CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t - 2)(t - 5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

(a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

(b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

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Continue question 1 on page 5.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.

(d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

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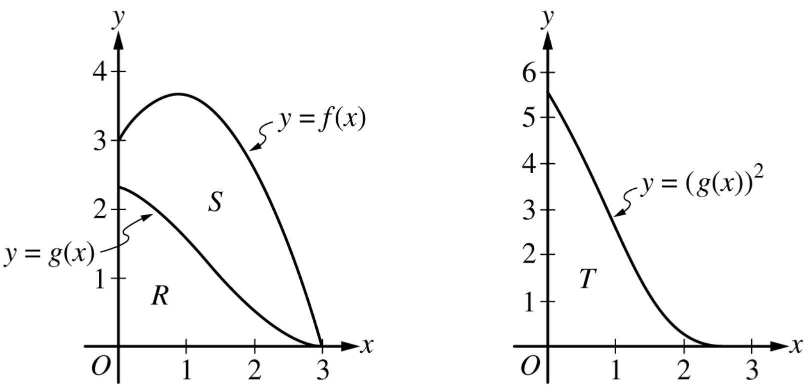

2. The function f is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(a) Find the area of region S.

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Continue question 2 on page 7.

(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.

(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.

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END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

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Continue question 3 on page 15.

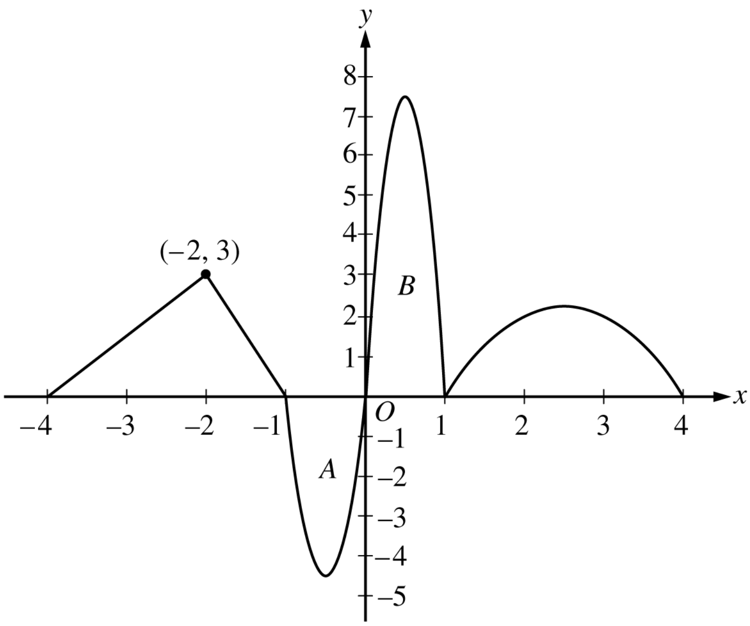
NO CALCULATOR ALLOWED

(c) Find the average value of f on the interval $-3 \leq x \leq 4$.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \leq x \leq 4$? Justify your answer.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED



Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\, dt$.

(a) Find $g(0)$ and $g(4)$.

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Continue question 4 on page 17.

(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

NO CALCULATOR ALLOWED

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	–5	11

5. Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

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Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\frac{dy}{dx} = \frac{2y - 2x}{3y - 2x}$.

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

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Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.