[-1,1]$

87. The inner function, $\cos x$, accepts values on the interval $(-\infty,\infty)$. The outer function returns values on the interval $[0,\pi]$
domain: $(-\infty,\infty)$; range: $[0,\pi]$

88. The inner function, $\sin x$, accepts values on the interval $(-\infty,\infty)$. The outer function returns values on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
domain: $(-\infty,\infty)$; range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

89. The inner function, $\cos x$, accepts values on the interval $(-\infty, \infty)$. The outer function returns values on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

domain: $(-\infty, \infty)$; range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

90. The inner function, $\sin x$, accepts values on the interval $(-\infty, \infty)$. The outer function returns values on the interval $[0, \pi]$

domain: $(-\infty, \infty)$; range: $[0, \pi]$

91. The functions $\sin^{-1} x$ and $\cos^{-1} x$ accept values on the interval $[-1,1]$. The sum of these values is always $\frac{\pi}{2}$.

domain: $[-1,1]$; range: $\left\{\frac{\pi}{2}\right\}$

92. The functions $\sin^{-1} x$ and $\cos^{-1} x$ accept values on the interval $[-1,1]$. The difference of these values range from $-\frac{\pi}{2}$ to $\frac{3\pi}{2}$

domain: $[-1,1]$; range: $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

93. $\theta = \tan^{-1}\frac{33}{x} - \tan^{-1}\frac{8}{x}$

x	θ
5	$\tan^{-1}\frac{33}{5} - \tan^{-1}\frac{8}{5} \approx 0.408$ radians
10	$\tan^{-1}\frac{33}{10} - \tan^{-1}\frac{8}{10} \approx 0.602$ radians
15	$\tan^{-1}\frac{33}{15} - \tan^{-1}\frac{8}{15} \approx 0.654$ radians
20	$\tan^{-1}\frac{33}{20} - \tan^{-1}\frac{8}{20} \approx 0.645$ radians
25	$\tan^{-1}\frac{33}{25} - \tan^{-1}\frac{8}{25} \approx 0.613$ radians

94. The viewing angle increases rapidly up to about 16 feet, then it decreases less rapidly; about 16 feet; when $x = 15$, $\theta = 0.6542$ radians; when $x = 17$, $\theta = 0.6553$ radians.

95. $\theta = 2\tan^{-1}\frac{21.634}{28} \approx 1.3157$ radians;
$1.3157\left(\frac{180}{\pi}\right) \approx 75.4°$

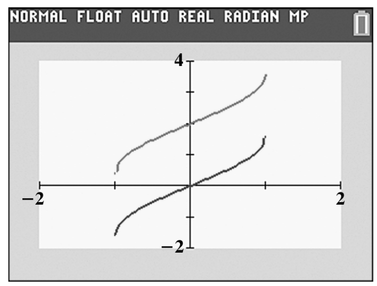
96. $\theta = 2\tan^{-1}\frac{21.634}{300} \approx 0.1440$ radians;
$0.1440\left(\frac{180}{\pi}\right) \approx 8.2°$

97. $\tan^{-1} b - \tan^{-1} a = \tan^{-1} 2 - \tan^{-1} 0$
≈ 1.1071 square units

98. $\tan^{-1} b - \tan^{-1} a = \tan^{-1} 1 - \tan^{-1}(-2)$
≈ 1.8925 square units

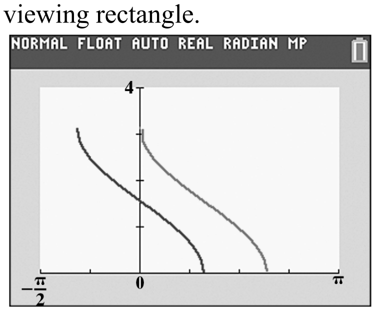
99. – 109. Answers may vary.

110. $y = \sin^{-1} x$
$y = \sin^{-1} x + 2$



The graph of the second equation is the graph of the first equation shifted up 2 units.

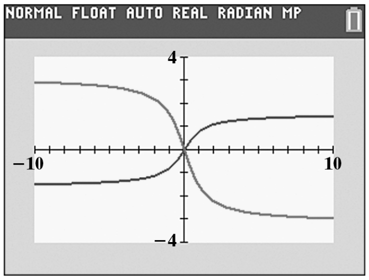
111. The domain of $y = \cos^{-1} x$ is the interval [–1, 1], and the range is the interval $[0, \pi]$. Because the second equation is the first equation with 1 subtracted from the variable, we will move our x max to π, and graph in a $\left[-\frac{\pi}{2}, \pi, \frac{\pi}{4}\right]$ by [0, 4, 1] viewing rectangle.



The graph of the second equation is the graph of the first equation shifted right 1 unit.

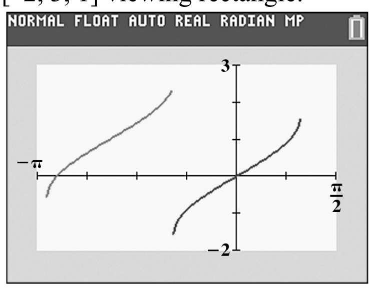
112. $y = \tan^{-1} x$

$y = -2\tan^{-1} x$



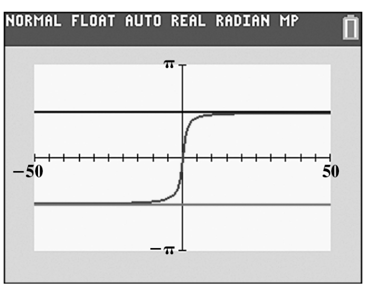
The graph of the second equation is the graph of the first equation reversed and stretched.

113. The domain of $y = \sin^{-1} x$ is the interval [–1, 1], and the range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Because the second equation is the first equation plus 1, and with 2 added to the variable, we will move our *y* max to 3, and move our *x* min to $-\pi$, and graph in a $\left[-\pi, \frac{\pi}{2}, \frac{\pi}{2}\right]$ by [–2, 3, 1] viewing rectangle.



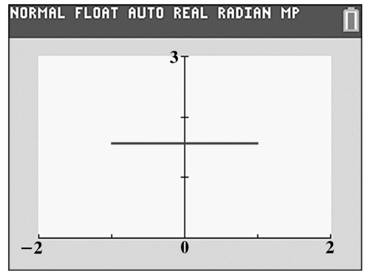
The graph of the second equation is the graph of the first equation shifted left 2 units and up 1 unit.

114. $y = \tan^{-1} x$



Observations may vary.

115.



It seems $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $-1 \le x \le 1$.

116. does not make sense; Explanations will vary. Sample explanation: The cosine’s inverse is not a function over that interval.

117. does not make sense; Explanations will vary. Sample explanation: Though this restriction works for tangent, it is not selected simply because it is easier to remember. Rather the restrictions are based on which intervals will have inverses.

118. makes sense

119. does not make sense; Explanations will vary. Sample explanation:

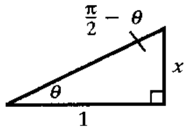
$$\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

120.
$$y = 2\sin^{-1}(x-5)$$
$$\frac{y}{2} = \sin^{-1}(x-5)$$
$$\sin\frac{y}{2} = x-5$$
$$x = \sin\frac{y}{2} + 5$$

121.
$$2\sin^{-1} x = \frac{\pi}{4}$$
$$\sin^{-1} x = \frac{\pi}{8}$$
$$x = \sin\frac{\pi}{8}$$

122. Prove: If $x > 0$, $\tan^{-1} x + \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$

Since $x > 0$, there is an angle θ with $0 < \theta < \frac{\pi}{2}$ as shown in the figure.



$\tan\theta = x$ and $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{x}$ thus

$\tan^{-1} x = \theta$ and $\tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \theta$ so

$\tan^{-1} x + \tan^{-1}\frac{1}{x} = \theta + \frac{\pi}{2} - \theta = \frac{\pi}{2}$

123. Let α equal the acute angle in the smaller right triangle.

$\tan\alpha = \frac{8}{x}$

so $\tan^{-1}\frac{8}{x} = \alpha$

$\tan(\alpha + \theta) = \frac{33}{x}$

so $\tan^{-1}\frac{33}{x} = \alpha + \theta$

$\theta = \alpha + \theta - \alpha = \tan^{-1}\frac{33}{x} - \tan^{-1}\frac{8}{x}$

124. $\log_b\left(x\sqrt[3]{y}\right)$

$= \log_b\left(xy^{1/3}\right)$

$= \log_b x + \log_b y^{1/3}$

$= \log_b x + \frac{1}{3}\log_b y$

125. $\frac{1}{2}\log x + 6\log(x-2)$

$= \log x^{1/2} + \log(x-2)^6$

$= \log\sqrt{x} + \log(x-2)^6$

$= \log\left(\sqrt{x}(x-2)^6\right)$

126. $f(x) = \frac{5x+1}{x-1}$

$f(-x) = \frac{5(-x)+1}{-x-1} = \frac{-5x+1}{-x-1} = \frac{5x-1}{x+1}$

no symmetry

$f(0) = \frac{5(0)+1}{0-1} = \frac{1}{-1} = -1$

The y-intercept is -1.

$5x + 1 = 0$

$5x = -1$

$x = -\frac{1}{5}$

The x-intercept is $-\frac{1}{5}$.

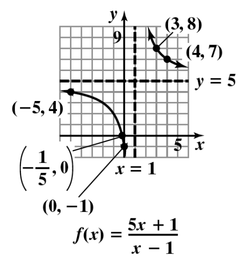
Vertical asymptote:

$x - 1 = 0$

$x = 1$

Horizontal asymptote:

$y = \frac{5}{1} = 5$



127. $\tan A = \frac{a}{b}$

$\tan 22.3^\circ = \frac{a}{12.1}$

$a = 12.1\tan 22.3^\circ$

$a \approx 4.96$

$\cos A = \frac{b}{c}$

$\cos 22.3^\circ = \frac{12.1}{c}$

$c = \frac{12.1}{\cos 22.3^\circ}$

$c \approx 13.08$

128. $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \frac{18}{25}$

$\theta = \tan^{-1}\left(\frac{18}{25}\right)$

$\theta \approx 35.8^\circ$

129. $10\cos\left(\frac{\pi}{6}x\right)$

amplitude: $|10| = 10$

period: $\frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi} = 12$

Section 4.8

Check Point Exercises

1. We begin by finding the measure of angle B. Because $C = 90°$ and the sum of a triangle's angles is 180°, we see that $A + B = 90°$. Thus, $B = 90° - A = 90° - 62.7° = 27.3°$.
Now we find b. Because we have a known angle, a known opposite side, and an unknown adjacent side, use the tangent function.

$$\tan 62.7° = \frac{8.4}{b}$$
$$b = \frac{8.4}{\tan 62.7°} \approx 4.34$$

Finally, we need to find c. Because we have a known angle, a known opposite side and an unknown hypotenuse, use the sine function.

$$\sin 62.7° = \frac{8.4}{c}$$
$$c = \frac{8.4}{\sin 62.7} \approx 9.45$$

In summary, $B = 27.3°$, $b \approx 4.34$, and $c \approx 9.45$.

2. Using a right triangle, we have a known angle, an unknown opposite side, a, and a known adjacent side. Therefore, use the tangent function.

$$\tan 85.4° = \frac{a}{80}$$
$$a = 80 \tan 85.4° \approx 994$$

The Eiffel tower is approximately 994 feet high.

3. Using a right triangle, we have an unknown angle, A, a known opposite side, and a known hypotenuse. Therefore, use the sine function.

$$\sin A = \frac{6.7}{13.8}$$
$$A = \sin^{-1} \frac{6.7}{13.8} \approx 29.0°$$

The wire makes an angle of approximately 29.0° with the ground.

4. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, an unknown opposite side, a in the smaller triangle, b in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 32° = \frac{a}{800}$$
$$a = 800 \tan 32° \approx 499.9$$
$$\tan 35° = \frac{b}{800}$$
$$b = 800 \tan 35° \approx 560.2$$

The height of the sculpture of Lincoln's face is 560.2 – 499.9, or approximately 60.3 feet.

5. a. We need the acute angle between ray OD and the north-south line through O.
The measurement of this angle is given to be 25°. The angle is measured from the south side of the north-south line and lies east of the north-south line. Thus, the bearing from O to D is S 25°E.

b. We need the acute angle between ray OC and the north-south line through O.
This angle measures $90° - 75° = 15°$.
This angle is measured from the south side of the north-south line and lies west of the north-south line. Thus the bearing from O to C is S 15° W.

6. a. Your distance from the entrance to the trail system is represented by the hypotenuse, c, of a right triangle. Because we know the length of the two sides of the right triangle, we find c using the Pythagorean Theorem.
We have

$$c^2 = a^2 + b^2 = (2.3)^2 + (3.5)^2 = 17.54$$
$$c = \sqrt{17.54} \approx 4.2$$

You are approximately 4.2 miles from the entrance to the trail system.

b. To find your bearing from the entrance to the trail system, consider a north-south line passing through the entrance. The acute angle from this line to the ray on which you lie is $31° + \theta$. Because we are measuring the angle from the south side of the line and you are west of the entrance, your bearing from the entrance is S $(31° + \theta)$ W. To find θ, Use a right triangle and the tangent function.

$$\tan \theta = \frac{3.5}{2.3}$$
$$\theta = \tan^{-1} \frac{3.5}{2.3} \approx 56.7°$$

Thus, $31° + \theta = 31° + 56.7° = 87.7°$. Your bearing from the entrance to the trail system is S 87.7° W.

7. When the object is released $(t = 0)$, the ball's distance, d, from its rest position is 6 inches down. Because it is down, d is negative: when $t = 0$, $d = -6$. Notice the greatest distance from rest position occurs at $t = 0$. Thus, we will use the equation with the cosine function, $y = a\cos \omega t$, to model the ball's motion. Recall that $|a|$ is the maximum distance. Because the ball initially moves down, $a = -6$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 4$$
$$2\pi = 4\omega$$
$$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$$

Substitute these values into $d = a\cos wt$. The equation for the ball's simple harmonic motion is

$$d = -6\cos\frac{\pi}{2}t.$$

8. We begin by identifying values for a and ω.

$$d = 12\cos\frac{\pi}{4}t,\ a = 12 \text{ and } \omega = \frac{\pi}{4}.$$

a. The maximum displacement from the rest position is the amplitude. Because $a = 12$, the maximum displacement is 12 centimeters.

b. The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{\pi}{4}\cdot\frac{1}{2\pi} = \frac{1}{8}$$

The frequency is $\frac{1}{8}$ cm per second.

c. The time required for one cycle is the period.

$$\text{period } = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi\cdot\frac{4}{\pi} = 8$$

The time required for one cycle is 8 seconds.

Concept and Vocabulary Check 4.8

1. sides; angles

2. north; south

3. simple harmonic; $|a|$; $\frac{2\pi}{\omega}$; $\frac{\omega}{2\pi}$

Exercise Set 4.8

1. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A = 90° - 23.5° = 66.5°$.
Because we have a known angle, a known adjacent side, and an unknown opposite side, use the tangent function.

$$\tan 23.5° = \frac{a}{10}$$
$$a = 10\tan 23.5° \approx 4.35$$

Because we have a known angle, a known adjacent side, and an unknown hypotenuse, use the cosine function.

$$\cos 23.5° = \frac{10}{c}$$
$$c = \frac{10}{\cos 23.5°} \approx 10.90$$

In summary, $B = 66.5°$, $a \approx 4.35$, and $c \approx 10.90$.

2. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 41.5° = 48.5°$.
Because we have a known angle, a known adjacent side, and an unknown opposite side, use the tangent function.

$$\tan 41.5° = \frac{a}{20}$$
$$a = 20\tan 41.5° \approx 17.69$$

Because we have a known angle, a known adjacent side, and an unknown hypotenuse, use the cosine function.

$$\cos 41.5° = \frac{20}{c}$$
$$c = \frac{20}{\cos 41.5°} \approx 26.70$$

In summary, $B = 48.5°$, $a \approx 17.69$, and $c \approx 26.70$.

3. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 52.6° = 37.4°$.
Because we have a known angle, a known hypotenuse, and an unknown opposite side, use the sine function.

$$\sin 52.6 = \frac{a}{54}$$
$$a = 54\sin 52.6° \approx 42.90$$

Because we have a known angle, a known hypotenuse, and an unknown adjacent side, use the cosine function.

$$\cos 52.6° = \frac{b}{54}$$
$$b = 54\cos 52.6° \approx 32.80$$

In summary, $B = 37.4°$, $a \approx 42.90$, and $b \approx 32.80$.

4. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 54.8° = 35.2°$.
Because we have a known angle, a known hypotenuse, and an unknown opposite side, use the sine function.

$$\sin 54.8° = \frac{a}{80}$$
$$a = 80 \sin 54.8° \approx 65.37$$

Because we have a known angle, a known hypotenuse, and an unknown adjacent side, use the cosine function.

$$\cos 54.8 = \frac{b}{80}$$
$$b = 80 \cos 54.8° \approx 46.11$$

In summary, $B = 35.2°$, $a \approx 65.37$, and $c \approx 46.11$.

5. Find the measure of angle A. Because $C = 90°$, $A + B = 90°$.
Thus, $A = 90° - B = 90° - 16.8° = 73.2°$.
Because we have a known angle, a known opposite side and an unknown adjacent side, use the tangent function.

$$\tan 16.8° = \frac{30.5}{a}$$
$$a = \frac{30.5}{\tan 16.8°} \approx 101.02$$

Because we have a known angle, a known opposite side, and an unknown hypotenuse, use the sine function.

$$\sin 16.8° = \frac{30.5}{c}$$
$$c = \frac{30.5}{\sin 16.8°} \approx 105.52$$

In summary, $A = 73.2°$, $a \approx 101.02$, and $c \approx 105.52$.

6. Find the measure of angle A. Because $C = 90°$, $A + B = 90°$.
Thus, $A = 90° - B = 90° - 23.8° = 66.2°$.
Because we have a known angle, a known opposite side, and an unknown adjacent side, use the tangent function.

$$\tan 23.8° = \frac{40.5}{a}$$
$$a = \frac{40.5}{\tan 23.8°} \approx 91.83$$

Because we have a known angle, a known opposite side, and an unknown hypotenuse, use the sine function.

$$\sin 23.8° = \frac{40.5}{c}$$
$$c = \frac{40.5}{\sin 23.8°} \approx 100.36$$

In summary, $A = 66.2°$, $a \approx 91.83$, and $c \approx 100.36$.

7. Find the measure of angle A. Because we have a known hypotenuse, a known opposite side, and an unknown angle, use the sine function.

$$\sin A = \frac{30.4}{50.2}$$
$$A = \sin^{-1}\left(\frac{30.4}{50.2}\right) \approx 37.3°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus,
$B = 90° - A \approx 90° - 37.3° = 52.7°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(30.4)^2 + b^2 = (50.2)^2$$
$$b^2 = (50.2)^2 - (30.4)^2 = 1595.88$$
$$b = \sqrt{1595.88} \approx 39.95$$

In summary, $A \approx 37.3°$, $B \approx 52.7°$, and $b \approx 39.95$.

8. Find the measure of angle A. Because we have a known hypotenuse, a known opposite side, and an unknown angle, use the sine function.

$$\sin A = \frac{11.2}{65.8}$$
$$A = \sin^{-1}\left(\frac{11.2}{65.8}\right) \approx 9.8°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 9.8° = 80.2°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(11.2)^2 + b^2 = (65.8)^2$$
$$b^2 = (65.8)^2 - (11.2)^2 = 4204.2$$
$$b = \sqrt{4204.2} \approx 64.84$$

In summary, $A \approx 9.8°$, $B \approx 80.2°$, and $b \approx 64.84$.

9. Find the measure of angle A. Because we have a known opposite side, a known adjacent side, and an unknown angle, use the tangent function.

$$\tan A = \frac{10.8}{24.7}$$
$$A = \tan^{-1}\left(\frac{10.8}{24.7}\right) \approx 23.6°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 23.6° = 66.4°$.
Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2 = (10.8)^2 + (24.7)^2 = 726.73$$
$$c = \sqrt{726.73} \approx 26.96$$

In summary, $A \approx 23.6°$, $B \approx 66.4°$, and $c \approx 26.96$.

10. Find the measure of angle *A*. Because we have a known opposite side, a known adjacent side, and an unknown angle, use the tangent function.

$$\tan A = \frac{15.3}{17.6}$$
$$A = \tan^{-1}\left(\frac{15.3}{17.6}\right) \approx 41.0°$$

Find the measure of angle *B*. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 41.0° = 49.0°$.
Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2 = (15.3)^2 + (17.6)^2 = 543.85$$
$$c = \sqrt{543.85} \approx 23.32$$

In summary, $A \approx 41.0°$, $B \approx 49.0°$, and $c \approx 23.32$.

11. Find the measure of angle *A*. Because we have a known hypotenuse, a known adjacent side, and unknown angle, use the cosine function.

$$\cos A = \frac{2}{7}$$
$$A = \cos^{-1}\left(\frac{2}{7}\right) \approx 73.4°$$

Find the measure of angle *B*. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 73.4° = 16.6°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$a^2 + (2)^2 = (7)^2$$
$$a^2 = (7)^2 - (2)^2 = 45$$
$$a = \sqrt{45} \approx 6.71$$

In summary, $A \approx 73.4°$, $B \approx 16.6°$, and $a \approx 6.71$.

12. Find the measure of angle *A*. Because we have a known hypotenuse, a known adjacent side, and an unknown angle, use the cosine function.

$$\cos A = \frac{4}{9}$$
$$A = \cos^{-1}\left(\frac{4}{9}\right) \approx 63.6°$$

Find the measure of angle *B*. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 63.6° = 26.4°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$a^2 + (4)^2 = (9)^2$$
$$a^2 = (9)^2 - (4)^2 = 65$$
$$a = \sqrt{65} \approx 8.06$$

In summary, $A \approx 63.6°$, $B \approx 26.4°$, and $a \approx 8.06$.

13. We need the acute angle between ray *OA* and the north-south line through *O*. This angle measure $90° - 75° = 15°$. This angle is measured from the north side of the north-south line and lies east of the north-south line. Thus, the bearing from *O* and *A* is N 15° E.

14. We need the acute angle between ray OB and the north-south line through *O*. This angle measures $90° - 60° = 30°$. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from *O* to *B* is N 30° W.

15. The measurement of this angle is given to be 80°. The angle is measured from the south side of the north-south line and lies west of the north-south line. Thus, the bearing from *O* to *C* is S 80° W.

16. We need the acute angle between ray OD and the north-south line through *O*. This angle measures $90° - 35° = 55°$. This angle is measured from the south side of the north-south line and lies east of the north-south line. Thus, the bearing from *O* to *D* is S 55° E.

17. When the object is released $(t = 0)$, the object's distance, *d*, from its rest position is 6 centimeters down. Because it is down, *d* is negative: When $t = 0$, $d = -6$. Notice the greatest distance from rest position occurs at $t = 0$. Thus, we will use the equation with the cosine function, $y = a\cos\omega t$ to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, $a = -6$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 4$$
$$2\pi = 4\omega$$
$$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$$

Substitute these values into $d = a\cos\omega t$. The equation for the object's simple harmonic motion is $d = -6\cos\frac{\pi}{2}t$.

18. When the object is released ($t = 0$), the object's distance, d, from its rest position is 8 inches down. Because it is down, d, is negative: When $t = 0$, $d = -8$. Notice the greatest distance from rest position occurs at $t = 0$. Thus, we will use the equation with the cosine function, $y = a\cos\omega t$, to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, $a = -8$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 2$$
$$2\pi = 2\omega$$
$$\omega = \frac{2\pi}{2} = \pi$$

Substitute these values into $d = a\cos\omega t$.
The equation for the object's simple harmonic motion is $d = -8\cos\pi t$.

19. When $t = 0$, $d = 0$. Therefore, we will use the equation with the sine function, $y = a\sin\omega t$, to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, and has an amplitude of 3 inches, $a = -3$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 1.5$$
$$2\pi = 1.5\omega$$
$$\omega = \frac{2\pi}{1.5} = \frac{4\pi}{3}$$

Substitute these values into $d = a\sin\omega t$. The equation for the object's simple harmonic motion is $d = -3\sin\frac{4\pi}{3}t$.

20. When $t = 0$, $d = 0$. Therefore, we will use the equation with the sine function, $y = a\sin\omega t$, to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, and has an amplitude of 5 centimeters, $a = -5$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 2.5$$
$$2\pi = 2.5\omega$$
$$\omega = \frac{2\pi}{2.5} = \frac{4\pi}{5}$$

Substitute these values into $d = a\sin\omega t$. The equation for the object's simple harmonic motion is $d = -5\sin\frac{4\pi}{5}t$.

21. We begin by identifying values for a and ω.

$d = 5\cos\frac{\pi}{2}t$, $a = 5$ and $\omega = \frac{\pi}{2}$

a. The maximum displacement from the rest position is the amplitude. Because $a = 5$, the maximum displacement is 5 inches.

b. The frequency, f, is $f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{2}}{2\pi} = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}$.

The frequency is $\frac{1}{4}$ inch per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 2\pi\cdot\frac{2}{\pi} = 4$$

The time required for one cycle is 4 seconds.

22. We begin by identifying values for a and ω.

$d = 10\cos 2\pi t$, $a = 10$ and $\omega = 2\pi$

a. The maximum displacement from the rest position is the amplitude.
Because $a = 10$, the maximum displacement is 10 inches.

b. The frequency, f, is $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1$.

The frequency is 1 inch per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$$

The time required for one cycle is 1 second.

23. We begin by identifying values for a and ω.

$d = -6\cos 2\pi t$, $a = -6$ and $\omega = 2\pi$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -6$, the maximum displacement is 6 inches.

b. The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1.$$

The frequency is 1 inch per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$$

The time required for one cycle is 1 second.

24. We begin by identifying values for a and ω.

$d=-8\cos\frac{\pi}{2}t,\ a=-8$ and $\omega=\frac{\pi}{2}$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -8$, the maximum displacement is 8 inches.

b. The frequency, f, is $f=\frac{\omega}{2\pi}=\frac{\frac{\pi}{2}}{2\pi}=\frac{1}{4}$.

The frequency is $\frac{1}{4}$ inch per second.

c. The time required for one cycle is the period.

period = $\frac{2\pi}{\omega}=\frac{2\pi}{\frac{\pi}{2}}=2\pi\cdot\frac{2}{\pi}=4$

The time required for one cycle is 4 seconds.

25. We begin by identifying values for a and ω.

$d=\frac{1}{2}\sin 2t,\ a=\frac{1}{2}$ and $\omega=2$

a. The maximum displacement from the rest position is the amplitude.

Because $a=\frac{1}{2}$, the maximum displacement is $\frac{1}{2}$ inch.

b. The frequency, f, is

$f=\frac{\omega}{2\pi}=\frac{2}{2\pi}=\frac{1}{\pi}\approx 0.32.$

The frequency is approximately 0.32 cycle per second.

c. The time required for one cycle is the period.

period = $\frac{2\pi}{\omega}=\frac{2\pi}{2}=\pi\approx 3.14$

The time required for one cycle is approximately 3.14 seconds.

26. We begin by identifying values for a and ω.

$d=\frac{1}{3}\sin 2t,\ a=\frac{1}{3}$ and $\omega=2$

a. The maximum displacement from the rest position is the amplitude.

Because $a=\frac{1}{3}$, the maximum displacement is $\frac{1}{3}$ inch.

b. The frequency, f, is $f=\frac{\omega}{2\pi}=\frac{2}{2\pi}=\frac{1}{\pi}\approx 0.32$.

The frequency is approximately 0.32 cycle per second.

c. The time required for one cycle is the period.

period = $\frac{2\pi}{\omega}=\frac{2\pi}{2}=\pi\approx 3.14$

The time required for one cycle is approximately 3.14 seconds.

27. We begin by identifying values for a and ω.

$d=-5\sin\frac{2\pi}{3}t,\ a=-5$ and $\omega=\frac{2\pi}{3}$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -5$, the maximum displacement is 5 inches.

b. The frequency, f, is

$f=\frac{\omega}{2\pi}=\frac{\frac{2\pi}{3}}{2\pi}=\frac{2\pi}{3}\cdot\frac{1}{2\pi}=\frac{1}{3}.$

The frequency is $\frac{1}{3}$ cycle per second.

c. The time required for one cycle is the period.

period = $\frac{2\pi}{\omega}=\frac{2\pi}{\frac{2\pi}{3}}=2\pi\cdot\frac{3}{2\pi}=3$

The time required for one cycle is 3 seconds.

28. We begin by identifying values for a and ω.

$d=-4\sin\frac{3\pi}{2}t,\ a=-4$ and $\omega=\frac{3\pi}{2}$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -4$, the maximum displacement is 4 inches.

b. The frequency, f, is

$f=\frac{\omega}{2\pi}=\frac{\frac{3\pi}{2}}{2\pi}=\frac{3\pi}{2}\cdot\frac{1}{2\pi}=\frac{3}{4}.$

The frequency is $\frac{3}{4}$ cycle per second.

c. The time required for one cycle is the period.

period = $\frac{2\pi}{\omega}=\frac{2\pi}{\frac{3\pi}{2}}=2\pi\cdot\frac{2}{3\pi}=\frac{4}{3}$

The required time for one cycle is $\frac{4}{3}$ seconds.

29. $x = 500\tan 40° + 500\tan 25°$
$x \approx 653$

30. $x = 100\tan 20° + 100\tan 8°$
$x \approx 50$

31. $x = 600\tan 28° - 600\tan 25°$
$x \approx 39$

32. $x = 400\tan 40° - 400\tan 28°$
$x \approx 123$

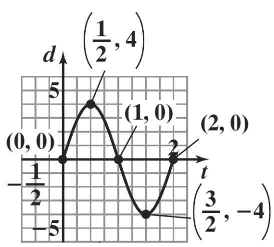
33. $x = \dfrac{300}{\tan 34°} - \dfrac{300}{\tan 64°}$
$x \approx 298$

34. $x = \dfrac{500}{\tan 20°} - \dfrac{500}{\tan 48°}$
$x \approx 924$

35. $x = \dfrac{400\tan 40°\tan 20°}{\tan 40° - \tan 20°}$
$x \approx 257$

36. $x = \dfrac{100\tan 43°\tan 38°}{\tan 43° - \tan 38°}$
$x \approx 482$

37. $d = 4\cos\left(\pi t - \dfrac{\pi}{2}\right)$



$d = 4\cos\left(\pi t - \frac{\pi}{2}\right)$

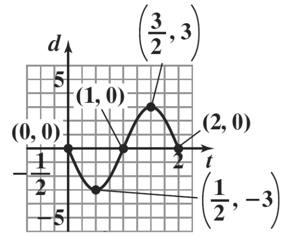
a. 4 in.

b. $\dfrac{1}{2}$ in. per sec

c. 2 sec

d. $\dfrac{1}{2}$

38. $d = 3\cos\left(\pi t + \dfrac{\pi}{2}\right)$



$d = 3\cos\left(\pi t + \frac{\pi}{2}\right)$

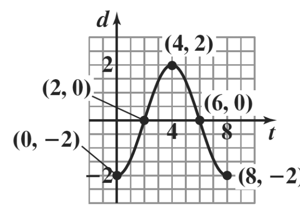
a. 3 in.

b. $\dfrac{1}{2}$ in. per sec

c. 2 sec

d. $-\dfrac{1}{2}$

39. $d = -2\sin\left(\dfrac{\pi t}{4} + \dfrac{\pi}{2}\right)$



$d = -2\sin\left(\frac{\pi}{4}t + \frac{\pi}{2}\right)$

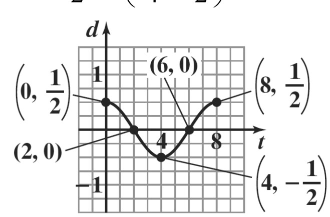
a. 2 in.

b. $\dfrac{1}{8}$ in. per sec

c. 8 sec

d. -2

40. $d = -\dfrac{1}{2}\sin\left(\dfrac{\pi t}{4} - \dfrac{\pi}{2}\right)$



$d = -\frac{1}{2}\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right)$

a. $\frac{1}{2}$ in.

b. $\frac{1}{8}$ in. per sec

c. 8 sec

d. 2

41. Using a right triangle, we have a known angle, an unknown opposite side, *a*, and a known adjacent side. Therefore, use tangent function.

$$\tan 21.3° = \frac{a}{5280}$$
$$a = 5280 \tan 21.3° \approx 2059$$

The height of the tower is approximately 2059 feet.

42. $30 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 90 \text{ ft}$

Using a right triangle, we have a known angle, an unknown opposite side, *a*, and a known adjacent side. Therefore, use the tangent function.

$$\tan 38.7° = \frac{a}{90}$$
$$a = 90 \tan 38.7° \approx 72$$

The height of the building is approximately 72 feet.

43. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *a*. Therefore, use the tangent function.

$$\tan 23.7° = \frac{305}{a}$$
$$a = \frac{305}{\tan 23.7°} \approx 695$$

The ship is approximately 695 feet from the statue's base.

44. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *a*. Therefore, use the tangent function.

$$\tan 22.3° = \frac{200}{a}$$
$$a = \frac{200}{\tan 22.3°} \approx 488$$

The ship is about 488 feet offshore.

45. The angle of depression from the helicopter to point *P* is equal to the angle of elevation from point *P* to the helicopter. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *d*. Therefore, use the tangent function.

$$\tan 36° = \frac{1000}{d}$$
$$d = \frac{1000}{\tan 36°} \approx 1376$$

The island is approximately 1376 feet off the coast.

46. The angle of depression from the helicopter to the stolen car is equal to the angle of elevation from the stolen car to the helicopter. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *d*. Therefore, use the tangent function.

$$\tan 72° = \frac{800}{d}$$
$$d = \frac{800}{\tan 72°} \approx 260$$

The stolen car is approximately 260 feet from a point directly below the helicopter.

47. Using a right triangle, we have an unknown angle, *A*, a known opposite side, and a known hypotenuse. Therefore, use the sine function.

$$\sin A = \frac{6}{23}$$
$$A = \sin^{-1}\left(\frac{6}{23}\right) \approx 15.1°$$

The ramp makes an angle of approximately 15.1° with the ground.

48. Using a right triangle, we have an unknown angle, *A*, a known opposite side, and a known adjacent side. Therefore, use the tangent function.

$$\tan A = \frac{250}{40}$$
$$A = \tan^{-1}\left(\frac{250}{40}\right) \approx 80.9°$$

The angle of elevation of the sun is approximately 80.9°.

49. Using the two right triangles, we have a known angle, an unknown opposite side, *a* in the smaller triangle, *b* in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 19.2° = \frac{a}{125}$$
$$a = 125 \tan 19.2° \approx 43.5$$
$$\tan 31.7° = \frac{b}{125}$$
$$b = 125 \tan 31.7° \approx 77.2$$

The balloon rises approximately 77.2 – 43.5 or 33.7 feet.

50. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, an unknown opposite side, *a* in the smaller triangle, *b* in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 53° = \frac{a}{330}$$
$$a = 330 \tan 53° = 437.9$$
$$\tan 63° = \frac{b}{330}$$
$$b = 330 \tan 63° \approx 647.7$$

The height of the flagpole is approximately 647.7 – 437.9, or 209.8 feet (or 209.7 feet).

51. Using a right triangle, we have a known angle, a known hypotenuse, and unknown sides. To find the opposite side, *a*, use the sine function.

$$\sin 53° = \frac{a}{150}$$
$$a = 150 \sin 53° \approx 120$$

To find the adjacent side, *b*, use the cosine function.

$$\cos 53° = \frac{b}{150}$$
$$b = 150 \cos 53° \approx 90$$

The boat has traveled approximately 90 miles north and 120 miles east.

52. Using a right triangle, we have a known angle, a known hypotenuse, and unknown sides. To find the opposite side, *a*, use the sine function.

$$\sin 64° = \frac{a}{40}$$
$$a = 40 \sin 64° \approx 36$$

To find the adjacent side, *b*, use the cosine function.

$$\cos 64° = \frac{b}{40}$$
$$b = 40 \cos 64° \approx 17.5$$

The boat has traveled about 17.5 mi south and 36 mi east.

53. The bearing from the fire to the second ranger is N 28° E. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *b*. Therefore, use the tangent function.

$$\tan 28° = \frac{7}{b}$$
$$b = \frac{7}{\tan 28°} \approx 13.2$$

The first ranger is 13.2 miles from the fire, to the nearest tenth of a mile.

54. The bearing from the lighthouse to the second ship is N 34° E. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *b*. Therefore, use the tangent function.

$$\tan 34° = \frac{9}{b}$$
$$b = \frac{9}{\tan 34°} \approx 13.3$$

The first ship is about 13.3 miles from the lighthouse, to the nearest tenth of a mile.

55. Using a right triangle, we have a known adjacent side, a known opposite side, and an unknown angle, *A*. Therefore, use the tangent function.

$$\tan A = \frac{1.5}{2}$$
$$A = \tan\left(\frac{1.5}{2}\right) \approx 37°$$

We need the acute angle between the ray that runs from your house through your location, and the north-south line through your house. This angle measures approximately $90° - 37° = 53°$. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from your house to you is N 53° W.

56. Using a right triangle, we have a known adjacent side, a known opposite side, and an unknown angle, *A*. Therefore, use the tangent function.

$$\tan A = \frac{6}{9}$$
$$A = \tan^{-1}\left(\frac{6}{9}\right) \approx 34°$$

We need the acute angle between the ray that runs from the ship through the harbor, and the north-south line through the ship. This angle measures $90° - 34° = 56°$. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from the ship to the harbor is N 56° W. The ship should use a bearing of N 56° W to sail directly to the harbor.

57. To find the jet's bearing from the control tower, consider a north-south line passing through the tower. The acute angle from this line to the ray on which the jet lies is $35° + \theta$. Because we are measuring the angle from the north side of the line and the jet is east of the tower, the jet's bearing from the tower is N $(35° + \theta)$ E. To find θ, use a right triangle and the tangent function.

$$\tan \theta = \frac{7}{5}$$
$$\theta = \tan^{-1}\left(\frac{7}{5}\right) \approx 54.5°$$

Thus, $35° + \theta = 35° + 54.5° = 89.5°$.
The jet's bearing from the control tower is N 89.5° E.

58. To find the ship's bearing from the port, consider a north-south line passing through the port. The acute angle from this line to the ray on which the ship lies is $40° + \theta$. Because we are measuring the angle from the south side of the line and the ship is west of the port, the ship's bearing from the port is S $(40° + \theta)$ W. To find θ, use a right triangle and the tangent function.

$$\tan\theta = \frac{11}{7}$$
$$\theta = \tan^{-1}\left(\frac{11}{7}\right) \approx 57.5°$$

Thus, $40° + \theta = 40° + 57.5 = 97.5°$. Because this angle is over 90° we subtract this angle from 180° to find the bearing from the north side of the north-south line. The bearing of the ship from the port is N 82.5° W.

59. The frequency, *f*, is $f = \frac{\omega}{2\pi}$, so

$$\frac{1}{2} = \frac{\omega}{2\pi}$$
$$\omega = \frac{1}{2} \cdot 2\pi = \pi$$

Because the amplitude is 6 feet, $a = 6$. Thus, the equation for the object's simple harmonic motion is $d = 6\sin \pi t$.

60. The frequency, *f*, is $f = \frac{\omega}{2\pi}$, so

$$\frac{1}{4} = \frac{\omega}{2\pi}$$
$$\omega = \frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$$

Because the amplitude is 8 feet, $a = 8$. Thus, the equation for the object's simple harmonic motion is $d = 8\sin\frac{\pi}{2}t$.

61. The frequency, *f*, is $f = \frac{\omega}{2\pi}$, so

$$264 = \frac{\omega}{2\pi}$$
$$\omega = 264 \cdot 2\pi = 528\pi$$

Thus, the equation for the tuning fork's simple harmonic motion is $d = \sin 528\pi t$.

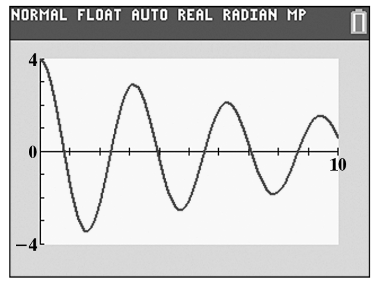
62. The frequency, *f*, is $f = \frac{\omega}{2\pi}$, so

$$98{,}100{,}000 = \frac{\omega}{2\pi}$$
$$\omega = 98{,}100{,}000 \cdot 2\pi = 196{,}200{,}000\pi$$

Thus, the equation for the radio waves' simple harmonic motion is $d = \sin 196{,}200{,}000\pi t$.

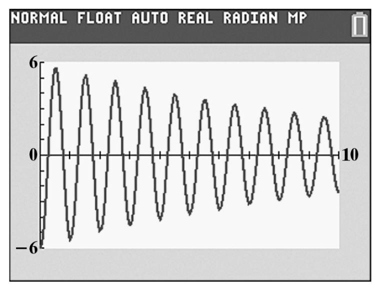
63. – 69. Answers may vary.

70. $y = 4e^{-0.1x}\cos 2x$



3 complete oscillations occur.

71. $y = -6e^{-0.09x}\cos 2\pi x$



10 complete oscillations occur.

72. makes sense

73. does not make sense; Explanations will vary. Sample explanation: When using bearings, the angle must be less than $90°$.

74. does not make sense; Explanations will vary. Sample explanation: When using bearings, north and south are listed before east and west.

75. does not make sense; Explanations will vary. Sample explanation: Frequency and Period are inverses of each other. If the period is 10 seconds then the frequency is $\frac{1}{10} = 0.1$ oscillations per second.

76. Using the right triangle, we have a known angle, an unknown opposite side, r, and an unknown hypotenuse, $r + 112$. Because both sides are in terms of the variable r, we can find r by using the sine function.

$$\sin 76.6° = \frac{r}{r+112}$$
$$\sin 76.6°(r+112) = r$$
$$r\sin 76.6° + 112\sin 76.6° = r$$
$$r - r\sin 76.6° = 112\sin 76.6°$$
$$r\left(1-\sin 76.6°\right) = 112\sin 76.6°$$
$$r = \frac{112\sin 76.6°}{1-\sin 76.6°} \approx 4002$$

The Earth's radius is approximately 4002 miles.

77. Let d be the adjacent side to the 40° angle. Using the right triangles, we have a known angle and unknown sides in both triangles. Use the tangent function.

$$\tan 20° = \frac{h}{75+d}$$
$$h = (75+d)\tan 20°$$

Also, $\tan 40° = \dfrac{h}{d}$
$$h = d\tan 40°$$

Using the transitive property we have
$$(75+d)\tan 20° = d\tan 40°$$
$$75\tan 20° + d\tan 20° = d\tan 40°$$
$$d\tan 40° - d\tan 20° = 75\tan 20°$$
$$d(\tan 40° - \tan 20°) = 75\tan 20°$$
$$d = \frac{75\tan 20°}{\tan 40° - \tan 20°}$$

Thus, $h = d\tan 40°$
$$= \frac{75\tan 20°}{\tan 40° - \tan 20°}\tan 40° \approx 48$$

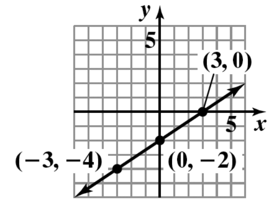
The height of the building is approximately 48 feet.

78. Answers may vary.

79. Let x = the amount invested at 6%.
Let $3000 - x$ = the amount invested at 8%.
$$0.06x + 0.08(3000 - x) = 230$$
$$0.06x + 240 - 0.08x = 230$$
$$-0.02x + 240 = 230$$
$$-0.02x = -10$$
$$x = \frac{-10}{-0.02}$$
$$x = 500$$
$$3000 - x = 2500$$
\$500 was invested at 6% and \$2500 was invested at 8%.

80. $(x-1)^2 = 5$
$$x - 1 = \pm\sqrt{5}$$
$$x = 1 \pm \sqrt{5}$$
The solution set is $\left\{1+\sqrt{5}, 1-\sqrt{5}\right\}$.

81. The slope m is $\frac{2}{3}$ and the y-intercept is –2, so one point on the line is (0, –2). We can find a second point on the line by using the slope $m = \frac{2}{3} = \frac{\text{Rise}}{\text{Run}}$: starting at the point (0, –2), move 2 units up and 3 units to the right, to obtain the point (3, 0).



$f(x) = \frac{2}{3}x - 2$

82. $\sec x \cot x = \dfrac{1}{\cos x}\cdot\dfrac{\cos x}{\sin x} = \dfrac{1}{\sin x}$ or $\csc x$

83. $\tan x \csc x \cos x = \dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\sin x}\cdot\dfrac{\cos x}{1} = 1$

84. $\sec x + \tan x = \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x} = \dfrac{1+\sin x}{\cos x}$

Chapter 4 Review Exercises

1. The radian measure of a central angle is the length of the intercepted arc divided by the circle's radius.
$$\theta = \frac{27}{6} = 4.5 \text{ radians}$$

2. $15° = 15°\cdot\dfrac{\pi \text{ radians}}{180°} = \dfrac{15\pi}{180}$ radian
$$= \frac{\pi}{12} \text{ radian}$$

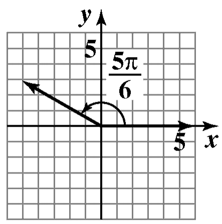
3. $120° = 120°\cdot\dfrac{\pi \text{ radians}}{180°} = \dfrac{120\pi}{180}$ radians
$$= \frac{2\pi}{3} \text{ radians}$$

4. $315° = 315°\cdot\dfrac{\pi \text{ radians}}{180°} = \dfrac{315\pi}{180}$ radians
$$= \frac{7\pi}{4} \text{ radians}$$

5. $\dfrac{5\pi}{3}$ radians $= \dfrac{5\pi}{3}$ radians $\cdot\dfrac{180°}{\pi \text{ radians}}$
$$= \frac{5\cdot 180°}{3} = 300°$$

6. $\frac{7\pi}{5}\text{ radians} = \frac{7\pi}{5}\text{ radians}\cdot\frac{180°}{\pi\text{ radians}}$
$= \frac{7\cdot 180°}{5} = 252°$

7. $-\frac{5\pi}{6}\text{ radians} = -\frac{5\pi}{6}\text{ radians}\cdot\frac{180°}{\pi\text{ radians}}$
$= -\frac{5\cdot 180°}{6} = -150°$

8.


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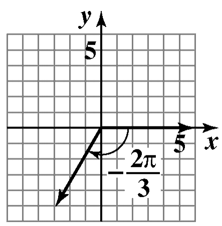

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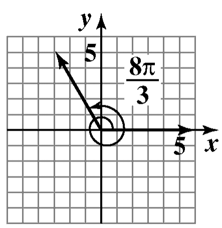

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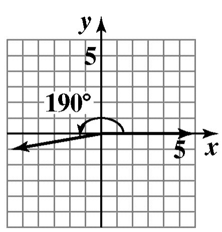
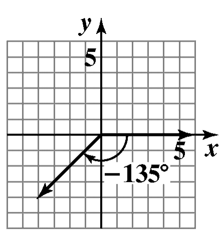


12.


13. $400° - 360° = 40°$

14. $-445° + (2)360° = 275°$

15. $\frac{13\pi}{4} - 2\pi = \frac{13\pi}{4} - \frac{8\pi}{4} = \frac{5\pi}{4}$

16. $\frac{31\pi}{6} - (2)2\pi = \frac{31\pi}{6} - \frac{24\pi}{6} = \frac{7\pi}{6}$

17. $-\frac{8\pi}{3} + (2)2\pi = -\frac{8\pi}{3} + \frac{12\pi}{3} = \frac{4\pi}{3}$

18. $135° = 135°\cdot\frac{\pi\text{ radians}}{180°} = \frac{135\cdot\pi}{180}\text{ radians}$
$= \frac{3\pi}{4}\text{ radians}$
$s = r\theta$
$s = (10\text{ ft})\left(\frac{3\pi}{4}\right) = \frac{15\pi}{2}\text{ ft} \approx 23.56\text{ ft}$

19. $\frac{10.3\text{ revolutions}}{1\text{ minute}}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$
$= \frac{20.6\pi\text{ radians}}{1\text{ minute}} = 20.6\pi\text{ radians per minute}$

20. Use $v = r\omega$ where v is the linear speed and ω is the angular speed in radians per minute.
$\omega = \frac{2250\text{ revolutions}}{1\text{ minute}}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$
$= 4500\pi\text{ radians per minute}$
$v = 3\text{ feet}\left(\frac{4500\pi}{\text{minute}}\right) = \frac{13{,}500\pi\text{ feet}}{\text{min}}$
$\approx 42{,}412\text{ ft per min}$

21. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$
$\sin t = y = -\frac{3}{5}$
$\cos t = x = -\frac{4}{5}$
$\tan t = \frac{y}{x} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$
$\csc t = \frac{1}{y} = -\frac{5}{3}$
$\sec t = \frac{1}{x} = -\frac{5}{4}$
$\cot t = \frac{x}{y} = \frac{4}{3}$

22. $P\left(\frac{8}{17},-\frac{15}{17}\right)$

$\sin t = y = -\frac{15}{17}$

$\cos t = x = \frac{8}{17}$

$\tan t = \frac{y}{x} = \frac{-\frac{15}{17}}{\frac{8}{17}} = -\frac{15}{8}$

$\csc t = \frac{1}{y} = -\frac{17}{15}$

$\sec t = \frac{1}{x} = \frac{17}{8}$

$\cot t = \frac{x}{y} = -\frac{8}{15}$

23. $\sec\frac{5\pi}{6} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2\sqrt{3}}{3}$

24. $\tan\frac{4\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

25. $\sec\frac{\pi}{2}$ is undefined.

26. cot π is undefined.

27. $\sin t = \frac{2}{\sqrt{7}}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{2}{\sqrt{7}}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{4}{7}$

$\cos t = \sqrt{\frac{3}{7}} = \frac{\sqrt{21}}{7}$

Because $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive.

$\tan t = \frac{\frac{2}{\sqrt{7}}}{\sqrt{\frac{3}{7}}} = \frac{2\sqrt{3}}{3}$

$\csc t = \frac{\sqrt{7}}{2}$

$\sec t = \frac{\sqrt{21}}{3}$

$\cot t = \frac{\sqrt{3}}{2}$

28. $\tan 4.7 \cot 4.7 = \tan 4.7\left(\frac{1}{\tan 4.7}\right) = 1$

29. $\sin^2\frac{\pi}{17} + \cos^2\frac{\pi}{17} = 1$ because $\sin^2 t + \cos^2 t = 1$.

30. $\tan^2 1.4 - \sec^2 1.4 = -\left(\sec^2 1.4 - \tan^2 1.4\right)$
$= -1$

31. Use the Pythagorean Theorem to find the hypotenuse, c.

$c^2 = a^2 + b^2$

$c = \sqrt{8^2 + 5^2} = \sqrt{64 + 25} = \sqrt{89}$

$\sin\theta = \frac{5}{\sqrt{89}} = \frac{5\sqrt{89}}{\sqrt{89}}$

$\cos\theta = \frac{8}{\sqrt{89}} = \frac{8\sqrt{89}}{\sqrt{89}}$

$\tan\theta = \frac{5}{8}$

$\csc\theta = \frac{\sqrt{89}}{5}$

$\sec\theta = \frac{\sqrt{89}}{8}$

$\cot\theta = \frac{3}{5}$

32. $\sin\frac{\pi}{6} + \tan^2\frac{\pi}{3} = \frac{1}{2} + \left(\sqrt{3}\right)^2$
$= \frac{1}{2} + 3$
$= \frac{7}{2}$

33. $\cos^2\frac{\pi}{4} + \tan^2\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}\right)^2 - (1)^2$
$= \frac{1}{2} - 1$
$= -\frac{1}{2}$

34. $\sec^2\frac{\pi}{5} - \tan^2\frac{\pi}{5} = 1$

35. $\cos\frac{2\pi}{9}\sec\frac{2\pi}{9} = 1$

36. $\sin 70° = \cos(90° - 70°) = \cos 20°$

37. $\cos\dfrac{\pi}{2} = \sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{2}\right) = \sin 0$

38. $\tan 23° = \dfrac{a}{100}$

$a = 100\tan 23°$

$a \approx 100(0.4245) \approx 42\,\text{mm}$

39. $\sin 61° = \dfrac{20}{c}$

$c = \dfrac{20}{\sin 61°}$

$c \approx \dfrac{20}{0.8746} \approx 23\,\text{cm}$

40. $\sin 48° = \dfrac{a}{50}$

$a = 50\sin 48°$

$a \approx 50(0.7431) \approx 37\,\text{in.}$

41. $\sin\theta = \dfrac{y}{r} = \dfrac{1}{4}$

$x^2 + y^2 = r^2$

$x^2 + 1^2 = 4^2$

$x^2 = 15$

$x = \sqrt{15}$

$\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta = \dfrac{x}{y} = \dfrac{\sqrt{15}}{1} = \sqrt{15}$

42. $\dfrac{1}{2}\text{ mi.} = \dfrac{1}{2}\cdot 5280\text{ ft} = 2640\text{ ft}$

$\sin 17° = \dfrac{a}{2640}$

$a = 2640\cdot\sin 17°$

$a \approx 2640(0.2924) \approx 772$

The hiker gains 772 feet of altitude.

43. $\tan 32° = \dfrac{d}{50}$

$d = 50\tan 32°$

$d \approx 50(0.6249) \approx 31$

The distance across the lake is about 31 meters.

44. $\tan\theta = \dfrac{6}{4}$

Use a calculator in degree mode to find θ.

Scientific Calculator
6 [÷] 4 [=] [TAN^{-1}]

Graphing Calculator
[TAN^{-1}] [(] 6 [÷] 4 [)] [ENTER]

The display should show approximately 56. Thus, the angle of elevation of the sun is approximately 56°.

45. We need values for x, y, and r. Because $P = (-1, -5)$ is a point on the terminal side of θ, $x = -1$ and $y = -5$. Furthermore,

$r = \sqrt{(-1)^2 + (-5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$\sin\theta = \dfrac{y}{r} = \dfrac{-5}{\sqrt{26}} = \dfrac{-5\sqrt{26}}{\sqrt{26}\cdot\sqrt{26}} = -\dfrac{5\sqrt{26}}{26}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-1}{\sqrt{26}} = \dfrac{-1\sqrt{26}}{\sqrt{26}\cdot\sqrt{26}} = -\dfrac{\sqrt{26}}{26}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-5}{-1} = 5$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{26}}{-5} = -\dfrac{\sqrt{26}}{5}$

$\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{26}}{-1} = -\sqrt{26}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-1}{-5} = \dfrac{1}{5}$

46. We need values for x, y, and r. Because $P = (0, -1)$ is a point on the terminal side of θ, $x = 0$ and $y = -1$. Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-1)^2}$$
$$= \sqrt{0+1} = \sqrt{1} = 1$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-1}{1} = -1$$
$$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$$
$$\tan\theta = \frac{y}{x} = \frac{-1}{0}, \text{ undefined}$$

$$\csc\theta = \frac{r}{y} = \frac{1}{-1} = -1$$
$$\sec\theta = \frac{r}{x} = \frac{1}{0}, \text{ undefined}$$
$$\cot\theta = \frac{x}{y} = \frac{0}{-1} = 0$$

47. Because $\tan\theta > 0$, θ cannot lie in quadrant II and quadrant IV; the tangent function is negative in those two quadrants. Thus, with $\tan\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\sec\theta > 0$. Because quadrant I is the only quadrant in which the tangent is positive and the secant is positive, we conclude that θ lies in quadrant I.

48. Because $\tan\theta > 0$, θ cannot lie in quadrant II and quadrant IV; the tangent function is negative in those two quadrants. Thus, with $\tan\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which the tangent is positive and the cosine is negative, we conclude that θ lies in quadrant III.

49. Because the cosine is positive and the sine is negative, θ lies in quadrant IV. In quadrant IV, x is positive and y is negative. Thus, $\cos\theta = \frac{2}{5} = \frac{x}{r}$, $x =$ 2, $r = 5$. Furthermore,

$$x^2 + y^2 = r^2$$
$$2^2 + y^2 = 5^2$$
$$y^2 = 25 - 4 = 21$$
$$y = -\sqrt{21}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{21}}{5} = -\frac{\sqrt{21}}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{21}}{2} = -\frac{\sqrt{21}}{2}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{-\sqrt{21}} = -\frac{5\cdot\sqrt{21}}{\sqrt{21}\cdot\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{2}$$
$$\cot\theta = \frac{x}{y} = \frac{2}{-\sqrt{21}} = -\frac{2\sqrt{21}}{\sqrt{21}\cdot\sqrt{21}} = -\frac{2\sqrt{21}}{21}$$

50. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II x is negative and y is positive. Thus,

$$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3},\ x = -3,\ y = 1.$$

Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1\cdot\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = \frac{\sqrt{10}}{10}$$
$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{-3}{1} = -3$$

51. Because the cotangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III x and y are both negative. Thus,

$$\cot\theta = \frac{3}{1} = \frac{x}{y} = \frac{-3}{-1},\ x = -3,\ y = -1.$$

Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Now that we know *x*, *y*, and *r*, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{-3} = \frac{1}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-1} = -\sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

52. Because $265°$ lies between 180° and 270°, it is in quadrant III.
The reference angle is $\theta' = 265° - 180° = 85°$.

53. Because $\frac{5\pi}{8}$ lies between $\frac{\pi}{2} = \frac{4\pi}{8}$ and $\pi = \frac{8\pi}{8}$, it is in quadrant II.
The reference angle is $\theta' = \pi - \frac{5\pi}{8} = \frac{8\pi}{8} - \frac{5\pi}{8} = \frac{3\pi}{8}$.

54. Find the coterminal angle: $-410^{\circ} + (2)360^{\circ} = 310^{\circ}$
Find the reference angle: $360^{\circ} - 310^{\circ} = 50^{\circ}$

55. Find the coterminal angle: $\frac{17\pi}{6} - 2\pi = \frac{5\pi}{6}$
Find the reference angle: $2\pi - \frac{5\pi}{6} = \frac{\pi}{6}$

56. Find the coterminal angle: $-\frac{11\pi}{3} + 4\pi = \frac{\pi}{3}$
Find the reference angle: $\frac{\pi}{3}$

57. 240° lies in quadrant III.
The reference angle is
$\theta' = 240° - 180° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
In quadrant III, $\sin\theta < 0$, so
$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$.

58. 120° lies in quadrant II.
The reference angle is
$\theta' = 180° - 120° = 60°$.
$\tan 60° = \sqrt{3}$
In quadrant II, $\tan\theta < 0$, so
$\tan 120° = -\tan 60° = -\sqrt{3}$.

59. $\frac{7\pi}{4}$ lies in quadrant IV.
The reference angle is
$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.
$\sec\frac{\pi}{4} = \sqrt{2}$
In quadrant IV, $\sec\theta > 0$, so
$\sec\frac{7\pi}{4} = \sec\frac{\pi}{4} = \sqrt{2}$.

60. $\frac{11\pi}{6}$ lies in quadrant IV.
The reference angle is
$\theta' = 2\pi - \frac{11\pi}{6} = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$.
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
In quadrant IV, $\cos\theta > 0$, so $\cos\frac{11\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

61. –210° lies in quadrant II.
The reference angle is
$\theta' = 210° - 180° = 30°$.
$\cot 30° = \sqrt{3}$
In quadrant II, $\cot\theta < 0$, so
$\cot(-210°) = -\cot 30° = -\sqrt{3}$.

62. $-\frac{2\pi}{3}$ lies in quadrant III.
The reference angle is
$\theta' = \pi + \frac{-2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.
$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$
In quadrant III, $\csc\theta < 0$, so
$\csc\left(-\frac{2\pi}{3}\right) = -\csc\frac{\pi}{3} = -\frac{2\sqrt{3}}{3}$.

63. $-\frac{\pi}{3}$ lies in quadrant IV.

The reference angle is

$\theta' = \frac{\pi}{3}.$

$\sin \quad \frac{\pi}{3} \quad = \frac{\sqrt{3}}{2}$

In quadrant IV, $\sin\theta < 0$, so

$\sin \quad -\frac{\pi}{3} \quad = -\sin \quad \frac{\pi}{3} \quad = -\frac{\sqrt{3}}{2}.$

64. 495° lies in quadrant II.

$495° - 360° = 135°$

The reference angle is

$\theta' = 180° - 135° = 45°.$

$\sin 45° = \frac{\sqrt{2}}{2}$

In quadrant II, $\sin\theta > 0$, so

$\sin 495° = \sin 45° = \frac{\sqrt{2}}{2}.$

65. $\frac{13\pi}{4}$ lies in quadrant III.

$\frac{13\pi}{4} - 2\pi = \frac{13\pi}{4} - \frac{8\pi}{4} = \frac{5\pi}{4}$

The reference angle is

$\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}.$

$\tan\frac{\pi}{4} = 1$

In quadrant III, $\tan\theta > 0$, so $\tan\frac{13\pi}{4} = \tan\frac{\pi}{4} = 1$.

66. $\sin\frac{22\pi}{3} = \sin \quad \frac{22\pi}{3} - 6\pi$

$= \sin\frac{4\pi}{3}$

$= -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

67. $\cos \quad -\frac{35\pi}{6} \quad = \cos \quad -\frac{35\pi}{6} + 6\pi$

$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

68. The equation $y = 3\sin 4x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 4$. The amplitude is $|A| = |3| = 3.$

The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

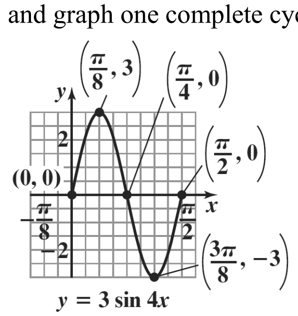
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{\pi}{8}$	$\frac{\pi}{8}$, 3
$\frac{\pi}{4}$	$\frac{\pi}{4}$, 0
$\frac{3\pi}{8}$	$\frac{3\pi}{8}$, −3
$\frac{\pi}{2}$	(2π, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



69. The equation $y = -2\cos 2x$ is of the form $y = A\cos Bx$ with $A = -2$ and $B = 2$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

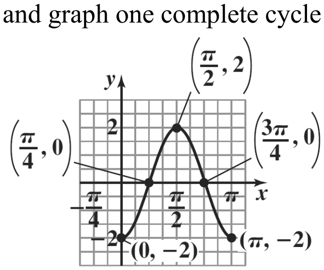
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	coordinates
0	$(0, -2)$
$\frac{\pi}{4}$	$\frac{\pi}{4}, 0$
$\frac{\pi}{2}$	$\frac{\pi}{2}, 2$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 0$
π	$(\pi, -2)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -2\cos 2x$

70. The equation $y = 2\cos\frac{1}{2}x$ is of the form $y = A\cos Bx$ with $A = 2$ and $B = \frac{1}{2}$. The amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$.

The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

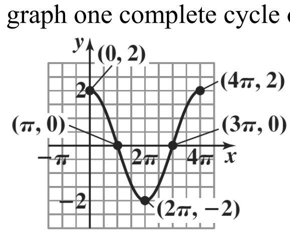
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	coordinates
0	$(0, 2)$
π	$(\pi, 0)$
2π	$(2\pi, -2)$
3π	$(3\pi, 0)$
4π	$(4\pi, 2)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 2\cos\frac{1}{2}x$

71. The equation $y = \frac{1}{2}\sin\frac{\pi}{3}x$ is of the form $y = A\sin Bx$ with $A = \frac{1}{2}$ and $B = \frac{\pi}{3}$. The amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6$. The quarter-period is $\frac{6}{4} = \frac{3}{2}$. The cycle begins at $x =$ 0. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{3}{2} = 3$

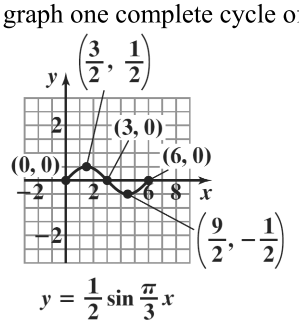
$x = 3 + \frac{3}{2} = \frac{9}{2}$

$x = \frac{9}{2} + \frac{3}{2} = 6$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{3}{2}$	$\frac{3}{2}, \frac{1}{2}$
3	(3, 0)
$\frac{9}{2}$	$\frac{9}{2}, -\frac{1}{2}$
6	(6, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



72. The equation $y = -\sin\pi x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \pi$. The amplitude is $|A| = |-1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

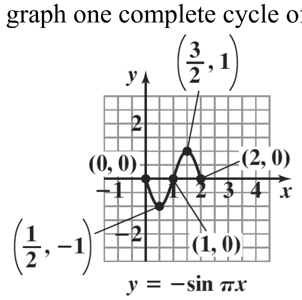
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{1}{2}$	$\frac{1}{2}, -1$
1	(1, 0)
$\frac{3}{2}$	$\frac{3}{2}, 1$
2	(2, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



73. The equation $y = 3\cos\frac{x}{3}$ is of the form $y = A\cos Bx$ with $A = 3$ and $B = \frac{1}{3}$. The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

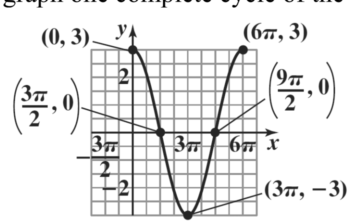
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	coordinates
0	$(0, 3)$
$\frac{3\pi}{2}$	$\frac{3\pi}{2}, 0$
3π	$(3\pi, -3)$
$\frac{9\pi}{2}$	$\frac{9\pi}{2}, 0$
6π	$(6\pi, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\cos\frac{x}{3}$

74. The equation $y = 2\sin(x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 2$, $B = 1$, and $C = \pi$. The amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{1} = \pi$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The cycle begins at $x = \pi$. Add quarter-periods to generate x-values for the key points.

$x = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

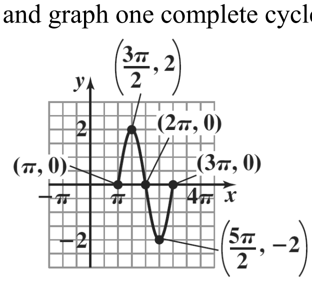
$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} + \frac{\pi}{2} = 3\pi$

Evaluate the function at each value of x.

x	coordinates
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\frac{3\pi}{2}, 2$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\frac{5\pi}{2}, -2$
3π	$(3\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 2\sin(x - \pi)$

75. $y=-3\cos(x+\pi)=-3\cos(x-(-\pi))$

The equation $y=-3\cos(x-(-\pi))$ is of the form $y=A\cos(Bx-C)$ with $A=-3$, $B=1$, and $C=-\pi$.

The amplitude is $|A|=|-3|=3$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$. The phase shift is $\frac{C}{B}=\frac{-\pi}{1}=-\pi$. The quarter-period is $\frac{2\pi}{4}=\frac{\pi}{2}$. The cycle begins at $x=-\pi$. Add quarter-periods to generate x-values for the key points.

$x=-\pi$

$x=-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$

$x=-\frac{\pi}{2}+\frac{\pi}{2}=0$

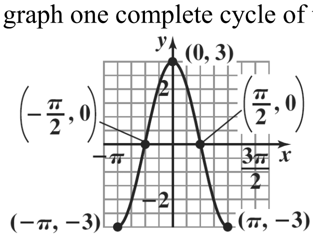
$x=0+\frac{\pi}{2}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

Evaluate the function at each value of x.

x	coordinates
$-\pi$	$(-\pi, -3)$
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, 0\right)$
0	$(0, 3)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
π	$(\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y=-3\cos(x+\pi)$

76. $y=\frac{3}{2}\cos\left(2x+\frac{\pi}{4}\right)=\frac{3}{2}\cos\left(2x-\left(-\frac{\pi}{4}\right)\right)$

The equation $y=\frac{3}{2}\cos\left(2x-\left(-\frac{\pi}{4}\right)\right)$ is of the form $y=A\cos(Bx-C)$ with $A=\frac{3}{2}$, $B=2$, and $C=-\frac{\pi}{4}$. The amplitude is $|A|=\left|\frac{3}{2}\right|=\frac{3}{2}$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$. The phase shift is $\frac{C}{B}=\frac{\frac{-\pi}{4}}{2}=-\frac{\pi}{4}\cdot\frac{1}{2}=-\frac{\pi}{8}$. The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x=-\frac{\pi}{8}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{\pi}{8}$

$x=-\frac{\pi}{8}+\frac{\pi}{4}=\frac{\pi}{8}$

$x=\frac{\pi}{8}+\frac{\pi}{4}=\frac{3\pi}{8}$

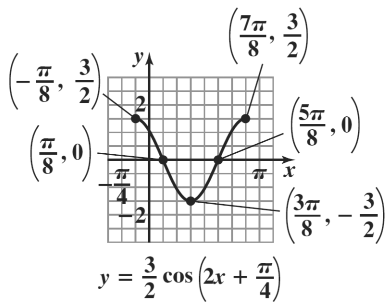
$x=\frac{3\pi}{8}+\frac{\pi}{4}=\frac{5\pi}{8}$

$x=\frac{5\pi}{8}+\frac{\pi}{4}=\frac{7\pi}{8}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{8}$	$\left(-\frac{\pi}{8}, \frac{3}{2}\right)$
$\frac{\pi}{8}$	$\left(\frac{\pi}{8}, 0\right)$
$\frac{3\pi}{8}$	$\left(\frac{3\pi}{8}, -\frac{3}{2}\right)$
$\frac{5\pi}{8}$	$\left(\frac{5\pi}{8}, 0\right)$
$\frac{7\pi}{8}$	$\left(\frac{7\pi}{8}, \frac{3}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



77. $y = \frac{5}{2}\sin\ 2x + \frac{\pi}{2}\ = \frac{5}{2}\sin\ 2x - \ -\frac{\pi}{2}$

The equation $y = \frac{5}{2}\sin\ 2x - \ -\frac{\pi}{2}$ is of

the form $y = A\sin(Bx - C)$ with $A = \frac{5}{2}$,

$B = 2$, and $C = -\frac{\pi}{2}$. The amplitude is

$|A| = \left|\frac{5}{2}\right| = \frac{5}{2}$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is

$\frac{C}{B} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2}\cdot\frac{1}{2} = -\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x = -\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

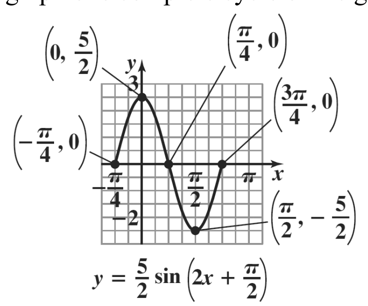
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{4}$	$(-\frac{\pi}{4}, 0)$
0	$0, \frac{5}{2}$
$\frac{\pi}{4}$	$\frac{\pi}{4}, 0$
$\frac{\pi}{2}$	$\frac{\pi}{2}, -\frac{5}{2}$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 0$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



78. The equation $y = -3\sin\ \frac{\pi}{3}x - 3\pi$ is of

the form $y = A\sin(Bx - C)$ with $A = -3$,

$B = \frac{\pi}{3}$, and $C = 3\pi$. The amplitude is $|A| = |-3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi\cdot\frac{3}{\pi} = 6$. The phase shift

is $\frac{C}{B} = \frac{3\pi}{\frac{\pi}{3}} = 3\pi\cdot\frac{3}{\pi} = 9$. The quarter-period is

$\frac{6}{4} = \frac{3}{2}$. The cycle begins at $x = 9$. Add quarter-periods to generate x-values for the key points.

$x = 9$

$x = 9 + \frac{3}{2} = \frac{21}{2}$

$x = \frac{21}{2} + \frac{3}{2} = 12$

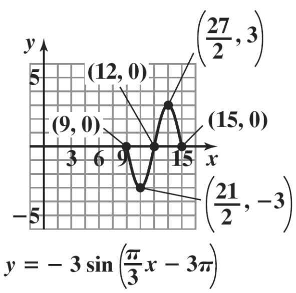
$x = 12 + \frac{3}{2} = \frac{27}{2}$

$x = \frac{27}{2} + \frac{3}{2} = 15$

Evaluate the function at each value of x.

x	coordinates
9	(9, 0)
$\frac{21}{2}$	$\frac{21}{2}, -3$
12	(12, 0)
$\frac{27}{2}$	$\frac{27}{2}, 3$
15	(15, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



79. The graph of $y = \sin 2x + 1$ is the graph of $y = \sin 2x$ shifted one unit upward. The period for both functions is $\frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

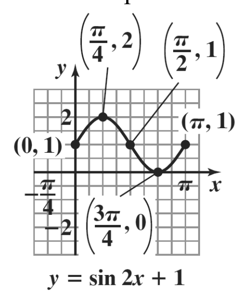
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	coordinates
0	(0, 1)
$\frac{\pi}{4}$	$\frac{\pi}{4}, 2$
$\frac{\pi}{2}$	$\frac{\pi}{2}, 1$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 0$
π	$(\pi, 1)$

By connecting the points with a smooth curve we obtain one period of the graph.



80. The graph of $y = 2\cos\frac{1}{3}x - 2$ is the graph of $y = 2\cos\frac{1}{3}x$ shifted two units downward. The period for both functions is $\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at x = 0. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

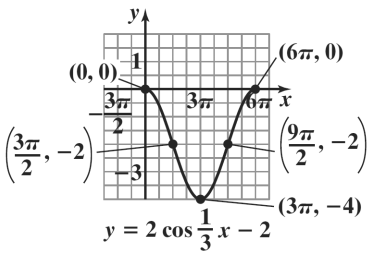
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{3\pi}{2}$	$\frac{3\pi}{2}, -2$
3π	$(3\pi, -4)$
$\frac{9\pi}{2}$	$\frac{9\pi}{2}, -2$
6π	$(6\pi, 0)$

By connecting the points with a smooth curve we obtain one period of the graph.



81. a. At midnight $x = 0$. Thus,

$$y = 98.6 + 0.3\sin \frac{\pi}{12}\cdot 0 - \frac{11\pi}{12}$$
$$= 98.6 + 0.3\sin -\frac{11\pi}{12}$$
$$\approx 98.6 + 0.3(-0.2588) \approx 98.52$$

The body temperature is about 98.52°F.

b. period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{12}} = 2\pi \cdot \frac{12}{\pi} = 24$ hours

c. Solve the equation

$$\frac{\pi}{12}x - \frac{11\pi}{12} = \frac{\pi}{2}$$
$$\frac{\pi}{12}x = \frac{\pi}{2} + \frac{11\pi}{12} = \frac{6\pi}{12} + \frac{11\pi}{12} = \frac{17\pi}{12}$$
$$x = \frac{17\pi}{12}\cdot\frac{12}{\pi} = 17$$

The body temperature is highest for $x = 17$.

$$y = 98.6 + 0.3\sin \frac{\pi}{12}\cdot 17 - \frac{11\pi}{12}$$
$$= 98.6 + 0.3\sin\frac{\pi}{2} = 98.6 + 0.3 = 98.9$$

17 hours after midnight, which is 5 P.M., the body temperature is 98.9°F.

d. Solve the equation

$$\frac{\pi}{12}x - \frac{11\pi}{12} = \frac{3\pi}{2}$$
$$\frac{\pi}{12}x = \frac{3\pi}{2} + \frac{11\pi}{12} = \frac{18\pi}{12} + \frac{11\pi}{12} = \frac{29\pi}{12}$$
$$x = \frac{29\pi}{12}\cdot\frac{12}{\pi} = 29$$

The body temperature is lowest for $x = 29$.

$$y = 98.6 + 0.3\sin \frac{\pi}{12}\cdot 29 - \frac{11\pi}{12}$$
$$= 98.6 + 0.3\sin \frac{3\pi}{2}$$
$$= 98.6 + 0.3(-1) = 98.3°$$

29 hours after midnight or 5 hours after midnight, at 5 A.M., the body temperature is 98.3°F.

e. The graph of $y = 98.6 + 0.3\sin \frac{\pi}{12}x - \frac{11\pi}{12}$ is of the form $y = D + A\sin(Bx - C)$ with $A = 0.3,\ B = \frac{\pi}{12},\ C = \frac{11\pi}{12},$ and $D = 98.6$. The amplitude is $|A| = |0.3| = 0.3$. The period from part (b) is 24. The quarter-period is $\frac{24}{4} = 6$. The phase shift is $\frac{C}{B} = \frac{\frac{11\pi}{12}}{\frac{\pi}{12}} = \frac{11\pi}{12}\cdot\frac{12}{\pi} = 11$. The cycle begins at $x = 11$. Add quarter-periods to generate x-values for the key points.

$x = 11$

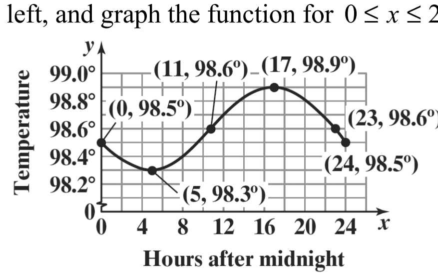
$x = 11 + 6 = 17$

$x = 17 + 6 = 23$

$x = 23 + 6 = 29$

$x = 29 + 6 = 35$

Evaluate the function at each value of x. The key points are (11, 98.6), (17, 98.9), (23, 98.6), (29, 98.3), (35, 98.6). Extend the pattern to the left, and graph the function for $0 \le x \le 24$.



82. Blue:
This is a sine wave with a period of 480.
Since the amplitude is 1, $A = 1$.

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{480} = \frac{\pi}{240}$$

The equation is $y = \sin\frac{\pi}{240}x$.

Red:
This is a sine wave with a period of 640.
Since the amplitude is 1, $A = 1$.

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{640} = \frac{\pi}{320}$$

The equation is $y = \sin\frac{\pi}{320}x$.

83. Solve the equations

$$2x = -\frac{\pi}{2} \quad \text{and} \quad 2x = \frac{\pi}{2}$$
$$x = \frac{-\frac{\pi}{2}}{2} \qquad x = \frac{\frac{\pi}{2}}{2}$$
$$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$$

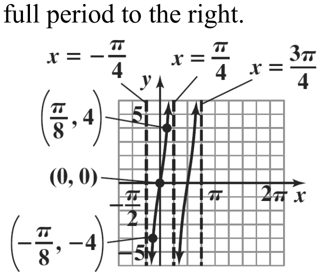
Thus, two consecutive asymptotes occur at
$x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{4} + \frac{\pi}{4}}{2} = \frac{0}{2} = 0$$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 4, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –4 and 4.

Use the two consecutive asymptotes. $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = 4\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$.

Continue the pattern and extend the graph another full period to the right.



$y = 4\tan 2x$

84. Solve the equations

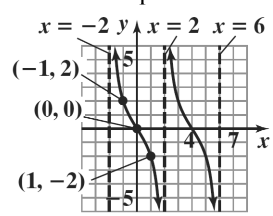
$$\frac{\pi}{4}x = -\frac{\pi}{2} \quad \text{and} \quad \frac{\pi}{4}x = \frac{\pi}{2}$$
$$x = -\frac{\pi}{2}\cdot\frac{4}{\pi} \qquad x = \frac{\pi}{2}\cdot\frac{4}{\pi}$$
$$x = -2 \qquad x = 2$$

Thus, two consecutive asymptotes occur at
$x = -2$ and $x = 2$.

$$x\text{-intercept} = \frac{-2+2}{2} = \frac{0}{2} = 0$$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is –2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2.

Use the two consecutive asymptotes, $x = -2$ and $x = 2$, to graph one full period of $y = -2\tan\frac{\pi}{4}x$ from –2 to 2. Continue the pattern and extend the graph another full period to the right.



$y = -2\tan\frac{\pi}{4}x$

85. Solve the equations

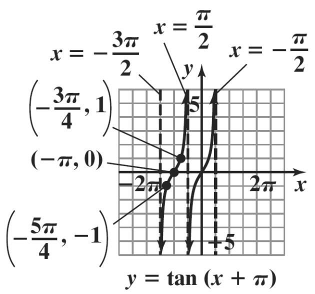
$$x + \pi = -\frac{\pi}{2} \quad \text{and} \quad x + \pi = \frac{\pi}{2}$$
$$x = -\frac{\pi}{2} - \pi \qquad x = \frac{\pi}{2} - \pi$$
$$x = -\frac{3\pi}{2} \qquad x = -\frac{\pi}{2}$$

Thus, two consecutive asymptotes occur at
$x = -\frac{3\pi}{2}$ and $x = -\frac{\pi}{2}$.

$$x\text{-intercept} = \frac{-\frac{3\pi}{2} - \frac{\pi}{2}}{2} = \frac{-2\pi}{2} = -\pi$$

An x-intercept is $-\pi$ and the graph passes through $(-\pi, 0)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes,
$x = -\frac{3\pi}{2}$ and $x = -\frac{\pi}{2}$, to graph one full period of
$y = \tan(x + \pi)$ from $-\frac{3\pi}{2}$ to $-\frac{\pi}{2}$.

Continue the pattern and extend the graph another full period to the right.



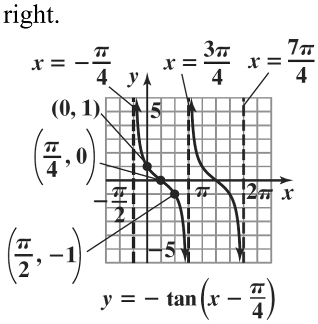
86. Solve the equations

$$x - \frac{\pi}{4} = -\frac{\pi}{2} \qquad \text{and} \qquad x - \frac{\pi}{4} = \frac{\pi}{2}$$
$$x = -\frac{\pi}{2} + \frac{\pi}{4} \qquad\qquad x = \frac{\pi}{2} + \frac{\pi}{4}$$
$$x = -\frac{\pi}{4} \qquad\qquad x = \frac{3\pi}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = -\frac{3\pi}{4}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{4} - \frac{3\pi}{4}}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$$

An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\frac{\pi}{4}, 0$. Because the coefficient of the tangent is –1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 1 and –1. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, to graph one full period of $y = -\tan\ x - \frac{\pi}{4}$ from $-\frac{\pi}{4}$ to $\frac{3\pi}{4}$. Continue the pattern and extend the graph another full period to the right.



87. Solve the equations

$$3x = 0 \quad \text{and} \quad 3x = \pi$$
$$x = 0 \qquad\qquad x = \frac{\pi}{3}$$

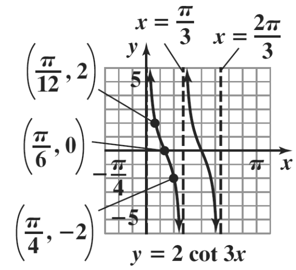
Thus, two consecutive asymptotes occur at $x = 0$ and $x = \frac{\pi}{3}$.

$$x\text{-intercept} = \frac{0 + \frac{\pi}{3}}{2} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$$

An x-intercept is $\frac{\pi}{6}$ and the graph passes through $\frac{\pi}{6}, 0$.

Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = 0$ and $x = \frac{\pi}{3}$, to graph one full period of $y = 2\cot 3x$ from 0 to $\frac{\pi}{3}$.

Continue the pattern and extend the graph another full period to the right.



88. Solve the equations

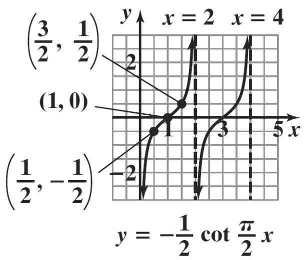
$$\frac{\pi}{2}x = 0 \quad \text{and} \quad \frac{\pi}{2}x = \pi$$
$$x = 0 \qquad\qquad x = \pi \cdot \frac{2}{\pi}$$
$$x = 2$$

Thus, two consecutive asymptotes occur at $x = 0$ and $x = 2$.

$$x\text{-intercept} = \frac{0+2}{2} = \frac{2}{2} = 1$$

An x-intercept is 1 and the graph passes through (1, 0). Because the coefficient of the cotangent is $-\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $-\frac{1}{2}$ and $\frac{1}{2}$. Use the two consecutive asymptotes,

$x = 0$ and $x = 2$, to graph one full period of $y = -\frac{1}{2}\cot\frac{\pi}{2}x$ from 0 to 2. Continue the pattern and extend the graph another full period to the right.



89. Solve the equations

$x + \frac{\pi}{2} = 0$ and $x + \frac{\pi}{2} = \pi$

$x = 0 - \frac{\pi}{2}$ $\qquad$ $x = \pi - \frac{\pi}{2}$

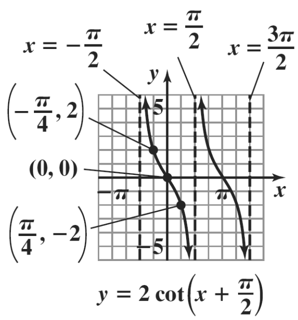
$x = -\frac{\pi}{2}$ $\qquad$ $x = \frac{\pi}{2}$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

x-intercept $= \frac{-\frac{\pi}{2} + \frac{\pi}{2}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2.

Use the two consecutive asymptotes, $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$, to graph one full period of $y = 2\cot\left(x + \frac{\pi}{2}\right)$ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. Continue the pattern and extend the graph another full period to the right.



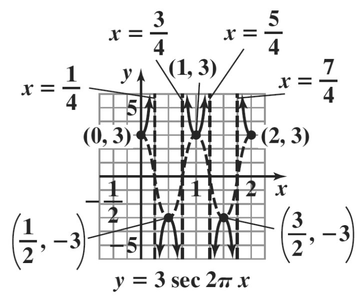
90. Graph the reciprocal cosine function, $y = 3\cos 2\pi x$. The equation is of the form $y = A\cos Bx$ with $A = 3$ and $B = 2\pi$.

amplitude: $|A| = |3| = 3$

period: $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$

Use quarter-periods, $\frac{1}{4}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1. Evaluating the function at each value of x, the key points are (0, 3), $\left(\frac{1}{4}, 0\right)$, $\left(\frac{1}{2}, -3\right)$, $\left(\frac{3}{4}, 0\right)$, (1, 3).

Use these key points to graph $y = 3\cos 2\pi x$ from 0 to 1. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\sec 2\pi x$.

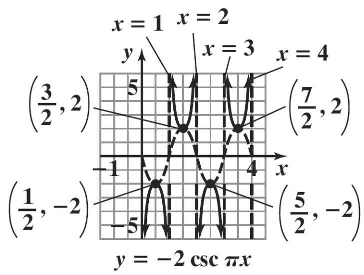


91. Graph the reciprocal sine function, $y = -2\sin\pi x$. The equation is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$.

amplitude: $|A| = |-2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2. Evaluating the function at each value of x, the key points are (0, 0), $\left(\frac{1}{2}, -2\right)$, (1, 0), $\left(\frac{3}{2}, 2\right)$, (2,0). Use these key points to graph $y = -2\sin\pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -2\csc \pi x$.



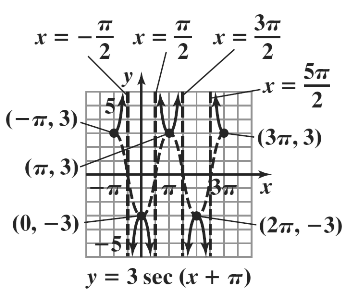
92. Graph the reciprocal cosine function, $y = 3\cos(x+\pi)$. The equation is of the form $y = A\cos(Bx - C)$ with $A = 3$, $B = 1$, and $C = -\pi$.

amplitude: $|A| = |3| = 3$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

phase shift: $\frac{C}{B} = \frac{-\pi}{1} = -\pi$

Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = -\pi$, the x-values are $-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$. Evaluating the function at each value of x, the key points are $(-\pi, 3)$, $\left(-\frac{\pi}{2}, 0\right)$, $(0, -3)$, $\left(\frac{\pi}{2}, 0\right)$, $(\pi, 3)$. Use these key points to graph $y = 3\cos(x+\pi)$ from $-\pi$ to π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\sec(x+\pi)$.



93. Graph the reciprocal sine function, $y = \frac{5}{2}\sin(x-\pi)$.

The equation is of the form $y = A\sin(Bx - C)$ with $A = \frac{5}{2}$, $B = 1$, and $C = \pi$.

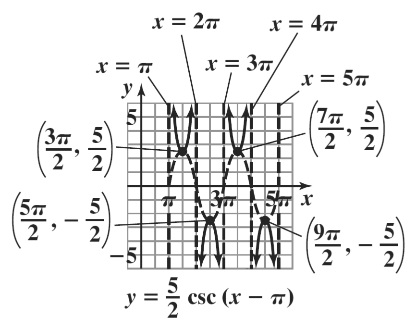
amplitude: $|A| = \left|\frac{5}{2}\right| = \frac{5}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

phase shift: $\frac{C}{B} = \frac{\pi}{1} = \pi$

Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = \pi$, the x-values are $\pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi$. Evaluating the function at each value of x, the key points are $(\pi, 0)$, $\left(\frac{3\pi}{2}, \frac{5}{2}\right)$, $(2\pi, 0)$, $\left(\frac{5\pi}{2}, -\frac{5}{2}\right)$, $(3\pi, 0)$.

Use these key points to graph $y = \frac{5}{2}\sin(x-\pi)$ from π to 3π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \frac{5}{2}\csc(x-\pi)$.



94. Let $\theta = \sin^{-1} 1$, then $\sin\theta = 1$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = 1$ is $\frac{\pi}{2}$. Thus $\theta = \frac{\pi}{2}$, or $\sin^{-1} 1 = \frac{\pi}{2}$.

95. Let $\theta = \cos^{-1} 1$, then $\cos\theta = 1$.

The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 1$ is 0. Thus $\theta = 0$, or $\cos^{-1} 1 = 0$.

96. Let $\theta = \tan^{-1} 1$, then $\tan\theta = 1$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\tan\theta = 1$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\tan^{-1} 1 = \frac{\pi}{4}$.

97. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{3}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{\sqrt{3}}{2}$ is $-\frac{\pi}{3}$. Thus $\theta = -\frac{\pi}{3}$, or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

98. Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$, then $\cos\theta = -\frac{1}{2}$.

The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{1}{2}$ is $\frac{2\pi}{3}$. Thus $\theta = \frac{2\pi}{3}$, or $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

99. Let $\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$, then $\tan\theta = -\frac{\sqrt{3}}{3}$.

The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -\frac{\sqrt{3}}{3}$ is $-\frac{\pi}{6}$.

Thus $\theta = -\frac{\pi}{6}$, or $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

100. Let $\theta = \sin^{-1}\frac{\sqrt{2}}{2}$, then $\sin\theta = \frac{\sqrt{2}}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$.

Thus, $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

101. Let $\theta = \cos^{-1} 0$, then $\cos\theta = 0$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 0$ is $\frac{\pi}{2}$. Thus,

$\sin\left(\cos^{-1} 0\right) = \sin\frac{\pi}{2} = 1$.

102. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{1}{2}$ is $-\frac{\pi}{6}$.

Thus, $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

103. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{3}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{\sqrt{3}}{2}$ is $\frac{5\pi}{6}$.

Thus, $\tan\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$.

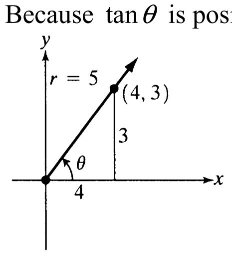
104. Let $\theta = \tan^{-1}\frac{\sqrt{3}}{3}$, then $\tan\theta = \frac{\sqrt{3}}{3}$.

The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = \frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

Thus $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) = \csc\frac{\pi}{6} = 2$.

105. Let $\theta = \tan^{-1}\frac{3}{4}$, then $\tan\theta = \frac{3}{4}$

Because $\tan\theta$ is positive, θ is in the first quadrant.



$r^2 = x^2 + y^2$

$r^2 = 4^2 + 3^2$

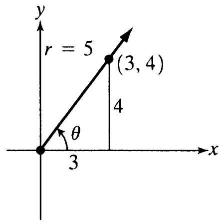
$r^2 = 25$

$r = 5$

$\cos\left(\tan^{-1}\frac{3}{4}\right) = \cos\theta = \frac{x}{r} = \frac{4}{5}$

106. Let $\theta = \cos^{-1}\frac{3}{5}$, then $\cos\theta = \frac{3}{5}$.

Because $\cos\theta$ is positive, θ is in the first quadrant.



$x^2 + y^2 = r^2$

$3^2 + y^2 = 5^2$

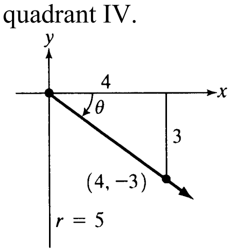
$y^2 = 25 - 9 = 16$

$y = \sqrt{16} = 4$

$\sin\ \cos^{-1}\frac{3}{5}\ = \sin\theta = \frac{y}{r} = \frac{4}{5}$

107. Let $\theta = \sin^{-1}\ -\frac{3}{5}$, then $\sin\theta = -\frac{3}{5}$.

Because $\sin\theta$ is negative, θ is in quadrant IV.



$x^2 + (-3)^2 = 5^2$

$x^2 + y^2 = r^2$

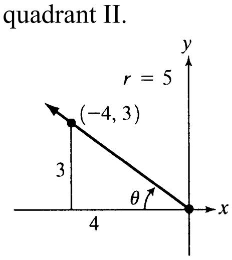
$x^2 = 25 - 9 = 16$

$x = \sqrt{16} = 4$

$\tan\ \sin^{-1}\ -\frac{3}{5}\ = \tan\theta = \frac{y}{x} = -\frac{3}{4}$

108. Let $\theta = \cos^{-1}\ -\frac{4}{5}$, then $\cos\theta = -\frac{4}{5}$.

Because $\cos\theta$ is negative, θ is in quadrant II.



$x^2 + y^2 = r^2$

$(-4)^2 + y^2 = 5^2$

$y^2 = 25 - 16 = 9$

$y = \sqrt{9} = 3$

Use the right triangle to find the exact value.

$\tan\ \cos^{-1}\ -\frac{4}{5}\ = \tan\theta = -\frac{3}{4}$

109. Let $\theta = \tan^{-1}\ -\frac{1}{3}$,

Because $\tan\theta$ is negative, θ is in quadrant IV and $x = 3$ and $y = -1$.

$r^2 = x^2 + y^2$

$r^2 = 3^2 + (-1)^2$

$r^2 = 10$

$r = \sqrt{10}$

$\sin\ \tan^{-1}\ -\frac{4}{5}\ = \sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

110. $x = \frac{\pi}{3}$, x is in $-\frac{\pi}{2}, \frac{\pi}{2}$, so $\sin^{-1}\ \sin\frac{\pi}{3}\ = \frac{\pi}{3}$

111. $x = \frac{2\pi}{3}$, x is not in $-\frac{\pi}{2}, \frac{\pi}{2}$. x is in the domain of $\sin x$, so

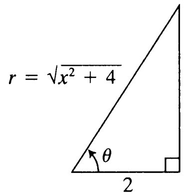
$\sin^{-1}\ \sin\frac{2\pi}{3}\ = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

112. $\sin^{-1}\ \cos\frac{2\pi}{3}\ = \sin^{-1}\ -\frac{1}{2}$

Let $\theta = \sin^{-1}\ -\frac{1}{2}$, then $\sin\theta = -\frac{1}{2}$. The only angle in the interval $-\frac{\pi}{2}, \frac{\pi}{2}$ that satisfies $\sin\theta = -\frac{1}{2}$ is $-\frac{\pi}{6}$. Thus, $\theta = -\frac{\pi}{6}$, or

$\sin^{-1}\ \cos\frac{2\pi}{3}\ = \sin^{-1}\ -\frac{1}{2}\ = -\frac{\pi}{6}$.

113. Let $\theta = \tan^{-1}\frac{x}{2}$, then $\tan\theta = \frac{x}{2}$.



$r^2 = x^2 + 2^2$

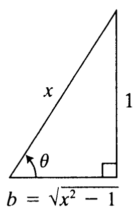
$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + 4}$

Use the right triangle to write the algebraic expression.

$$\cos\ \tan^{-1}\frac{x}{2} = \cos\theta = \frac{2}{\sqrt{x^2+4}} = \frac{2\sqrt{x^2+4}}{x^2+4}$$

114. Let $\theta = \sin^{-1}\frac{1}{x}$, then $\sin\theta = \frac{1}{x}$.



Use the Pythagorean theorem to find the third side, b.

$1^2 + b^2 = x^2$

$b^2 = x^2 - 1$

$b = \sqrt{x^2 - 1}$

Use the right triangle to write the algebraic expression.

$$\sec\ \sin^{-1}\frac{1}{x} = \sec\theta = \frac{x}{\sqrt{x^2-1}} = \frac{x\sqrt{x^2-1}}{x^2-1}$$

115. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus,
$B = 90° - A = 90° - 22.3° = 67.7°$
We have a known angle, a known hypotenuse, and an unknown opposite side. Use the sine function.

$\sin 22.3° = \frac{a}{10}$

$a = 10\sin 22.3° \approx 3.79$

We have a known angle, a known hypotenuse, and an unknown adjacent side. Use the cosine function.

$\cos 22.3° = \frac{b}{10}$

$b = 10\cos 22.3° \approx 9.25$

In summary, $B = 67.7°$, $a \approx 3.79$, and $b \approx 9.25$.

116. Find the measure of angle A. Because $C = 90°$, $A + B = 90°$. Thus, $A = 90° - B = 90° - 37.4° = 52.6°$
We have a known angle, a known opposite side, and an unknown adjacent side. Use the tangent function.

$\tan 37.4° = \frac{6}{a}$

$a = \frac{6}{\tan 37.4°} \approx 7.85$

We have a known angle, a known opposite side, and an unknown hypotenuse. Use the sine function.

$\sin 37.4° = \frac{6}{c}$

$c = \frac{6}{\sin 37.4°} \approx 9.88$

In summary, $A = 52.6°$, $a \approx 7.85$, and $c \approx 9.88$.

117. Find the measure of angle A. We have a known hypotenuse, a known opposite side, and an unknown angle. Use the sine function.

$\sin A = \frac{2}{7}$

$A = \sin^{-1}\ \frac{2}{7}\ \approx 16.6°$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A \approx 90° - 16.6° = 73.4°$
We have a known hypotenuse, a known opposite side, and an unknown adjacent side. Use the Pythagorean theorem.

$a^2 + b^2 = c^2$

$2^2 + b^2 = 7^2$

$b^2 = 7^2 - 2^2 = 45$

$b = \sqrt{45} \approx 6.71$

In summary, $A \approx 16.6°$, $B \approx 73.4°$, and $b \approx 6.71$.

118. Find the measure of angle A. We have a known opposite side, a known adjacent side, and an unknown angle. Use the tangent function.

$\tan A = \frac{1.4}{3.6}$

$A = \tan^{-1}\ \frac{1.4}{3.6}\ \approx 21.3°$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus,
$B = 90° - A \approx 90° - 21.3° = 68.7°$

We have a known opposite side, a known adjacent side, and an unknown hypotenuse.
Use the Pythagorean theorem.

$c^2 = a^2 + b^2 = (1.4)^2 + (3.6)^2 = 14.92$

$c = \sqrt{14.92} \approx 3.86$

In summary, $A \approx 21.3°$, $B \approx 68.7°$, and $c \approx 3.86$.

119. Using a right triangle, we have a known angle, an unknown opposite side, h, and a known adjacent side. Therefore, use the tangent function.

$$\tan 25.6° = \frac{h}{80}$$
$$h = 80\tan 25.6°$$
$$\approx 38.3$$

The building is about 38 feet high.

120. Using a right triangle, we have a known angle, an unknown opposite side, h, and a known adjacent side. Therefore, use the tangent function.

$$\tan 40° = \frac{h}{60}$$
$$h = 60\tan 40° \approx 50 \text{ yd}$$

The second building is 50 yds taller than the first.
Total height = $40 + 50 = 90$ yd .

121. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, a known opposite side, and an unknown adjacent side, d, in the smaller triangle. Therefore, use the tangent function.

$$\tan 68° = \frac{125}{d}$$
$$d = \frac{125}{\tan 68°} \approx 50.5$$

We now have a known angle, a known adjacent side, and an unknown opposite side, h, in the larger triangle. Again, use the tangent function.

$$\tan 71° = \frac{h}{50.5}$$
$$h = 50.5\tan 71° \approx 146.7$$

The height of the antenna is $146.7 - 125$, or 21.7 ft, to the nearest tenth of a foot.

122. We need the acute angle between ray OA and the north-south line through O. This angle measures $90° - 55° = 35°$. This angle measured from the north side of the north-south line and lies east of the north-south line. Thus the bearing from O to A is N35°E.

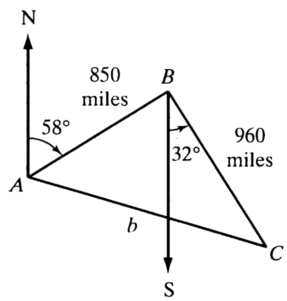
123. We need the acute angle between ray OA and the north-south line through O. This angle measures $90° - 55° = 35°$. This angle measured from the south side of the north-south line and lies west of the north-south line. Thus the bearing from O to A is S35°W.

124. Using a right triangle, we have a known angle, a known adjacent side, and an unknown opposite side, d. Therefore, use the tangent function.

$$\tan 64° = \frac{d}{12}$$
$$d = 12\tan 64° \approx 24.6$$

The ship is about 24.6 miles from the lighthouse.

125.



a. Using the figure,
$B = 58° + 32° = 90°$
Thus, use the Pythagorean Theorem to find the distance from city A to city C.

$$850^2 + 960^2 = b^2$$
$$b^2 = 722{,}500 + 921{,}600$$
$$b^2 = 1{,}644{,}100$$
$$b = \sqrt{1{,}644{,}100} \approx 1282.2$$

The distance from city A to city B is about 1282.2 miles.

b. Using the figure,

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{960}{850} \approx 1.1294$$
$$A \approx \tan^{-1}(1.1294) \approx 48°$$
$$180° - 58° - 48° = 74°$$

The bearing from city A to city C is S74°E.

126. $d = 20\cos\frac{\pi}{4}t$

$a = 20$ and $\omega = \frac{\pi}{4}$

a. maximum displacement:
$|\, a \,| = |\, 20 \,| = 20\,\text{cm}$

b. $f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{\pi}{4}\cdot\frac{1}{2\pi} = \frac{1}{8}$

frequency: $\frac{1}{8}$ cm per second

c. period: $\frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi\cdot\frac{4}{\pi} = 8$

The time required for one cycle is 8 seconds.

127. $d = \frac{1}{2}\sin 4t$

$a = \frac{1}{2}$ and $\omega = 4$

a. maximum displacement:

$|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$ cm

b. $f = \frac{\omega}{2\pi} = \frac{4}{2\pi} = \frac{2}{\pi} \approx 0.64$

frequency: 0.64 cm per second

c. period: $\frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2} \approx 1.57$

The time required for one cycle is about 1.57 seconds.

128. Because the distance of the object from the rest position at $t = 0$ is a maximum, use the form $d = a\cos\omega t$. The period is $\frac{2\pi}{\omega}$ so,

$2 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{2} = \pi$

Because the amplitude is 30 inches, $|a| = 30$. because the object starts below its rest position $a = -30$. the equation for the object's simple harmonic motion is $d = -30\cos\pi t$.

129. Because the distance of the object from the rest position at $t = 0$ is 0, use the form $d = a\sin\omega t$. The period is $\frac{2\pi}{\omega}$ so

$5 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{5}$

Because the amplitude is $\frac{1}{4}$ inch, $|a| = \frac{1}{4}$. a is negative since the object begins pulled down. The equation for the object's simple harmonic motion is

$d = -\frac{1}{4}\sin\frac{2\pi}{5}t.$

Chapter 4 Test

1. $135° = 135° \cdot \frac{\pi \text{ radians}}{180°}$

$= \frac{135\pi}{180}$ radians

$= \frac{3\pi}{4}$ radians

2. $75° = 75° \cdot \frac{\pi \text{ radians}}{180°} = \frac{75\pi}{180}$ radians

$= \frac{5\pi}{12}$ radians

$s = r\theta$

$s = 20\left(\frac{5\pi}{12}\right) = \frac{25\pi}{3}$ ft ≈ 26.18 ft

3. **a.** $\frac{16\pi}{3} - 4\pi = \frac{16\pi}{3} - \frac{12\pi}{3} = \frac{4\pi}{3}$

b. $\frac{16\pi}{3}$ is coterminal with $\frac{4\pi}{3}$.

$\frac{4\pi}{3} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

4. P = (–2, 5) is a point on the terminal side of θ, $x = -2$ and $y = 5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (5)^2}$

$= \sqrt{4 + 25} = \sqrt{29}$

Use *x, y,* and *r,* to find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{\sqrt{29}\sqrt{29}} = \frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{\sqrt{29}\sqrt{29}} = -\frac{2\sqrt{29}}{29}$

$\tan\theta = \frac{y}{x} = \frac{5}{-2} = -\frac{5}{2}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{5}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$

$\cot\theta = \frac{x}{y} = \frac{-2}{5} = -\frac{2}{5}$

5. Because $\cos\theta < 0$, θ cannot lie in quadrant I and quadrant IV; the cosine function is positive in those two quadrants. Thus, with $\cos\theta < 0$, θ lies in quadrant II or quadrant III. We are also given that $\cot\theta > 0$. Because quadrant III is the only quadrant in which the cosine is negative and the cotangent is positive, θ lies in quadrant III.

6. Because the cosine is positive and the tangent is negative, θ lies in quadrant IV. In quadrant IV x is positive and y is negative. Thus,

$\cos\theta = \frac{1}{3} = \frac{x}{r},\ x = 1,\ r = 3.$ Furthermore,

$$x^2 + y^2 = r^3$$
$$1^2 + y^2 = 3^2$$
$$y^2 = 9 - 1 = 8$$
$$y = -\sqrt{8} = -2\sqrt{2}$$

Use x, y, and r, to find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$$
$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{1} = -2\sqrt{2}$$
$$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = -\frac{3\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{3}{1} = 3$$
$$\cot\theta = \frac{x}{y} = \frac{1}{-2\sqrt{2}} = -\frac{1\cdot\sqrt{2}}{2\sqrt{2}\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

7. $\tan\frac{\pi}{6}\cos\frac{\pi}{3} - \cos\frac{\pi}{2} = \frac{\sqrt{3}}{3}\cdot\frac{1}{2} - 0 = \frac{\sqrt{3}}{6}$

8. 300° lies in quadrant IV.
The reference angle is

$$\theta' = 360° - 300° = 60°$$
$$\tan 60° = \sqrt{3}$$

In quadrant IV, $\tan\theta < 0$, so

$\tan 300° = -\tan 60 = -\sqrt{3}$.

9. $\frac{7\pi}{4}$ lies in quadrant IV.
The reference angle is

$$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$$
$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

In quadrant IV, $\sin\theta < 0$, so

$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

10. $\sec\frac{22\pi}{3} = \sec\frac{4\pi}{3} = -\sec\frac{\pi}{3}$
$$= \frac{1}{-\cos\frac{\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2$$

11. $\cot\left(-\frac{8\pi}{3}\right) = \cot\left(\frac{4\pi}{3}\right) = \cot\frac{\pi}{3}$
$$= \frac{1}{\tan\frac{\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

12. $\tan\left(\frac{7\pi}{3} + n\pi\right) = \tan\frac{7\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$

13. **a.** $\sin(-\theta) + \cos(-\theta) = -\sin(\theta) + \cos(\theta)$
$$= -a + b$$

b. $\tan\theta - \sec\theta = \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}$
$$= \frac{a}{b} - \frac{1}{b}$$
$$= \frac{a-1}{b}$$

14. The equation $y = 3\sin 2x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 2$. The amplitude is $|A| = |3| = 3$.

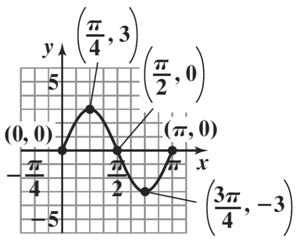
The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$$x = 0$$
$$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$$
$$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$
$$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$
$$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 3\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, -3\right)$
π	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin 2x$

15. The equation $y = -2\cos\left(x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -2$, $B = 1$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

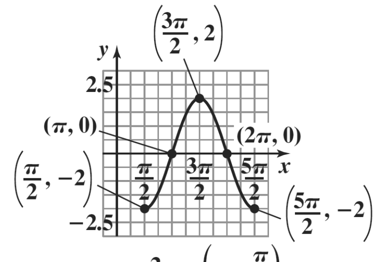
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -2\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 2\right)$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\left(\frac{5\pi}{2}, -2\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -2\cos\left(x - \frac{\pi}{2}\right)$

16. Solve the equations

$\frac{x}{2} = -\frac{\pi}{2}$ and $\frac{x}{2} = \frac{\pi}{2}$

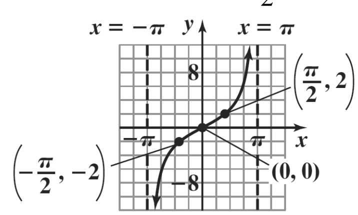
$x = -\frac{\pi}{2} \cdot 2$ $\qquad x = \frac{\pi}{2} \cdot 2$

$x = -\pi$ $\qquad x = \pi$

Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

x-intercept $= \frac{-\pi + \pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, to graph one full period of $y = 2\tan\frac{x}{2}$ from $-\pi$ to π.



$y = 2\tan\frac{x}{2}$

17. Graph the reciprocal sine function, $y = -\frac{1}{2}\sin \pi x$.

The equation is of the form $y = A\sin Bx$ with $A = -\frac{1}{2}$ and $B = \pi$.

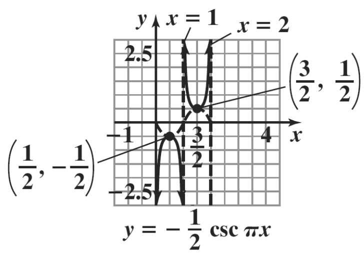
amplitude: $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2. Evaluating the function at

each value of x, the key points are
$(0, 3), \left(\frac{1}{2}, -\frac{1}{2}\right), (1, 0), \left(\frac{3}{2}, \frac{1}{2}\right), (2, 0)$.

Use these key points to graph $y = -\frac{1}{2}\sin \pi x$ from 0 to 2. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{1}{2}\csc \pi x$.



18. Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$, then $\cos\theta = -\frac{1}{2}$.
Because $\cos\theta$ is negative, θ is in quadrant II.

$$x^2 + y^2 = r^2$$
$$(-1)^2 + y^2 = 2^2$$
$$y^2 = 4 - 1 = 3$$
$$y = \sqrt{3}$$
$$\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

19. Let $\theta = \cos^{-1}\left(\frac{x}{3}\right)$, then $\cos\theta = \frac{x}{3}$.
Because $\cos\theta$ is positive, θ is in quadrant I.

$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = 3^2$$
$$y^2 = 9 - x^2$$
$$y = \sqrt{9 - x^2}$$
$$\sin\left[\cos^{-1}\left(\frac{x}{3}\right)\right] = \sin\theta = \frac{y}{r} = \frac{\sqrt{9 - x^2}}{3}$$

20. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 21° = 69°$.
We have a known angle, a known hypotenuse, and an unknown opposite side. Use the sine function.

$$\sin 21° = \frac{a}{13}$$
$$a = 13\sin 21° \approx 4.7$$

We have a known angle, a known hypotenuse, and an unknown adjacent side. Use the cosine function.

$$\cos 21° = \frac{b}{13}$$
$$b = 13\cos 21° \approx 12.1$$

In summary, $B = 69°$, $a \approx 4.7$, and $b \approx 12.1$.

21. Using a right triangle, we have a known angle, an unknown opposite side, h, and a known adjacent side. Therefore, use the tangent function.

$$\tan 37° = \frac{h}{30}$$
$$h = 30\tan 37° \approx 23$$

The building is about 23 yards high.

22. Using a right triangle, we have a known hypotenuse, a known opposite side, and an unknown angle. Therefore, use the sine function.

$$\sin\theta = \frac{43}{73}$$
$$\theta = \sin^{-1}\left(\frac{43}{73}\right) \approx 36.1°$$

The rope makes an angle of about 36.1° with the pole.

23. We need the acute angle between ray OP and the north-south line through O. This angle measures 90° – 10°. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus the bearing from O to P is N80°W.

24. $d = -6\cos \pi t$
$a = -6$ and $\omega = \pi$

a. maximum displacement: $|a| = |-6| = 6$ in.

b. $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$

frequency: $\frac{1}{2}$ in. per second

c. period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

The time required for one cycle is 2 seconds.

25. Trigonometric functions are periodic.

Cumulative Review Exercises (Chapters P-4)

1.
$$x^2 = 18 + 3x$$
$$x^2 - 3x - 18 = 0$$
$$(x - 6)(x + 3) = 0$$
$$x - 6 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 6 \qquad x = -3$$

The solution set is {–3,6 }.

2. $x^3+5x^2-4x-20=0$

$x^2(x+5)-4(x+5)=0$

$(x^2-4)(x+5)=0$

$(x-2)(x+2)(x+5)=0$

$x-2=0$ or $x+2=0$ or $x+5=0$

$x=2$ $\quad x=-2$ $\quad x=-5$

The solution set is $\{-5,-2,2\}$.

3. $\log_2 x+\log_2(x-2)=3$

$\log_2 x(x-2)=3$

$x(x-2)=2^3$

$x^2-2x=2^3$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x-4=0$ or $x+2=0$

$x=4$ $\quad x=-2$

$x=-2$ is extraneous

The solution set is $\{4\}$

4. $\sqrt{x-3}+5=x$

$\sqrt{x-3}=x-5$

$\left(\sqrt{x-3}\right)^2=(x-5)^2$

$x-3=x^2-10x+25$

$x^2-11x+28=0$

$(x-4)(x-7)=0$

$x-4=0$ or $x-7=0$

$x=4$ $\quad x=7$

$\sqrt{4-3}+5=4$

$\sqrt{1}+5=4$

$1+5=4$ false

$x=4$ is not a solution

$\sqrt{7-3}+5=7$

$\sqrt{4}+5=7$

$2+5=7$ true

The solution set is $\{7\}$.

5. $x^3-4x^2+x+6=0$

$p: \pm1,\pm2,\pm3,\pm6$

$q: \pm1$

$\frac{p}{q}: \pm1,\pm2,\pm3,\pm6$

2	1	–4	1	6
		2	–4	–6
	1	–2	–3	0

$x^3-4x^2+x+6=(x-2)\left(x^2-2x-3\right)$

Thus,

$x^3-4x^2+x+6=0$

$(x-2)\left(x^2-2x-3\right)=0$

$(x-2)(x-3)(x+1)=0$

$x-2=0$ or $x-3=0$ or $x+1=0$

$x=2$ $\quad x=3$ $\quad x=-1$

The solution set is $\{-1,2,3\}$

6. $|2x-5|\le 11$

$-11\le 2x-5\le 11$

$-6\le 2x\le 16$

$-3\le x\le 8$

The solution set is $\{x\mid -3\le x\le 8\}$

7. $f(x)=\sqrt{x-6}$

$x=\sqrt{y-6}$

$x^2=y-6$

$y=x^2+6$

$f^{-1}(x)=x^2+6$

8.

$$\begin{array}{r} 4x^2-\frac{14}{5}x-\frac{17}{25} \\ 5x+2\overline{)20x^3-6x^2-9x+10} \\ \underline{20x^3+8x^2} \\ -14x^2-9x \\ \underline{-14x^2-\frac{28}{5}x} \\ -\frac{17}{5}x+10 \\ \underline{-\frac{17}{5}x-\frac{34}{25}} \\ \frac{284}{25} \end{array}$$

The quotient is $4x^2-\frac{14}{5}x-\frac{17}{25}+\frac{284}{125x+50}$.

9. $\log 25+\log 40=\log(25\cdot 40)$

$=\log 1000$

$=\log 10^3$

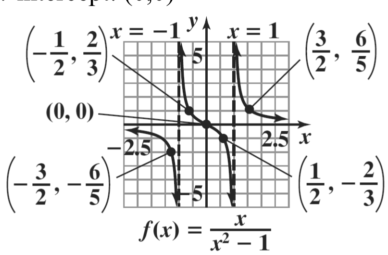
$=3$

10. $\frac{14\pi}{9}$ radians $=\frac{14\pi}{9}$ radians $\cdot\frac{180°}{\pi \text{ radians}}$

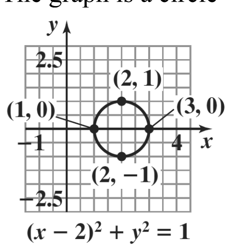
$=\frac{14\cdot 180°}{9}=280°$

11. $3x^4 - 2x^3 + 5x^2 + x - 9 = 0$
The sign changes 3 times so the equation has at most 3 positive real roots;
$f(-x) = 3x^4 + 2x^3 + 5x^2 - x - 9$
The sign changes 1 time, so the equation has at most 1 negative real root.

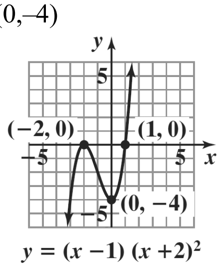
12. $f(x) = \dfrac{x}{x^2 - 1}$
vertical asymptotes: $x^2 - 1 = 0,\ x = 1$ and $x = -1$
horizontal asymptote: $m = 1$ and $n = 2$ so $m < n$ and the x-axis is a horizontal asymptote.
x-intercept: (0,0)



13. $(x-2)^2 + y^2 = 1$
The graph is a circle with center (2,0) and $r = 1$.



14. $y = (x-1)(x+2)^2$
x-intercepts: (1,0) and (–2,0)
y-intercept: $y = (-1)(2)^2 = -4$
(0,–4)



15. $y = \sin\left(2x + \dfrac{\pi}{2}\right) = \sin\left(2x - \left(-\dfrac{\pi}{2}\right)\right)$

The equation $y = \sin\left(2x - \left(-\dfrac{\pi}{2}\right)\right)$ is of the form

$y = A\sin(Bx - C)$ with $A = 1,\ B = 2$, and $C = -\dfrac{\pi}{2}$.

The amplitude is $|A| = |1| = 1$

The period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2} = \pi$. The phase shift is

$\dfrac{C}{B} = \dfrac{-\frac{\pi}{2}}{2} = -\dfrac{\pi}{2}\cdot\dfrac{1}{2} = -\dfrac{\pi}{4}$. The quarter-period is $\dfrac{\pi}{4}$.

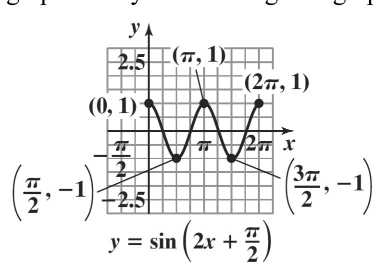
The cycle begins at $x = -\dfrac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\dfrac{\pi}{4},\ x = -\dfrac{\pi}{4} + \dfrac{\pi}{4} = 0,\ x = 0 + \dfrac{\pi}{4} = \dfrac{\pi}{4}$,

$x = \dfrac{\pi}{4} + \dfrac{\pi}{4} = \dfrac{\pi}{2},\ x = \dfrac{\pi}{2} + \dfrac{\pi}{4} = \dfrac{3\pi}{4}$ To graph from 0 to π, evaluate the function at the last four key points and at $x = \pi$.

x	coordinates
0	(0, 1)
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -1\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, 1)$

Connect the points with a smooth curve and extend the graph one cycle to the right to graph from 0 to 2π.



16. Solve the equations

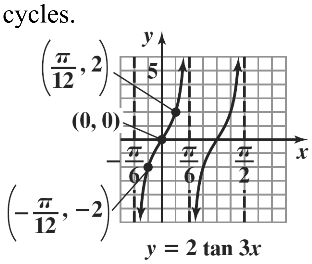
$$3x = -\frac{\pi}{2} \quad \text{and} \quad 3x = \frac{\pi}{2}$$
$$x = \frac{-\frac{\pi}{2}}{3} \qquad x = \frac{\frac{\pi}{2}}{3}$$
$$x = -\frac{\pi}{6} \qquad x = \frac{\pi}{6}$$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{6}$ and $x = \frac{\pi}{6}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{6} + \frac{\pi}{6}}{2} = \frac{0}{2} = 0$$

An x-intercept is 0 and the graph passes through $(0,0)$. Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = -\frac{\pi}{6}$ and $x = \frac{\pi}{6}$, to graph one full period of $y = 2\tan 3x$ from $-\frac{\pi}{6}$ to $\frac{\pi}{6}$.

Extend the pattern to the right to graph two complete cycles.



17. $C(p) = 30{,}000 + 2500p$
$R(p) = 3125p$
$30{,}000 + 2500p = 3125p$
$30{,}000 = 625p$
$p = 48$

48 performances must be played for you to break even.

18. a. Let t be the number of years after 2000.

$$A = A_0 e^{kt}$$
$$A = 110e^{kt}$$
$$303 = 110e^{k(10)}$$
$$\frac{303}{110} = e^{k(10)}$$
$$\ln\frac{303}{110} = \ln e^{k(10)}$$
$$\ln\frac{303}{110} = 10k$$
$$\frac{\ln\frac{303}{110}}{10} = k$$
$$k \approx 0.1013$$

Thus, $A = 110e^{0.1013t}$

b.
$$A = 110e^{0.1013t}$$
$$400 = 110e^{0.1013t}$$
$$\frac{400}{110} = e^{0.1013t}$$
$$\ln\frac{400}{110} = \ln e^{0.1013t}$$
$$\ln\frac{400}{110} = 0.1013t$$
$$\frac{\ln\frac{400}{110}}{0.1013} = t$$
$$t \approx 13$$

There will be 400 million cell phone subscribers in the United States 13 years after 2000, or 2013.

19. $2200 = \frac{k}{3.5}$
$k = 7700$
$h = \frac{7700}{5} = 1540$

The rate of heat loss is 1540 Btu per hour.

20. Using a right triangle, we have a known opposite side, a known adjacent side, and an unknown angle. Therefore, use the tangent function.

$$\tan\theta = \frac{200}{50} = 4$$
$$\theta = \tan^{-1}(4) \approx 76°$$

The angle of elevation is about $76°$.