Chapter 3
Exponential and Logarithmic Functions

Section 3.1

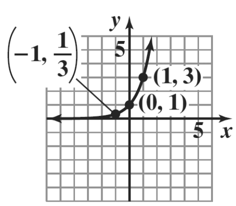
Check Point Exercises

1. $f(x) = 42.2(1.56)^x$
$f(3) = 42.2(1.56)^3 \approx 160.20876 \approx 160$
According to the function, the average amount spent after three hours of shopping at the mall is $160. This overestimates the actual amount shown by $11.

2. Begin by setting up a table of coordinates.

x	$f(x) = 3^x$
–3	$f(-3) = 3^{-3} = \frac{1}{27}$
–2	$f(-2) = 3^{-2} = \frac{1}{9}$
–1	$f(-1) = 3^{-1} = \frac{1}{3}$
0	$f(0) = 3^0 = 1$
1	$f(1) = 3^1 = 3$
2	$f(2) = 3^2 = 9$
3	$f(3) = 3^3 = 27$

Plot a few of these points, connecting them with a continuous curve.

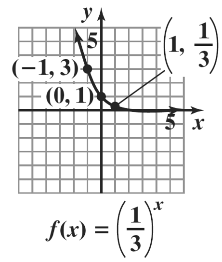


$f(x) = 3^x$

3. Begin by setting up a table of coordinates.

x	$f(x) = \left(\frac{1}{3}\right)^x$
–2	$\left(\frac{1}{3}\right)^{-2} = 9$
–1	$\left(\frac{1}{3}\right)^{-1} = 3$
0	$\left(\frac{1}{3}\right)^{0} = 1$
1	$\left(\frac{1}{3}\right)^{1} = \frac{1}{3}$
2	$\left(\frac{1}{3}\right)^{2} = \frac{1}{9}$

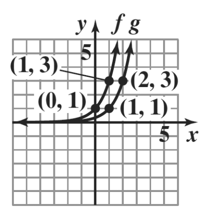
Plot a few of these points, connecting them with a continuous curve.



$f(x) = \left(\frac{1}{3}\right)^x$

4. Note that the function $g(x) = 3^{x-1}$ has the general form $g(x) = b^{x+c}$ where $c = -1$. Because $c < 0$, we graph $g(x) = 3^{x-1}$ by shifting the graph of $f(x) = 3^x$ one unit to the right. Construct a table showing some of the coordinates for f and g.

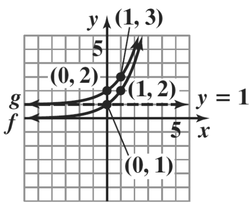
x	$f(x) = 3^x$	$g(x) = 3^{x-1}$
–2	$3^{-2} = \frac{1}{9}$	$3^{-2-1} = 3^{-3} = \frac{1}{27}$
–1	$3^{-1} = \frac{1}{3}$	$3^{-1-1} = 3^{-2} = \frac{1}{9}$
0	$3^0 = 1$	$3^{0-1} = 3^{-1} = \frac{1}{3}$
1	$3^1 = 3$	$3^{1-1} = 3^0 = 1$
2	$3^2 = 9$	$3^{2-1} = 3^1 = 3$



$f(x) = 3^x$
$g(x) = 3^{x-1}$

5. Note that the function $g(x) = 2^x + 1$ has the general form $g(x) = b^x + c$ where $c = 1$. Because $c > 0$, we graph $g(x) = 2^x + 1$ by shifting the graph of $f(x) = 2^x$ up one unit. Construct a table showing some of the coordinates for f and g.

x	$f(x) - 2^x$	$g(x) = 2^x + 1$
-2	$2^{-2} = \frac{1}{4}$	$2^{-2} + 1 = \frac{1}{4} + 1 = \frac{5}{4}$
-1	$2^{-1} = \frac{1}{2}$	$2^{-1} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$
0	$2^0 = 1$	$2^0 + 1 = 1 + 1 = 2$
1	$2^1 = 2$	$2^1 + 1 = 2 + 1 = 3$
2	$2^2 = 4$	$2^2 + 1 = 4 + 1 = 5$



$f(x) = 2^x$
$g(x) = 2^x + 1$

6. 2017 is 39 years after 1978.

$f(x) = 1145e^{0.0325x}$

$f(39) = 1145e^{0.0325(39)} \approx 4067$

In 2017 the gray wolf population of the Western Great Lakes is projected to be about 4067.

7. **a.** $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 10,000\left(1 + \frac{0.08}{4}\right)^{4(5)}$
$= \$14,859.47$

b. $A = Pe^{rt}$

$A = 10,000e^{0.08(5)}$
$= \$14,918.25$

Concept and Vocabulary Check 3.1

1. b^x; $(-\infty, \infty)$; $(0, \infty)$

2. x; $y = 0$; horizontal

3. e; natural; 2.72

4. A; P; r; n

5. semiannually; quarterly; continuous

Exercise Set 3.1

1. $2^{3.4} \approx 10.556$

2. $3^{2.4} \approx 13.967$

3. $3^{\sqrt{5}} \approx 11.665$

4. $5^{\sqrt{3}} \approx 16.242$

5. $4^{-1.5} = 0.125$

6. $6^{-1.2} \approx 0.116$

7. $e^{2.3} \approx 9.974$

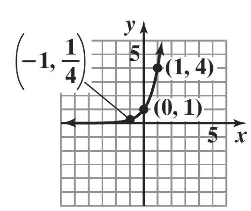
8. $e^{3.4} \approx 29.964$

9. $e^{-0.95} \approx 0.387$

10. $e^{-0.75} \approx 0.472$

11.

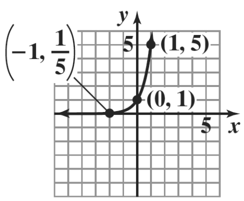
x	$f(x) = 4^x$
-2	$4^{-2} = \frac{1}{16}$
-1	$4^{-1} = \frac{1}{4}$
0	$4^0 = 1$
1	$4^1 = 4$
2	$4^2 = 16$



$f(x) = 4^x$

12.

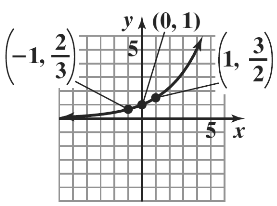
x	$g(x)=5^x$
-2	$5^{-2}=\frac{1}{25}$
-1	$5^{-1}=\frac{1}{5}$
0	$5^0=1$
1	$5^1=5$
2	$5^2=25$



$f(x) = 5^x$

13.

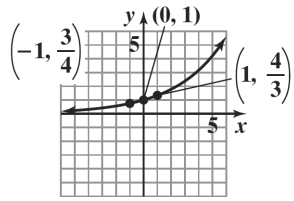
x	$g(x)=\left(\frac{3}{2}\right)^x$
-2	$\left(\frac{3}{2}\right)^{-2}=\frac{4}{9}$
-1	$\left(\frac{3}{2}\right)^{-1}=\frac{2}{3}$
0	$\left(\frac{3}{2}\right)^{0}=1$
1	$\left(\frac{3}{2}\right)^{1}=\frac{3}{2}$
2	$\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$



$g(x) = \left(\frac{3}{2}\right)^x$

14.

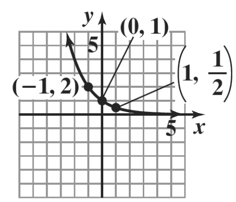
x	$g(x)=\left(\frac{4}{3}\right)^x$
-2	$\left(\frac{4}{3}\right)^{-2}=\frac{9}{16}$
-1	$\left(\frac{4}{3}\right)^{-1}=\frac{3}{4}$
0	$\left(\frac{4}{3}\right)^{0}=1$
1	$\left(\frac{4}{3}\right)^{1}=\frac{4}{3}$
2	$\left(\frac{4}{3}\right)^{2}=\frac{16}{9}$



$g(x) = \left(\frac{4}{3}\right)^x$

15.

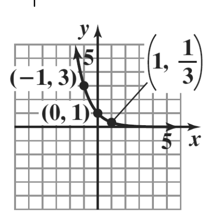
x	$h(x)=\left(\frac{1}{2}\right)^x$
-2	$\left(\frac{1}{2}\right)^{-2}=4$
-1	$\left(\frac{1}{2}\right)^{-1}=2$
0	$\left(\frac{1}{2}\right)^{0}=1$
1	$\left(\frac{1}{2}\right)^{1}=\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2}=\frac{1}{4}$



$h(x) = \left(\frac{1}{2}\right)^x$

16.

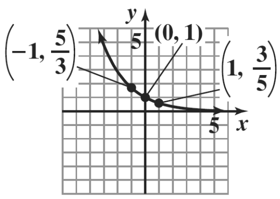
x	$h(x)=\left(\frac{1}{3}\right)^x$
-2	$\left(\frac{1}{3}\right)^{-2}=9$
-1	$\left(\frac{1}{3}\right)^{-1}=3$
0	$\left(\frac{1}{3}\right)^{0}=1$
1	$\left(\frac{1}{3}\right)^{1}=\frac{1}{3}$
2	$\left(\frac{1}{3}\right)^{2}=\frac{1}{9}$



$h(x) = \left(\frac{1}{3}\right)^x$

17.

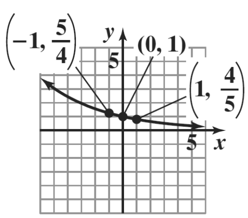
x	$f(x) = (0.6)^x$
–2	$(0.6)^{-2} = 2.\overline{7}$
–1	$(0.6)^{-1} = 1.\overline{6}$
0	$(0.6)^0 = 1$
1	$(0.6)^1 = 0.6$
2	$(0.6)^2 = 0.36$



$f(x) = (0.6)^x$

18.

x	$f(x) = (0.8)^x$
–2	$(0.8)^{-2} = 1.5625$
–1	$(0.8)^{-1} = 1.25$
0	$(0.8)^0 = 1$
1	$(0.8)^1 = 0.8$
2	$(0.8)^2 = 0.64$



$f(x) = (0.8)^x$

19. This is the graph of $f(x) = 3^x$ reflected about the *x*-axis and about the *y*-axis, so the function is $H(x) = -3^{-x}$.

20. This is the graph of $f(x) = 3^x$ shifted one unit to the right, so the function is $g(x) = 3^{x-1}$.

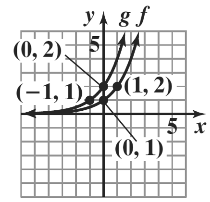
21. This is the graph of $f(x) = 3^x$ reflected about the *x*-axis, so the function is $F(x) = -3^x$.

22. This is the graph of *f(x)* = 3^x.

23. This is the graph of $f(x) = 3^x$ shifted one unit downward, so the function is $h(x) = 3^x - 1$.

24. This is the graph of $f(x) = 3^x$ reflected about the y-axis, so the function is $G(x) = 3^{-x}$.

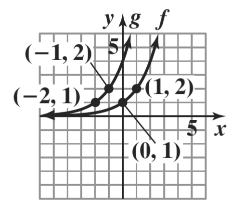
25. The graph of $g(x) = 2^{x+!}$ can be obtained by shifting the graph of $f(x) = 2^x$ one unit to the left.



$f(x) = 2^x$
$g(x) = 2^{x+1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

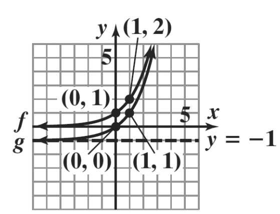
26. The graph of $g(x) = 2^{x+2}$ can be obtained by shifting the graph of $f(x) = 2^x$ two units to the left.



$f(x) = 2^x$
$g(x) = 2^{x+2}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

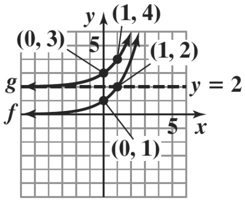
27. The graph of $g(x) = 2^x - 1$ can be obtained by shifting the graph of $f(x) = 2^x$ downward one unit.



$f(x) = 2^x$
$g(x) = 2^x - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

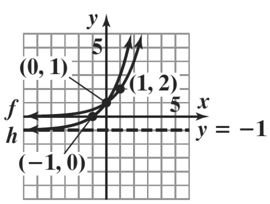
28. The graph of $g(x) = 2^x + 2$ can be obtained by shifting the graph of $f(x) = 2^x$ two units upward.



$f(x) = 2^x$
$g(x) = 2^x + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$; range: $(2, \infty)$

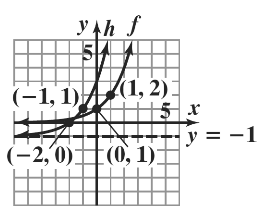
29. The graph of $h(x) = 2^{x+1} - 1$ can be obtained by shifting the graph of $f(x) = 2^x$ one unit to the left and one unit downward.



$f(x) = 2^x$
$h(x) = 2^{x+1} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

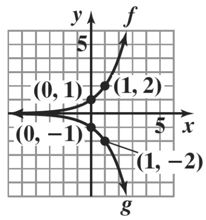
30. The graph of $h(x) = 2^{x+2} - 1$ can be obtained by shifting the graph of $f(x) = 2^x$ two units to the left and one unit downward.



$f(x) = 2^x$
$h(x) = 2^{x+2} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

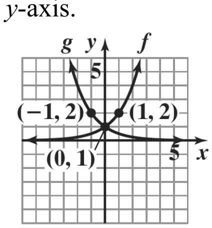
31. The graph of $g(x) = -2^x$ can be obtained by reflecting the graph of $f(x) = 2^x$ about the *x*-axis.



$f(x) = 2^x$
$g(x) = -2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(-\infty, 0)$

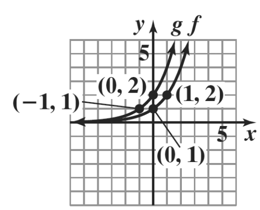
32. The graph of $g(x) = 2^{-x}$ can be obtained by reflecting the graph of $f(x) = 2^x$ about the *y*-axis.



$f(x) = 2^x$
$g(x) = 2^{-x}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

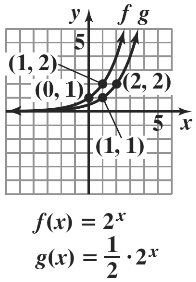
33. The graph of $g(x) = 2 \cdot 2^x$ can be obtained by vertically stretching the graph of $f(x) = 2^x$ by a factor of two.



$f(x) = 2^x$
$g(x) = 2 \cdot 2^x$

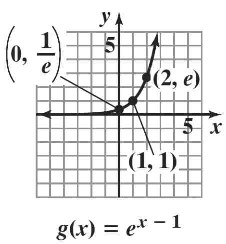
asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

34. The graph of $g(x)=\frac{1}{2}\cdot 2^x$ can be obtained by vertically shrinking the graph of $f(x)=2^x$ by a factor of one-half.



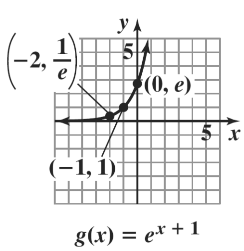
asymptote: $y=0$
domain: $(-\infty,\infty)$; range: $(0,\infty)$

35. The graph of $g(x)=e^{x-1}$ can be obtained by moving $f(x)=e^x$ 1 unit right.



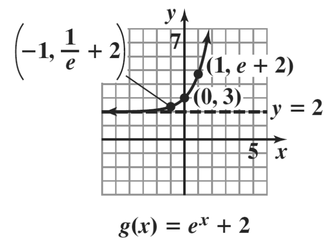
asymptote: $y=0$
domain: $(-\infty,\infty)$; range: $(0,\infty)$

36. The graph of $g(x)=e^{x+1}$ can be obtained by moving $f(x)=e^x$ 1 unit left.



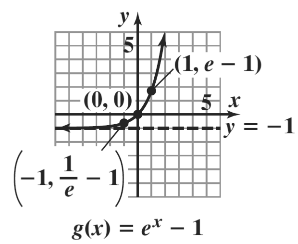
asymptote: $y=0$
domain: $(-\infty,\infty)$; range: $(0,\infty)$

37. The graph of $g(x)=e^x+2$ can be obtained by moving $f(x)=e^x$ 2 units up.



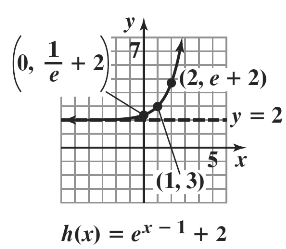
asymptote: $y=2$
domain: $(-\infty,\infty)$; range: $(2,\infty)$

38. The graph of $g(x)=e^x-1$ can be obtained by moving $f(x)=e^x$ 1 unit down.



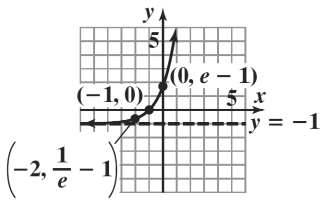
asymptote: $y=-1$
domain: $(-\infty,\infty)$; range: $(-1,\infty)$

39. The graph of $h(x)=e^{x-1}+2$ can be obtained by moving $f(x)=e^x$ 1 unit right and 2 units up.



asymptote: $y=2$
domain: $(-\infty,\infty)$; range: $(2,\infty)$

40. The graph of $h(x)=e^{x+1}-1$ can be obtained by moving $f(x)=e^x$ 1 unit left and 1 unit down.

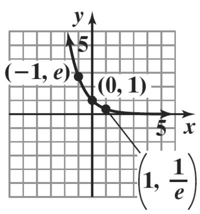


$h(x) = e^{x+1} - 1$

asymptote: $y=-1$

domain: $(-\infty,\infty)$; range: $(-1,\infty)$

41. The graph of $h(x)=e^{-x}$ can be obtained by reflecting $f(x)=e^x$ about the y-axis.

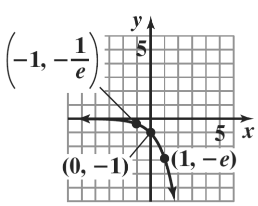


$h(x) = e^{-x}$

asymptote: $y=0$

domain: $(-\infty,\infty)$; range: $(0,\infty)$

42. The graph of $h(x)=-e^x$ can be obtained by reflecting $f(x)=e^x$ about the x-axis.

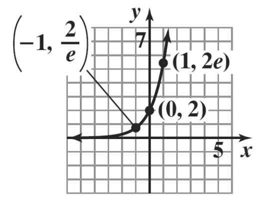


$h(x) = -e^x$

asymptote: $y=0$

domain: $(-\infty,\infty)$; range: $(-\infty,0)$

43. The graph of $g(x)=2e^x$ can be obtained by stretching $f(x)=e^x$ vertically by a factor of 2.

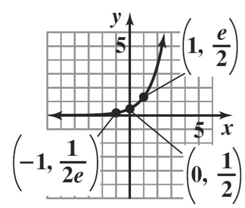


$g(x) = 2e^x$

asymptote: $y=0$

domain: $(-\infty,\infty)$; range: $(0,\infty)$

44. The graph of $g(x)=\frac{1}{2}e^x$ can be obtained by shrinking $f(x)=e^x$ vertically by a factor of $\frac{1}{2}$.

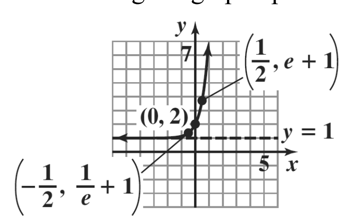


$g(x) = \frac{1}{2}e^x$

asymptote: $y=0$

domain: $(-\infty,\infty)$; range: $(0,\infty)$

45. The graph of $h(x)=e^{2x}+1$ can be obtained by stretching $f(x)=e^x$ horizontally by a factor of 2 and then moving the graph up 1 unit.

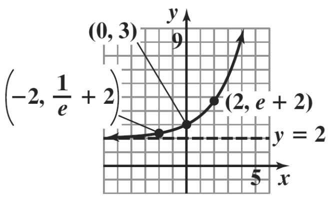


$h(x) = e^{2x} + 1$

asymptote: $y=1$

domain: $(-\infty,\infty)$; range: $(1,\infty)$

46. The graph of $h(x)=e^{\frac{1}{2}x}+2$ can be obtained by shrinking $f(x)=e^{x}$ horizontally by a factor of $\frac{1}{2}$ and then moving the graph up 2 units.

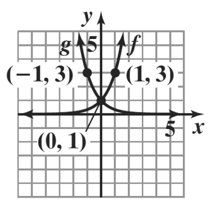


$h(x) = e^{x/2} + 2$

asymptote: $y=2$

domain: $(-\infty,\infty)$; range: $(2,\infty)$

47. The graph of $g(x)$ can be obtained by reflecting $f(x)$ about the y-axis.

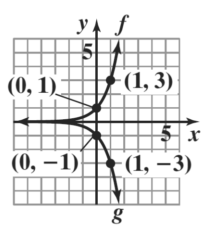


$f(x) = 3^x$
$g(x) = 3^{-x}$

asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

48. The graph of $g(x)$ can be obtained by reflecting $f(x)$ about the x-axis.

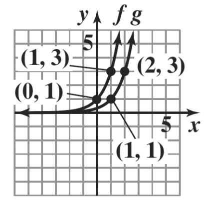


$f(x) = 3^x$
$g(x) = -3^x$

asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

49. The graph of $g(x)$ can be obtained by vertically shrinking $f(x)$ by a factor of $\frac{1}{3}$.

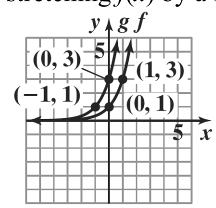


$f(x) = 3^x$
$g(x) = \frac{1}{3} \cdot 3^x$

asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

50. The graph of $g(x)$ can be obtained by horizontally stretching $f(x)$ by a factor of 3..

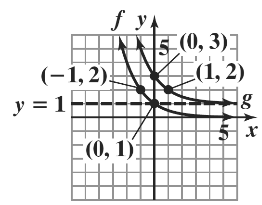


$f(x) = 3^x$
$g(x) = 3 \cdot 3^x$

asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

51. The graph of $g(x)$ can be obtained by moving the graph of $f(x)$ one space to the right and one space up.



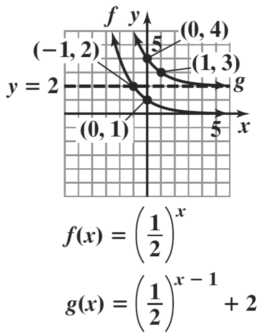
$f(x) = \left(\frac{1}{2}\right)^x$

$g(x) = \left(\frac{1}{2}\right)^{x-1} + 1$

asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=1$

52. The graph of $g(x)$ can be obtained by moving the graph of $f(x)$ one space to the right and two spaces up.



asymptote of $f(x)$: $y = 0$
asymptote of $g(x)$: $y = 2$

53. **a.** $A = 10,000\left(1+\frac{0.055}{2}\right)^{2(5)}$
$\approx \$13,116.51$

b. $A = 10,000\left(1+\frac{0.055}{4}\right)^{4(5)}$
$\approx \$13,140.67$

c. $A = 10,000\left(1+\frac{0.055}{12}\right)^{12(5)}$
$\approx \$13,157.04$

d. $A = 10,000e^{0.055(5)}$
$\approx \$13,165.31$

54. **a.** $A = 5000\left(1+\frac{0.065}{2}\right)^{2(10)} \approx \$9479.19$

b. $A = 5000\left(1+\frac{0.065}{4}\right)^{4\cdot 10} \approx \$9527.79$

c. $A = 5000\left(1+\frac{0.065}{12}\right)^{12(10)} \approx = \$9560.92$

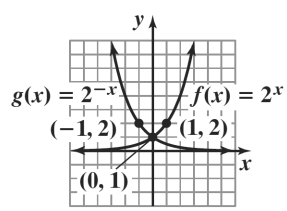
d. $A = 5000(e)^{0.065(10)} \approx 9577.70$

55. $A = 12,000\left(1+\frac{0.07}{12}\right)^{12(3)}$
$\approx 14,795.11$ (7% yield)
$A = 12,000e^{0.0685(3)}$
$\approx 14,737.67$ (6.85% yield)
Investing \$12,000 for 3 years at 7% compounded monthly yields the greater return.

56. $A = 6000\left(1+\frac{0.0825}{4}\right)^{4(4)}$
$\approx$ \$8317.84 (8.25% yield)
$A = 6000\left(1+\frac{0.083}{2}\right)^{2(4)}$
$\approx$ \$ 8306.64 (8.3% yield)
Investing \$6000 for 4 years at 8.25% compounded quarterly yields the greater return.

57.

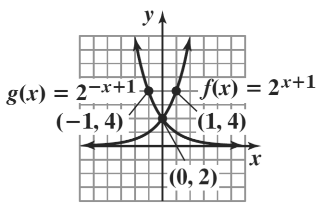
x	$f(x) = 2^x$	$g(x) = 2^{-x}$
−2	$\frac{1}{4}$	4
−1	$\frac{1}{2}$	2
0	1	1
1	2	$\frac{1}{2}$
2	4	$\frac{1}{4}$



The point of intersection is $(0,1)$.

58.

x	$f(x)=2^{x+1}$	$g(x)=2^{-x+1}$
–2	$\frac{1}{2}$	8
–1	1	4
0	2	2
1	4	1
2	8	$\frac{1}{2}$

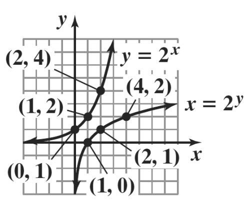


The point of intersection is $(0,2)$.

59.

x	$y=2^x$
–2	$\frac{1}{4}$
–1	$\frac{1}{2}$
0	1
1	2
2	4

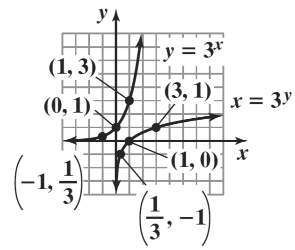
y	$x=2^y$
–2	$\frac{1}{4}$
–1	$\frac{1}{2}$
0	1
1	2
2	4



60.

x	$y=3^x$
–2	$\frac{1}{9}$
–1	$\frac{1}{3}$
0	1
1	3
2	9

y	$x=3^y$
–2	$\frac{1}{9}$
–1	$\frac{1}{3}$
0	1
1	3
2	9



61. The graph is of the form $y=b^x$.
Substitute values from the point (1, 4) to find b.

$$y=b^x$$
$$4=b^1$$
$$4=b$$

The equation of the graph is $y=4^x$

62. The graph is of the form $y=b^x$.
Substitute values from the point (1, 6) to find b.

$$y=b^x$$
$$6=b^1$$
$$6=b$$

The equation of the graph is $y=6^x$

63. The graph is of the form $y=-b^x$.
Substitute values from the point (1, –*e*) to find b.

$$y=-b^x$$
$$-e=-b^1$$
$$e=b$$

The equation of the graph is $y=-e^x$

64. The graph is of the form $y=b^x$.
Substitute values from the point (–1, *e*) to find b.

$$y=b^x$$
$$e=b^{-1}$$
$$e=\frac{1}{b}$$
$$eb=1$$
$$b=\frac{1}{e}$$

The equation of the graph is $y=\left(\frac{1}{e}\right)^x=e^{-x}$

65. **a.**
$$f(0)=574(1.026)^0$$
$$=574(1)=574$$

India's population in 1974 was 574 million.

b.
$$f(27)=574(1.026)^{27}\approx 1148$$

India's population in 2001 will be 1148 million.

c. Since $2028-1974=54$, find
$f(54)=574(1.026)^{54}\approx 2295$.
India's population in 2028 will be 2295 million.

d. $2055-1974=81$, find
$f(54)=574(1.026)^{81}\approx 4590$.
India's population in 2055 will be 4590 million.

e. India's population appears to be doubling every 27 years.

66. $f(80)=1000(0.5)^{\frac{80}{30}}=157.49$
Chernobyl will not be safe for human habitation by 2066. There will still be 157.5 kilograms of cesium-137 in Chernobyl's atmosphere.

67. $S=465,000(1+0.06)^{10}$
$=465,000(1.06)^{10}\approx \$832,744$

68. $S=510,000(1+0.03)^{5}$
$=510,000(1.03)^{5}$
$\approx \$591,230$

69. $2^{1.7}\approx 3.249009585$
$2^{1.73}\approx 3.317278183$
$2^{1.732}\approx 3.321880096$
$2^{1.73205}\approx 3.321995226$
$2^{1.7320508}\approx 3.321997068$
$2^{\sqrt{3}}\approx 3.321997085$
The closer the exponent is to $\sqrt{3}$, the closer the value is to $2^{\sqrt{3}}$.

70. $2^{3}\approx 8$
$2^{3.1}\approx 8.5741877$
$2^{3.14}\approx 8.815240927$
$2^{3.141}\approx 8.821353305$
$2^{3.1415}\approx 8.824411082$
$2^{3.14159}\approx 8.824961595$
$2^{3.141593}\approx 8.824979946$
$2^{\pi}\approx 8.824977827$
The closer the exponent gets to π, the closer the value is to 2^{π}.

71. **a.** $f(x)=x+31$
$f(33)=33+31$
$=64$
According to the linear model, 64% of high school seniors applied to more than three colleges in 2013.

b. $g(x)=32.7e^{0.0217x}$
$g(33)=32.7e^{0.0217(33)}$
≈ 67
According to the exponential model, about 67% of high school seniors applied to more than three colleges in 2013.

c. The exponential model is the better model for the data in 2013.

72. **a.** $f(x)=x+31$
$f(30)=30+31$
$=61$
According to the linear model, 61% of high school seniors applied to more than three colleges in 2013.

b. $g(x)=32.7e^{0.0217x}$
$g(30)=32.7e^{0.0217(30)}$
≈ 63
According to the exponential model, about 63% of high school seniors applied to more than three colleges in 2013.

c. The linear model is the better model for the data in 2010.

73. **a.** $f(0)=80e^{-0.5(0)}+20$
$=80e^{0}+20$
$=80(1)+20$
$=100$
100% of the material is remembered at the moment it is first learned.

b. $f(1)=80e^{-0.5(1)}+20\approx 68.5$
68.5% of the material is remembered 1 week after it is first learned.

c. $f(4)=80e^{-0.5(4)}+20\approx 30.8$
30.8% of the material is remembered 4 weeks after it is first learned.

d. $f(52)=80e^{-0.5(52)}+20\approx 20$
20% of the material is remembered 1 year after it is first learned.

74. **a.** $24\left(1+\frac{0.05}{12}\right)^{12(384)} \approx \$5{,}027{,}378{,}919$

b. $24e^{0.05(384)} \approx \$5{,}231{,}970{,}592$

75. $f(x) = 6.25(1.029)^x$
$f(59) = 6.25(1.029)^{59} \approx 33.8$
$g(x) = \frac{38.8}{1+6.3e^{-0.051x}}$
$g(59) = \frac{38.8}{1+6.3e^{-0.051(59)}} \approx 29.6$
Function $g(x)$ is a better model for the graph's value of 29.5 in 2009.

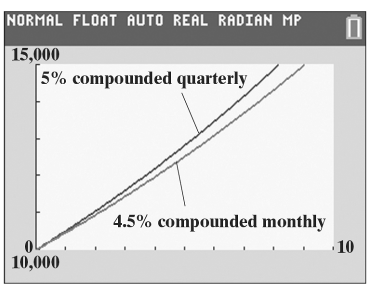
76. $f(x) = 6.25(1.029)^x$
$f(40) = 6.25(1.029)^{40} \approx 19.6$
$g(x) = \frac{38.8}{1+6.3e^{-0.051x}}$
$g(40) = \frac{38.8}{1+6.3e^{-0.051(40)}} \approx 21.3$
Function $g(x)$ is a better model for the graph's value of 21.3 in 1990.

77. – 80. Answers will vary.

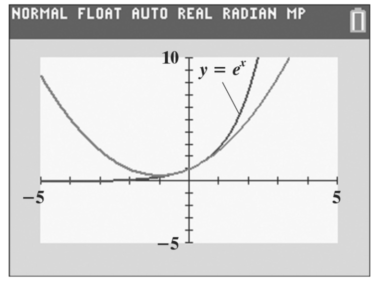
81. **a.** $A = 10{,}000\left(1+\frac{0.05}{4}\right)^{4t}$

$A = 10{,}000\left(1+\frac{0.045}{12}\right)^{12t}$

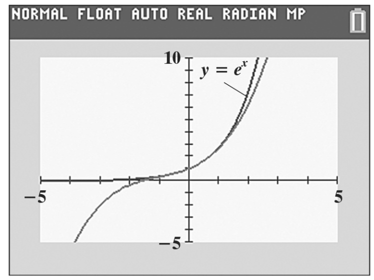
b. 5% compounded quarterly offers the better return.



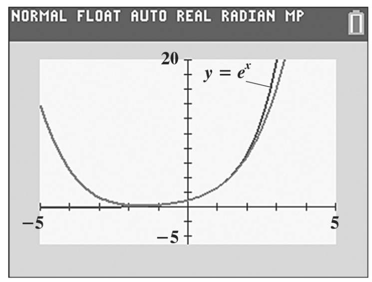
82. **a.**



b.



c.



d. Answers will vary.

83. does not make sense; Explanations will vary. Sample explanation: The horizontal asymptote is $y = 0$.

84. makes sense

85. does not make sense; Explanations will vary. Sample explanation: An exponential model appears to be a better choice.

86. makes sense

87. false; Changes to make the statement true will vary. A sample change is: The amount of money will not increase without bound.

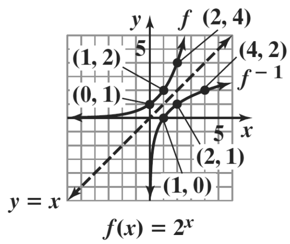
88. false; Changes to make the statement true will vary. A sample change is: The functions do not have the same graph. $f(x) = 3^{-x}$ reflects the graph of $y = 3^x$ about the y-axis while $f(x) = -3^x$ reflects the graph of $y = 3^x$ about the x-axis.

89. false; Changes to make the statement true will vary. A sample change is: If $f(x) = 2^x$ then $f(a+b) = f(a)\cdot f(b)$.

90. true

91. $y = 3^x$ is (d). y increases as x increases, but not as quickly as $y = 5^x$. $y = 5^x$ is (c). $y = \left(\frac{1}{3}\right)^x$ is (a). $y = \left(\frac{1}{3}\right)^x$ is the same as $y = 3^{-x}$, so it is (d) reflected about the y-axis. $y = \left(\frac{1}{5}\right)^x$ is (b). $y = \left(\frac{1}{5}\right)^x$ is the same as $y = 5^{-x}$, so it is (c) reflected about the y-axis.

92.



93. **a.**
$$\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^{x}}{2} = \frac{e^{x} + e^{-x}}{2} = \cosh x$$

b.
$$\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = \frac{-\left(-e^{-x} + e^{x}\right)}{2} = -\frac{e^{x} - e^{-x}}{2} = -\sinh x$$

c.
$$(\cosh x)^2 - (\sinh x)^2 \stackrel{?}{=} 1$$
$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \stackrel{?}{=} 1$$
$$\frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \stackrel{?}{=} 1$$
$$\frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} \stackrel{?}{=} 1$$
$$\frac{4}{4} \stackrel{?}{=} 1$$
$$1 = 1$$

94.
$$7x + 3y = 18$$
$$3y = -7x + 18$$
$$y = -\frac{7}{3}x + 6$$

95. $\pm 1, \ \pm 2$ are possible rational zeros

$$\begin{array}{c|cccc} 1 & 1 & 5 & -8 & 2 \\ & & 1 & 6 & -2 \\ \hline & 1 & 6 & -2 & 0 \end{array}$$

1 is a zero.

$x^2 + 6x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{44}}{2}$$

$$= \frac{-6 \pm 2\sqrt{11}}{2}$$

$$= -3 \pm \sqrt{11}$$

The zeros are $1, -3 + \sqrt{11}$, and $-3 - \sqrt{11}$.

96.

$$2x^2 + 5x < 12$$

$$2x^2 + 5x - 12 < 0$$

$$(2x - 3)(x + 4) < 0$$

Solve the related quadratic equation.

$(2x - 3)(x + 4) = 0$

Apply the zero product principle.

$2x - 3 = 0$ or $x + 4 = 0$

$x = \frac{3}{2}$ $\quad x = -4$

The boundary points are –4 and 1.5.

Test Interval	Test Number	Test	Conclusion
$(-\infty, -4)$	–5	$2(-5)^2 + 5(-5) < 12$ $25 < 12$, false	$(-\infty, -4)$ belongs to the solution set.
$(-4, 1.5)$	0	$2(0)^2 + 5(0) < 12$ $0 < 12$, true	$(-4, 5)$ does not belong to the solution set.
$(1.5, \infty)$	10	$2(10)^2 + 5(10) < 12$ $250 < 12$, false	$(5, \infty)$ belongs to the solution set.

The solution set is $\left(-4, \frac{3}{2}\right)$ or $\left\{ x \middle| -4 < x < \frac{3}{2} \right\}$.

97. We do not know how to solve $x = 2^y$ for y.

98. $\frac{1}{2}$; i.e. $25^{1/2} = 5$

99. $(x-3)^2 > 0$

Solving the related equation, $(x-3)^2 = 0$, gives $x = 3$.
Note that the boundary value $x = 3$ does not satisfy the inequality.
Testing each interval gives a solution set of $(-\infty, 3) \cup (3, \infty)$.

Section 3.2

Check Point Exercises

1. a. $3 = \log_7 x$ written in exponential form is $7^3 = x$.

b. $2 = \log_b 25$ written in exponential form is $b^2 = 25$.

c. $\log_4 26 = y$ written in exponential form is $4^y = 26$.

2. a. $2^5 = x$ written in logarithmic form is $5 = \log_2 x$.

b. $b^3 = 27$ written in logarithmic form is $3 = \log_b 27$.

c. $e^y = 33$ written in logarithmic form is $y = \log_e 33$.

3. a. Question: 10 to what power gives 100?
$\log_{10} 100 = 2$ because $10^2 = 100$.

b. Question: 5 to what power gives $\frac{1}{125}$?
$\log_5 \frac{1}{125} = -3$ because $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$.

c. Question: 36 to what power gives 6?
$\log_{36} 6 = \frac{1}{2}$ because $36^{1/2} = \sqrt{36} = 6$

d. Question: 3 to what power gives $\sqrt[7]{3}$?
$\log_3 \sqrt[7]{3} = \frac{1}{7}$ because $3^{1/7} = \sqrt[7]{3}$.

4. a. Because $\log_b b = 1$, we conclude $\log_9 9 = 1$.

b. Because $\log_b 1 = 0$, we conclude $\log_8 1 = 0$.

5. a. Because $\log_b b^x = x$, we conclude $\log_7 7^8 = 8$.

b. Because $b^{\log_b x} = x$, we conclude $3^{\log_3 17} = 17$.

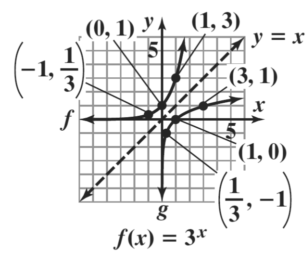
6. First, set up a table of coordinates for $f(x) = 3^x$.

x	-2	-1	0	1	2	3
$f(x) = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_3 x$.

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27
$g(x) = \log_3 x$	-2	-1	0	1	2	3

The graph of the inverse can also be drawn by reflecting the graph of $f(x) = 3^x$ about the line $y = x$.



7. The domain of h consists of all x for which $x-5>0$. Solving this inequality for x, we obtain $x > 5$. Thus, the domain of h is $(5, \infty)$.

8. Substitute the boy's age, 10, for x and evaluate the function at 10.
$$\begin{aligned} f(10) &= 29 + 48.8\log(10+1) \\ &= 29 + 48.8\log(11) \\ &\approx 80 \end{aligned}$$
A 10-year-old boy is approximately 80% of his adult height.

9. Because $I = 10{,}000\, I_0$,
$$\begin{aligned} R &= \log \frac{10{,}000 I_0}{I_0} \\ &= \log 10{,}000 \\ &= 4 \end{aligned}$$
The earthquake registered 4.0 on the Richter scale.

10. a. The domain of f consists of all x for which $4 - x > 0$. Solving this inequality for x, we obtain $x < 4$.
The domain of f is $(-\infty, 4)$

b. The domain of g consists of all x for which $x^2 > 0$. Solving this inequality for x, we obtain $x < 0$ or $x > 0$.
The domain of g is $(-\infty, 0) \cup (0, \infty)$.

11. Find the temperature increase after 30 minutes by substituting 30 for x and evaluating the function at 30.

$$f(x) = 13.4\ln x - 11.6$$
$$f(30) = 13.4\ln 30 - 11.6$$
$$\approx 34$$

The temperature increase after 30 minutes is $34°$. The function models the actual increase shown in the graph extremely well.

Concept and Vocabulary Check 3.2

1. $b^y = x$

2. logarithmic; b

3. 1

4. 0

5. x

6. x

7. $(0, \infty)$; $(-\infty, \infty)$

8. y; $x = 0$; vertical

9. up 5 units

10. to the left 5 units

11. x-axis

12. y-axis

13. $5 - x > 0$

14. common; $\log x$

15. natural; $\ln x$

Exercise Set 3.2

1. $2^4 = 16$

2. $2^6 = 64$

3. $3^2 = x$

4. $9^2 = x$

5. $b^5 = 32$

6. $b^3 = 27$

7. $6^y = 216$

8. $5^y = 125$

9. $\log_2 8 = 3$

10. $\log_5 625 = 4$

11. $\log_2 \frac{1}{16} = -4$

12. $\log_5 \frac{1}{125} = -3$

13. $\log_8 2 = \frac{1}{3}$

14. $\log_{64} 4 = \frac{1}{3}$

15. $\log_{13} x = 2$

16. $\log_{15} x = 2$

17. $\log_b 1000 = 3$

18. $\log_b 343 = 3$

19. $\log_7 200 = y$

20. $\log_8 300 = y$

21. $\log_4 16 = 2$ because $4^2 = 16$.

22. $\log_7 49 = 2$ because $7^2 = 49$.

23. $\log_2 64 = 6$ because $2^6 = 64$.

24. $\log_3 27 = 3$ because $3^3 = 27$.

25. $\log_5 \frac{1}{5} = -1$ because $5^{-1} = \frac{1}{5}$.

26. $\log_6 \frac{1}{6} = -1$ because $6^{-1} = \frac{1}{6}$.

27. $\log_2 \frac{1}{8} = -3$ because $2^{-3} = \frac{1}{8}$.

28. $\log_3 \frac{1}{9} = -2$ because $3^{-2} = \frac{1}{9}$.

29. $\log_7 \sqrt{7} = \frac{1}{2}$ because $7^{\frac{1}{2}} = \sqrt{7}$.

30. $\log_6 \sqrt{6} = \frac{1}{2}$ because $6^{\frac{1}{2}} = \sqrt{6}$.

31. $\log_2 \frac{1}{\sqrt{2}} = -\frac{1}{2}$ because $2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}$.

32. $\log_3 \frac{1}{\sqrt{3}} = -\frac{1}{2}$ because $3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$.

33. $\log_{64} 8 = \frac{1}{2}$ because $64^{1/2} = \sqrt{64} = 8$.

34. $\log_{81} 9 = \frac{1}{2}$ because $81^{1/2} = \sqrt{81} = 9$.

35. Because $\log_b b = 1$, we conclude $\log_5 5 = 1$.

36. Because $\log_b b = 1$, we conclude $\log_{11} 11 = 1$.

37. Because $\log_b 1 = 0$, we conclude $\log_4 1 = 0$.

38. Because $\log_b 1 = 0$, we conclude $\log_6 1 = 0$.

39. Because $\log_b b^x = x$, we conclude $\log_5 5^7 = 7$.

40. Because $\log_b b^x = x$, we conclude $\log_4 4^6 = 6$.

41. Because $b^{\log_b x} = x$, we conclude $8^{\log_8 19} = 19$.

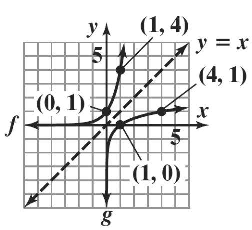
42. Because $b^{\log_b x} = x$, we conclude $7^{\log_7 23} = 23$.

43. First, set up a table of coordinates for $f(x) = 4^x$.

x	-2	-1	0	1	2	3
$f(x) = 4x$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_4 x$.

x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64
$g(x) = \log_{4x}$	-2	-1	0	1	2	3



$f(x) = 4^x$

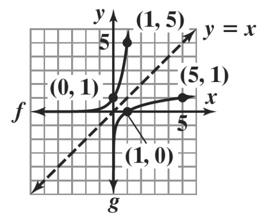
$g(x) = \log_4 x$

44. First, set up a table of coordinates for $f(x) = 5^x$.

x	-2	-1	0	1	2	3
$f(x) = 5^x$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_5 x$.

x	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125
$g(x) = \log_5 x$	-2	-1	0	1	2	3



$f(x) = 5^x$

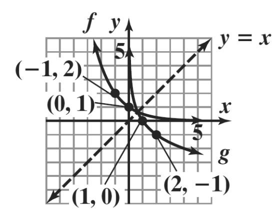
$g(x) = \log_5 x$

45. First, set up a table of coordinates for $f(x) = \left(\frac{1}{2}\right)^x$.

x	-2	-1	0	1	2	3
$f(x) = \left(\frac{1}{2}\right)^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_{1/2} x$.

x	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$g(x) = \log_{1/2} x$	-2	-1	0	1	2	3



$f(x) = \left(\frac{1}{2}\right)^x$

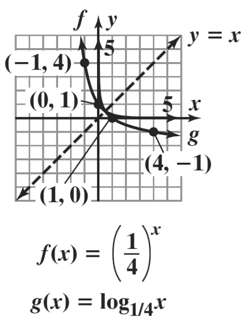
$g(x) = \log_{1/2} x$

46. First, set up a table of coordinates for $f(x)=\left(\frac{1}{4}\right)^x$.

x	-2	-1	0	1	2	3
$f(x)=\left(\frac{1}{4}\right)^x$	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$

Reversing these coordinates gives the coordinates for the inverse function $g(x)=\log_{1/4} x$.

x	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
$g(x)=\log_{1/4} x$	-2	-1	0	1	2	3



47. This is the graph of $f(x)=\log_3 x$ reflected about the x-axis and shifted up one unit, so the function is $H(x)=1-\log_3 x$.

48. This is the graph of $f(x)=\log_3 x$ reflected about the y-axis, so the function is $G(x)=\log_3(-x)$.

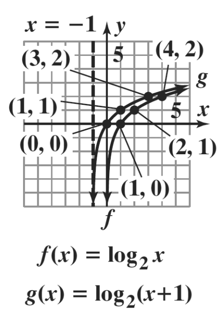
49. This is the graph of $f(x)=\log_3 x$ shifted down one unit, so the function is $h(x)=\log_3 x-1$.

50. This is the graph of $f(x)=\log_3 x$ reflected about the x-axis, so the function is $F(x)=-\log_3 x$.

51. This is the graph of $f(x)=\log_3 x$ shifted right one unit, so the function is $g(x)=\log_3(x-1)$.

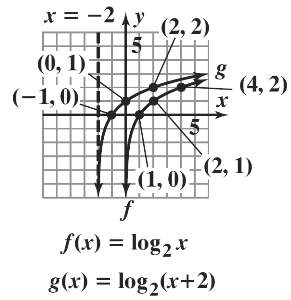
52. This is the graph of $f(x)=\log_3 x$.

53.



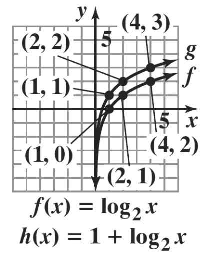
vertical asymptote: $x=-1$
domain: $(-1,\infty)$; range: $(-\infty,\infty)$

54.



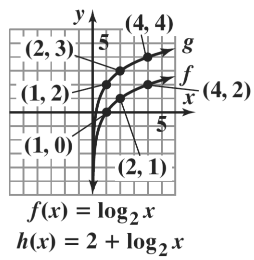
vertical asymptote: $x=-2$
domain: $(-2,\infty)$; range: $(-\infty,\infty)$

55.



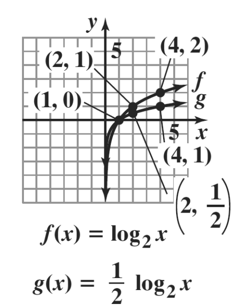
vertical asymptote: $x=0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

56.



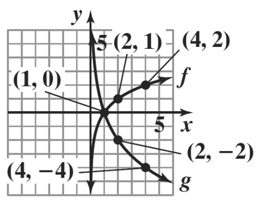
vertical asymptote: $x=0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

57.



vertical asymptote: $x=0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

58.

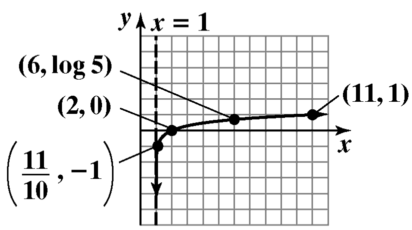


$f(x) = \log_2 x$

$g(x) = -2\log_2 x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

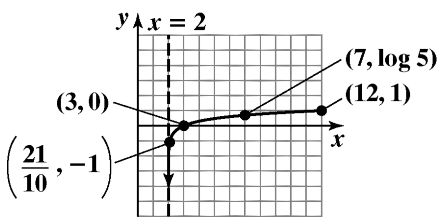
59.



$g(x) = \log(x-1)$

vertical asymptote: $x = 1$
domain: $(1, \infty)$; range: $(-\infty, \infty)$

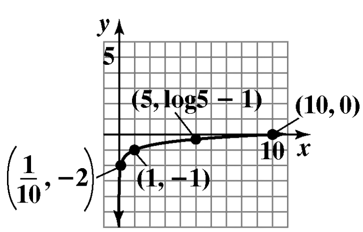
60.



$g(x) = \log(x-2)$

vertical asymptote: $x = 2$
domain: $(2, \infty)$; range: $(-\infty, \infty)$

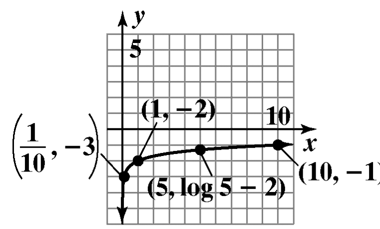
61.



$h(x) = \log x - 1$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

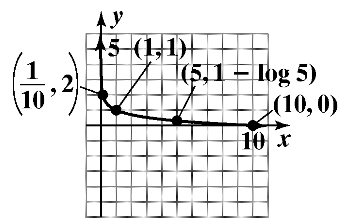
62.



$h(x) = \log x - 2$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

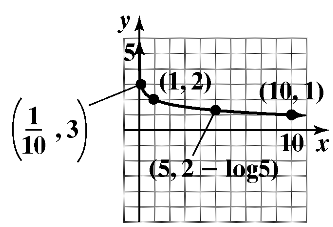
63.



$g(x) = 1 - \log x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

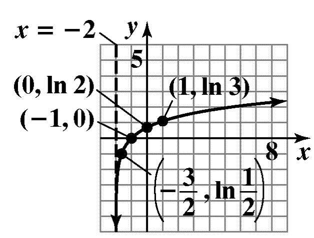
64.



$g(x) = 2 - \log x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

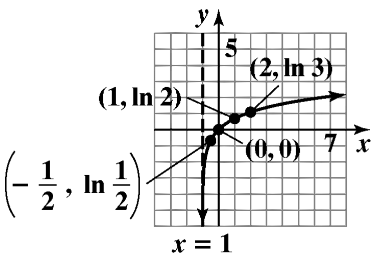
65.



$g(x) = \ln(x+2)$

vertical asymptote: $x = -2$
domain: $(-2, \infty)$; range: $(-\infty, \infty)$

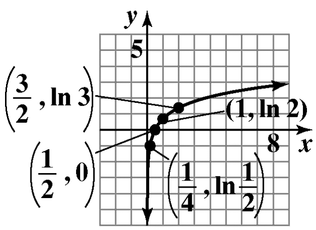
66.



$g(x) = \ln(x+1)$

vertical asymptote: $x = -1$
domain: $(-1, \infty)$; range: $(-\infty, \infty)$

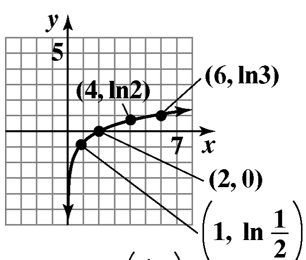
67.



$h(x) = \ln(2x)$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

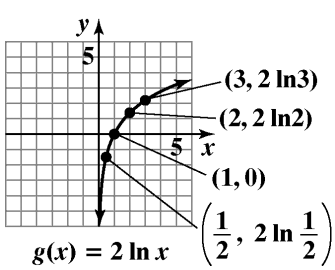
68.



$h(x) = \ln\left(\frac{1}{2}x\right)$

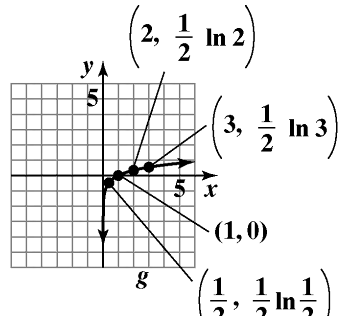
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

69.



vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

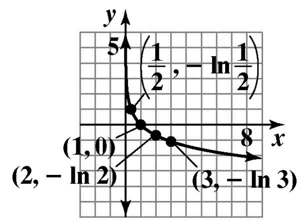
70.



$g(x) = \frac{1}{2}\ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

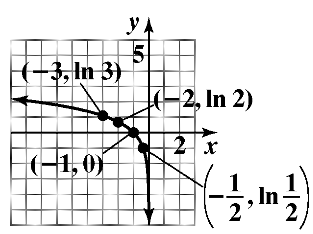
71.



$h(x) = -\ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

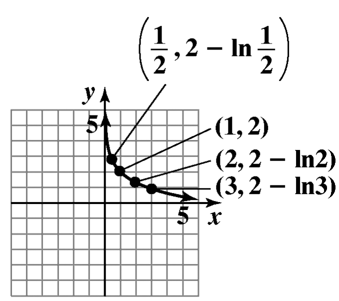
72.



$h(x) = \ln(-x)$

vertical asymptote: $x = 0$
domain: $(-\infty, 0)$; range: $(-\infty, \infty)$

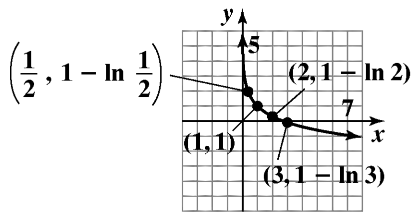
73.



$g(x) = 2 - \ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

74.



$g(x) = 1 - \ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

75. The domain of f consists of all x for which $x + 4 > 0$. Solving this inequality for x, we obtain $x > -4$. Thus, the domain of f is $(-4, \infty)$.

76. The domain of f consists of all x for which $x + 6 > 0$. Solving this inequality for x, we obtain $x > -6$. Thus, the domain of f is $(-6, \infty)$.

77. The domain of f consists of all x for which $2 - x > 0$. Solving this inequality for x, we obtain $x < 2$. Thus, the domain of f is $(-\infty, 2)$.

78. The domain of f consists of all x for which $7 - x > 0$. Solving this inequality for x, we obtain $x < 7$. Thus, the domain of f is $(-\infty, 7)$.

79. The domain of f consists of all x for which $(x - 2)^2 > 0$. Solving this inequality for x, we obtain $x < 2$ or $x > 2$. Thus, the domain of f is $(-\infty, 2)$ or $(2, \infty)$.

80. The domain of f consists of all x for which $(x-7)^2 > 0$. Solving this inequality for x, we obtain $x < 7$ or $x > 7$. Thus, the domain of f is $(-\infty, 7)$ or $(7, \infty)$.

81. $\log 100 = \log_{10} 100 = 2$ because $10^2 = 100$.

82. $\log 1000 = \log_{10} 1000 = 3$ because $10^3 = 1000$.

83. Because $\log 10^x = x$, we conclude $\log 10^7 = 7$.

84. Because $\log 10^x = x$, we conclude $\log 10^8 = 8$.

85. Because $10^{\log x} = x$, we conclude $10^{\log 33} = 33$.

86. Because $10^{\log x} = x$, we conclude $10^{\log 53} = 53$.

87. $\ln 1 = 0$ because $e^0 = 1$.

88. $\ln e = \log_e e = 1$ because $e^1 = e$.

89. Because $\ln e^x = x$, we conclude $\ln e^6 = 6$.

90. Because $\ln e^x = x$, we conclude $\ln e^7 = 7$.

91. $\ln \frac{1}{e^6} = \ln e^{-6}$

Because $\ln e^x = x$ we conclude

$\ln e^{-6} = -6$, so $\ln \frac{1}{e^6} = -6$.

92. $\ln \frac{1}{e^7} = \ln e^{-7}$ Because $\ln e^x = x$, we conclude

$\ln e^{-7} = -7$, so $\ln \frac{1}{e^7} = -7$.

93. Because $e^{\ln x} = x$, we conclude $e^{\ln 125} = 125$.

94. Because $e^{\ln x} = x$, we conclude $e^{\ln 300} = 300$.

95. Because $\ln e^x = x$, we conclude $\ln e^{9x} = 9x$.

96. Because $\ln e^x = x$, we conclude $\ln e^{13x} = 13x$.

97. Because $e^{\ln x} = x$, we conclude $e^{\ln 5x^2} = 5x^2$.

98. Because $e^{\ln x} = x$, we conclude $e^{\ln 7x^2} = 7x^2$.

99. Because $10^{\log x} = x$, we conclude $10^{\log \sqrt{x}} = \sqrt{x}$.

100. Because $10^{\log x} = x$, we conclude $10^{\log \sqrt[3]{x}} = \sqrt[3]{x}$.

101. $\log_3(x-1) = 2$

$$3^2 = x - 1$$
$$9 = x - 1$$
$$10 = x$$

The solution is 10, and the solution set is $\{10\}$.

102. $\log_5(x+4) = 2$

$$5^2 = x + 4$$
$$25 = x + 4$$
$$21 = x$$

The solution is 21, and the solution set is $\{21\}$.

103. $\log_4 x = -3$
$4^{-3} = x$
$x = \frac{1}{4^3} = \frac{1}{64}$
The solution is $\frac{1}{64}$, and the solution set is $\left\{\frac{1}{64}\right\}$.

104. $\log_{64} x = \frac{2}{3}$
$64^{\frac{2}{3}} = x$
$x = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$
The solution is 16, and the solution set is {16}.

105. $\log_3(\log_7 7) = \log_3 1 = 0$

106. $\log_5(\log_2 32) = \log_5\left(\log_2 2^5\right) = \log_5 5 = 1$

107. $\log_2(\log_3 81) = \log_2\left(\log_3 3^4\right)$
$= \log_2 4 = \log_2 2^2 = 2$

108. $\log(\ln e) = \log 1 = 0$

109. For $f(x) = \ln(x^2 - x - 2)$ to be real, $x^2 - x - 2 > 0$.
Solve the related equation to find the boundary points:
$x^2 - x - 2 = 0$
$(x+1)(x-2) = 0$
The boundary points are –1 and 2. Testing each interval gives a domain of $(-\infty, -1) \cup (2, \infty)$.

110. For $f(x) = \ln(x^2 - 4x - 12)$ to be real,
$x^2 - 4x - 12 > 0$.
Solve the related equation to find the boundary points:
$x^2 - 4x - 12 = 0$
$(x+2)(x-6) = 0$
The boundary points are –2 and 6. Testing each interval gives a domain of $(-\infty, -2) \cup (6, \infty)$.

111. For $f(x) = \ln\left(\frac{x+1}{x-5}\right)$ to be real, $\frac{x+1}{x-5} > 0$.
The boundary points are –1 and 5. Testing each interval gives a domain of $(-\infty, -1) \cup (5, \infty)$.

112. For $f(x) = \ln\left(\frac{x-2}{x+5}\right)$ to be real, $\frac{x-2}{x+5} > 0$.
The boundary points are –5 and 2. Testing each interval gives a domain of $(-\infty, -5) \cup (2, \infty)$.

113. $f(13) = 62 + 35\log(13-4) \approx 95.4$
She is approximately 95.4% of her adult height.

114. $f(10) = 62 + 35\log(10-4) \approx 89.2$.
She is approximately 89.2% of her adult height.

115. **a.** 2010 is 46 years after 1964.
$f(x) = -3.52\ln x + 34.5$
$f(46) = -3.52\ln 46 + 34.5 \approx 21$
According to the function, wives engaged in 21 hours of weekly housework in 2010. This overestimates the value in the graph by 1 hour.

b. 2025 is 61 years after 1964.
$f(x) = -3.52\ln x + 34.5$
$f(61) = -3.52\ln 61 + 34.5 \approx 20$
According to the function, wives will engage in 20 hours of weekly housework in 2025.

116. **a.** 2010 is 46 years after 1964.
$f(x) = 1.8\ln x + 3.42$
$f(39) = 1.8\ln 39 + 3.42 \approx 10$
According to the function, husbands engaged in 10 hours of weekly housework in 2010. This underestimates the value in the graph by 1 hour.

b. 2025 is 61 years after 1964.
$f(x) = 1.8\ln x + 3.42$
$f(61) = 1.8\ln 61 + 3.42 \approx 11$
According to the function, husbands will engage in 11 hours of weekly housework in 2025.

117. $D = 10\log\left[10^{12}(6.3\times10^6)\right] \approx 188$
Yes, the sound can rupture the human eardrum.

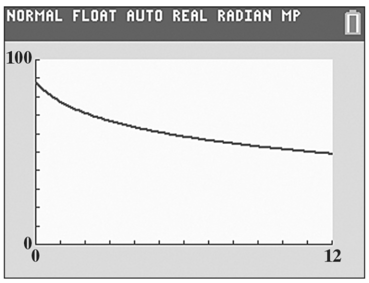
118. $D = 10\log\left[10^{12}\left(3.2\times10^{-6}\right)\right] \approx 65.05$
A normal conversation is about 65 decibels.

119. a. $f(0) = 88 - 15\ln(0 + 1) = 88$
The average score on the original exam was 88.

b. $f(2) = 88 - 15\ln(2 + 1) = 71.5$
$f(4) = 88 - 15\ln(4 + 1) = 63.9$
$f(6) = 88 - 15\ln(6 + 1) = 58.8$
$f(8) = 88 - 15\ln(8 + 1) = 55$
$f(10) = 88 - 15\ln(10 + 1) = 52$
$f(12) = 88 - 15\ln(12 + 1) = 49.5$

The average score after 2 months was about 71.5, after 4 months was about 63.9, after 6 months was about 58.8, after 8 months was about 55, after 10 months was about 52, and after one year was about 49.5.

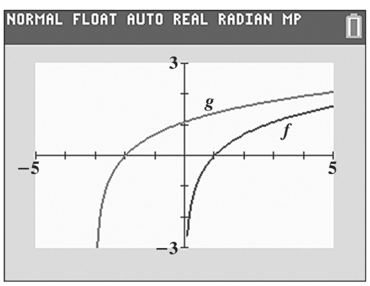
c.



Material retention decreases as time passes.

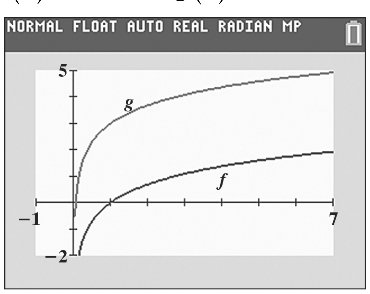
120. – 127. Answers will vary.

128. $f(x) = \ln x \qquad g(x) = \ln(x+3)$



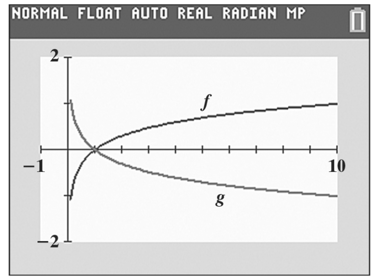
The graph of *g* is the graph of *f* shifted 3 units to the left.

129. $f(x) = \ln x \qquad g(x) = \ln x + 3$



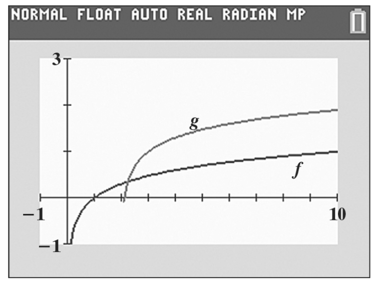
The graph of *g* is the graph of *f* shifted up 3 units.

130. $f(x) = \log x \qquad g(x) = -\log x$



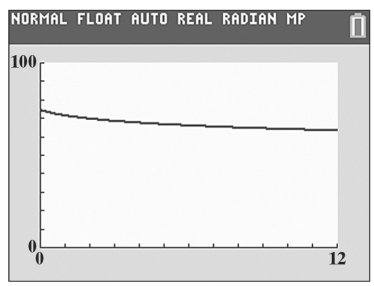
The graph of *g* is the graph of *f* reflected across the *x*–axis.

131. $f(x) = \log x \qquad g(x) = \log(x-2)+1$



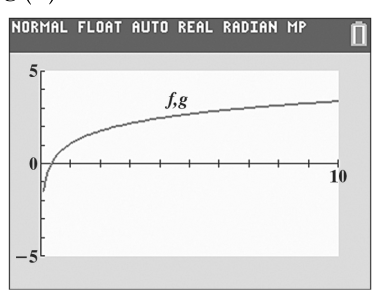
The graph of *g* is the graph of *f* shifted 2 units to the right and 1 unit up.

132. $f(t) = 75 - 10\log(t+1)$



After approximately 9 months, the average score falls below 65.

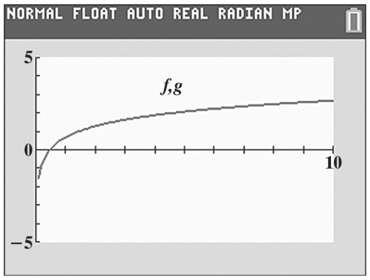
133. a. $f(x) = \ln(3x)$
$g(x) = \ln 3 + \ln x$



The graphs coincide.

b. $f(x)=\log(5x^2)$

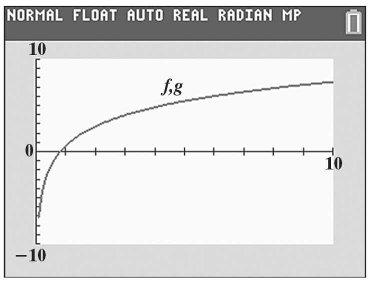
$g(x)=\log 5+\log x^2$



The graphs coincide.

c. $f(x)=\ln(2x^3)$

$g(x)=\ln 2+\ln x^3$

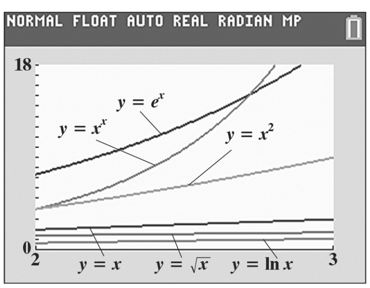


The graphs coincide.

d. In each case, the function, *f*, is equivalent to *g*. This means that $\log_b(MN)=\log_b M+\log_b N$.

e. The logarithm of a product is equal to <u>the sum of the logarithms of the factors</u>.

134.



Use the trace function to compare how quickly the functions increase. In order from slowest to fastest, the functions are:

$y=\ln x,\ y=\sqrt{x},\ y=x,\ y=x^2,\ y=e^x,$ and $y=x^x$.

135. makes sense

136. makes sense

137. makes sense

138. does not make sense; Explanations will vary. Sample explanation: An earthquake of magnitude 8 on the Richter scale is $10^{8-4}=10^4=10,000$ times as intense as an earthquake of magnitude 4.

139. false; Changes to make the statement true will vary. A sample change is: $\frac{\log_2 8}{\log_2 4}=\frac{3}{2}$

140. false; Changes to make the statement true will vary. A sample change is: We cannot take the log of a negative number.

141. false; Changes to make the statement true will vary. A sample change is: The domain of $f(x)=\log_2 x$ is $(0,\infty)$.

142. true

143. $\frac{\log_3 81-\log_\pi 1}{\log_{2\sqrt{2}} 8-\log 0.001}=\frac{4-0}{2-(-3)}=\frac{4}{5}$

144. $\log_4[\log_3(\log_2 8)]$

$=\log_4\left[\log_3\left(\log_2 2^3\right)\right]$

$=\log_4[\log_3 3]=\log_4 1=0$

145. $\log_4 60<\log_4 64=3$ so $\log_4 60<3$.

$\log_3 40>\log_3 27=3$ so $\log_3 40>3$.

$\log_4 60<3<\log_3 40$

$\log_3 40>\log_4 60$

146. Answers will vary.

147. Let *x* = the amount, in millions of dollars, that Trey Parker is worth

Let *x* + 450 = the amount, in millions of dollars, that Larry David is worth

Let *x* + 150 = the amount, in millions of dollars, that Matt Groening is worth

$$x+(x+450)+(x+150)=1650$$
$$x+x+450+x+150=1650$$
$$3x+600=1650$$
$$3x=1050$$
$$x=350$$

$x=350$, Parker is worth: \$350 million

$x+450=800$, David is worth: \$800 million

$x+150=500$, Groening is worth: \$500 million

148. $f(x) = mx + b$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{m(x+h)+b-(mx+b)}{h}$$
$$= \frac{mx+mh+b-mx-b}{h}$$
$$= \frac{mh}{h}$$
$$= m, \quad h \neq 0$$

149. $f(x) = x^2 + 4$

Replace $f(x)$ with y:

$y = x^2 + 4$

Interchange x and y:

$x = y^2 + 4$

Solve for y:

$$x = y^2 + 4$$
$$x - 4 = y^2$$
$$\sqrt{x-4} = y$$

Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \sqrt{x-4}$

150. **a.** $\log_2 32 = \log_2 2^5 = 5$

b. $\log_2 8 + \log_2 4 = \log_2 2^3 + \log_2 2^2 = 3 + 2 = 5$

c. $\log_2 (8 \cdot 4) = \log_2 8 + \log_2 4$

151. **a.** $\log_2 16 = \log_2 2^4 = 4$

b. $\log_2 32 - \log_2 2 = \log_2 2^5 - \log_2 2 = 5 - 1 = 4$

c. $\log_2 \left(\frac{32}{2}\right) = \log_2 32 - \log_2 2$

152. **a.** $\log_3 81 = \log_3 3^4 = 4$

b. $2\log_3 9 = 2\log_3 3^2 = 2 \cdot 2 = 4$

c. $\log_3 9^2 = 2\log_3 9$

Section 3.3

Check Point Exercises

1. **a.** $\log_6 (7 \cdot 11) = \log_6 7 + \log_6 11$

b. $\log(100x) = \log 100 + \log x$
$= 2 + \log x$

2. **a.** $\log_8 \left(\frac{23}{x}\right) = \log_8 23 - \log_8 x$

b. $\ln\left(\frac{e^5}{11}\right) = \ln e^5 - \ln 11$
$= 5 - \ln 11$

3. **a.** $\log_6 3^9 = 9\log_6 3$

b. $\ln \sqrt[3]{x} = \ln x^{1/3} = \frac{1}{3}\ln x$

c. $\log(x+4)^2 = 2\log(x+4)$

4. **a.** $\log_b x^4 \sqrt[3]{y}$

$$= \log_b x^4 y^{1/3}$$
$$= \log_b x^4 + \log_b y^{1/3}$$
$$= 4\log_b x + \frac{1}{3}\log_b y$$

b. $\log_5 \frac{\sqrt{x}}{25y^3}$

$$= \log_5 \frac{x^{1/2}}{25y^3}$$
$$= \log_5 x^{1/2} - \log_5 25y^3$$
$$= \log_5 x^{1/2} - \left(\log_5 5^2 + \log_5 y^3\right)$$
$$= \tfrac{1}{2}\log_5 x - \log_5 5^2 - \log_5 y^3$$
$$= \tfrac{1}{2}\log_5 x - 2\log_5 5 - 3\log_5 y$$
$$= \tfrac{1}{2}\log_5 x - 2 - 3\log_5 y$$

5. **a.** $\log 25 + \log 4 = \log(25 \cdot 4) = \log 100 = 2$

b. $\log(7x+6) - \log x = \log \frac{7x+6}{x}$

6. **a.** $2\ln x+\frac{1}{3}\ln(x+5)$

$=\ln x^2+\ln(x+5)^{1/3}$
$=\ln x^2(x+5)^{1/3}$
$=\ln\left[x^2\sqrt[3]{x+5}\right]$

b. $2\log(x-3)-\log x$

$=\log(x-3)^2-\log x$

$=\log\frac{(x-3)^2}{x}$

c. $\frac{1}{4}\log_b x-2\log_b 5-10\log_b y$

$=\log_b x^{1/4}-\log_b 5^2-\log_b y^{10}$
$=\log_b x^{1/4}-\left(\log_b 25+\log_b y^{10}\right)$
$=\log_b x^{1/4}-\log_b 25y^{10}$
$=\log_b\frac{x^{1/4}}{25y^{10}}$ or $\log_b\frac{\sqrt[4]{x}}{25y^{10}}$

7. $\log_7 2506=\frac{\log 2506}{\log 7}\approx 4.02$

8. $\log_7 2506=\frac{\ln 2506}{\ln 7}\approx 4.02$

Concept and Vocabulary Check 3.3

1. $\log_b M+\log_b N$; sum

2. $\log_b M-\log_b N$; difference

3. $p\log_b M$; product

4. $\log_a M$; $\log_a b$

Exercise Set 3.3

1. $\log_5(7\cdot 3)=\log_5 7+\log_5 3$

2. $\log_8(13\cdot 7)=\log_8 13+\log_8 7$

3. $\log_7(7x)=\log_7 7+\log_7 x=1+\log_7 x$

4. $\log_9 9x=\log_9 9+\log_9 x=1+\log_9 x$

5. $\log(1000x)=\log 1000+\log x=3+\log x$

6. $\log(10{,}000x)=\log 10{,}000+\log x=4+\log x$

7. $\log_7\left(\frac{7}{x}\right)=\log_7 7-\log_7 x=1-\log_7 x$

8. $\log_9\left(\frac{9}{x}\right)=\log_9 9-\log_9 x=1-\log_9 x$

9. $\log\left(\frac{x}{100}\right)=\log x-\log 100=\log_x-2$

10. $\log\left(\frac{x}{1000}\right)=\log x-\log 1000=\log x-3$

11. $\log_4\left(\frac{64}{y}\right)=\log_4 64-\log_4 y$
$=3-\log_4 y$

12. $\log_5\left(\frac{125}{y}\right)=\log_5 125-\log_5 y=3-\log_5 y$

13. $\ln\left(\frac{e^2}{5}\right)=\ln e^2-\ln 5=2\ln e-\ln 5=2-\ln 5$

14. $\ln\left(\frac{e^4}{8}\right)=\ln e^4-\ln 8=4\ln e-\ln 8=4-\ln 8$

15. $\log_b x^3=3\log_b x$

16. $\log_b x^7=7\log_b x$

17. $\log N^{-6}=-6\log N$

18. $\log M^{-8}=-8\log M$

19. $\ln\sqrt[5]{x}=\ln x^{(1/5)}=\frac{1}{5}\ln x$

20. $\ln\sqrt[7]{x}=\ln x^{\frac{1}{7}}=\frac{1}{7}\ln x$

21. $\log_b x^2y=\log_b x^2+\log_b y=2\log_b x+\log_b y$

22. $\log_b xy^3=\log_b x+\log_b y^3=\log_b x+3\log_b y$

23. $\log_4\left(\frac{\sqrt{x}}{64}\right)=\log_4 x^{1/2}-\log_4 64=\frac{1}{2}\log_4 x-3$

24. $\log_5\left(\frac{\sqrt{x}}{25}\right) = \log_5 x^{1/2} - \log_5 25 = \frac{1}{2}\log_5 x - 2$

25. $\log_6\left(\frac{36}{\sqrt{x+1}}\right) = \log_6 36 - \log_6 (x+1)^{1/2}$
$= 2 - \frac{1}{2}\log_6 (x+1)$

26. $\log_8\left(\frac{64}{\sqrt{x+1}}\right) = \log_8 64 - \log_8 (x+1)^{1/2}$
$= 2 - \frac{1}{2}\log_8 (x+1)$

27. $\log_b\left(\frac{x^2 y}{z^2}\right) = \log_b\left(x^2 y\right) - \log_b z^2$
$= \log_b x^2 + \log_b y - \log_b z^2$
$= 2\log_b x + \log_b y - 2\log_b z$

28. $\log_b\left(\frac{x^3 y}{z^2}\right) = \log_b\left(x^3 y\right) - \log_b z^2$
$\log_b\left(\frac{x^3 y}{z^2}\right) = \log_b x^3 + \log_b y - \log_b z^2$
$= 3\log_b x + \log_b y - 2\log_b z$

29. $\log\sqrt{100x} = \log(100x)^{1/2}$
$= \frac{1}{2}\log(100x)$
$= \frac{1}{2}(\log 100 + \log x)$
$= \frac{1}{2}(2 + \log x)$
$= 1 + \frac{1}{2}\log x$

30. $\ln\sqrt{ex} = \ln(ex)^{1/2}$
$= \frac{1}{2}\ln(ex)$
$= \frac{1}{2}(\ln e + \ln x)$
$= \frac{1}{2}(1 + \ln x)$
$= \frac{1}{2} + \frac{1}{2}\ln x$

31. $\log\sqrt[3]{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/3}$
$= \frac{1}{3}\left[\log\left(\frac{x}{y}\right)\right]$
$= \frac{1}{3}(\log x - \log y)$
$= \frac{1}{3}\log x - \frac{1}{3}\log y$

32. $\log\sqrt[5]{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/5}$
$= \frac{1}{5}\left[\log\left(\frac{x}{y}\right)\right]$
$= \frac{1}{5}(\log x - \log y)$
$= \frac{1}{5}\log x - \frac{1}{5}\log y$

33. $\log_b \frac{\sqrt{x}y^3}{z^3}$
$= \log_b x^{1/2} + \log_b y^3 - \log_b z^3$
$= \frac{1}{2}\log_b x + 3\log_b y - 3\log_b z$

34. $\log_b \frac{\sqrt[3]{x}y^4}{z^5}$
$= \log_b x^{1/3} + \log_b y^4 - \log_b z^5$
$= \frac{1}{3}\log_b x + 4\log_b y - 5\log_b z$

35. $\log_5 \sqrt[3]{\frac{x^2 y}{25}}$
$= \log_5 x^{2/3} + \log_5 y^{1/3} - \log_5 25^{1/3}$
$= \frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \log_5 5^{2/3}$
$= \frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \frac{2}{3}$

36. $\log_2 \sqrt[5]{\frac{xy^4}{16}}$
$= \log_2 x^{1/5} + \log_2 y^{4/5} - \log_2 16^{1/5}$
$= \frac{1}{5}\log_2 x + \frac{4}{5}\log_2 y - \frac{1}{5}\log_2 16$
$= \frac{1}{5}\log_2 x + \frac{4}{5}\log_2 y - \frac{4}{5}$

37. $\ln\left[\dfrac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$
$= \ln x^3 + \ln\sqrt{x^2+1} - \ln(x+1)^4$
$= 3\ln x + \dfrac{1}{2}\ln(x^2+1) - 4\ln(x+1)$

38. $\ln\left[\dfrac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$
$= \ln\left[\dfrac{x^4(x^2+3)^{1/2}}{(x+3)^5}\right]$
$= \ln x^4 + \ln\left(x^2+3\right)^{1/2} - \ln\left(x+3\right)^5$
$= 4\ln x + \dfrac{1}{2}\ln\left(x^2+3\right) - 5\ln\left(x+3\right)$

39. $\log\left[\dfrac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$
$= \log 10 + \log x^2 + \log\sqrt[3]{1-x} - \log 7 - \log(x+1)^2$
$= 1 + 2\log x + \dfrac{1}{3}\log(1-x) - \log 7 - 2\log(x+1)$

40. $\log\left[\dfrac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$
$= \log 100 + \log x^3 + \log\left(5-x\right)^{\frac{1}{3}} - \log 3 - \log\left(x+7\right)^2$
$= 2 + 3\log x + \dfrac{1}{3}\log(5-x) - \log 3 - 2\log(x+7)$

41. $\log 5 + \log 2 = \log(5\cdot 2) = \log 10 = 1$

42. $\log 250 + \log 4 = \log 1000 = 3$

43. $\ln x + \ln 7 = \ln(7x)$

44. $\ln x + \ln 3 = \ln(3x)$

45. $\log_2 96 - \log_2 3 = \log_2\left(\dfrac{96}{3}\right) = \log_2 32 = 5$

46. $\log_3 405 - \log_3 5 = \log_3\left(\dfrac{405}{5}\right)$
$= \log_3 81$
$= 4$

47. $\log(2x+5) - \log x = \log\left(\dfrac{2x+5}{x}\right)$

48. $\log(3x+7) - \log x = \log\left(\dfrac{3x+7}{x}\right)$

49. $\log x + 3\log y = \log x + \log y^3 = \log(xy^3)$

50. $\log x + 7\log y = \log x + \log y^7 = \log(xy^7)$

51. $\dfrac{1}{2}\ln x + \ln y = \ln x^{1/2} + \ln y$
$= \ln\left(x^{\frac{1}{2}}y\right)$ or $\ln\left(y\sqrt{x}\right)$

52. $\dfrac{1}{3}\ln x + \ln y = \ln x^{1/3} + \ln y$
$= \ln\left(x^{1/3}y\right)$ or $\ln\left(y\sqrt[3]{x}\right)$

53. $2\log_b x + 3\log_b y = \log_b x^2 + \log_b y^3$
$= \log_b(x^2y^3)$

54. $5\log_b x + 6\log_b y = \log_b x^5 + \log_b y^6$
$= \log_b\left(x^5y^6\right)$

55. $5\ln x - 2\ln y = \ln x^5 - \ln y^2 = \ln\left(\dfrac{x^5}{y^2}\right)$

56. $7\ln x - 3\ln y = \ln x^7 - \ln y^3 = \ln\left(\dfrac{x^7}{y^3}\right)$

57. $3\ln x - \dfrac{1}{3}\ln y = \ln x^3 - \ln y^{1/3}$
$= \ln\left(\dfrac{x^3}{y^{1/3}}\right)$ or $\ln\left(\dfrac{x^3}{\sqrt[3]{y}}\right)$

58. $2\ln x - \dfrac{1}{2}\ln y = \ln x^2 - \ln y^{1/2}$
$= \ln\left(\dfrac{x^2}{y^{1/2}}\right)$ or $\ln\left(\dfrac{x^2}{\sqrt{y}}\right)$

59. $4\ln(x+6) - 3\ln x = \ln(x+6)^4 - \ln x^3$
$= \ln\dfrac{(x+6)^4}{x^3}$

60. $8\ln(x+9)-4\ln x=\ln(x+9)^8-\ln x^4$

$=\ln\dfrac{(x+9)^8}{x^4}$

61. $3\ln x+5\ln y-6\ln z$

$=\ln x^3+\ln y^5-\ln z^6$

$=\ln\dfrac{x^3y^5}{z^6}$

62. $4\ln x+7\ln y-3\ln z$

$=\ln x^4+\ln y^7-\ln z^3$

$=\ln\dfrac{x^4y^7}{z^3}$

63. $\dfrac{1}{2}\left(\log x+\log y\right)$

$=\dfrac{1}{2}(\log xy)$

$=\log(xy)^{1/2}$

$=\log\sqrt{xy}$

64. $\dfrac{1}{3}\left(\log_4 x-\log_4 y\right)$

$=\dfrac{1}{3}\log_4\dfrac{x}{y}$

$=\log_4\left(\dfrac{x}{y}\right)^{1/3}$

$=\log_4\sqrt[3]{\dfrac{x}{y}}$

65. $\dfrac{1}{2}(\log_5 x+\log_5 y)-2\log_5(x+1)$

$=\dfrac{1}{2}\log_5 xy-\log_5(x+1)^2$

$=\log_5(xy)^{1/2}-\log_5(x+1)^2$

$=\log_5\dfrac{(xy)^{1/2}}{(x+1)^2}$

$=\log_5\dfrac{\sqrt{xy}}{(x+1)^2}$

66. $\dfrac{1}{3}\left(\log_4 x-\log_4 y\right)+2\log_4(x+1)$

$=\dfrac{1}{3}\log_4\dfrac{x}{y}+\log_4(x+1)^2$

$=\log_4\left[\left(\dfrac{x}{y}\right)^{1/3}(x+1)^2\right]$

$=\log_4\left[(x+1)^2\sqrt[3]{\dfrac{x}{y}}\right]$

67. $\dfrac{1}{3}[2\ln(x+5)-\ln x-\ln(x^2-4)]$

$=\dfrac{1}{3}[\ln(x+5)^2-\ln x-\ln(x^2-4)]$

$=\dfrac{1}{3}\left[\ln\dfrac{(x+5)^2}{x(x^2-4)}\right]$

$=\ln\left[\dfrac{(x+5)^2}{x(x^2-4)}\right]^{1/3}$

$=\ln\sqrt[3]{\dfrac{(x+5)^2}{x(x^2-4)}}$

68. $\dfrac{1}{3}\left[5\ln(x+6)-\ln x-\ln(x^2-25)\right]$

$=\dfrac{1}{3}\ln\left[\dfrac{(x+6)^5}{x(x^2-25)}\right]$

$=\ln\left[\dfrac{(x+6)^5}{x(x^2-25)}\right]^{\frac{1}{3}}$

69. $\log x+\log\left(x^2-1\right)-\log 7-\log(x+1)$

$=\log x+\log\left(x^2-1\right)-\left(\log 7+\log(x+1)\right)$

$=\log\left(x\left(x^2-1\right)\right)-\log\left(7(x+1)\right)$

$=\log\dfrac{x\left(x^2-1\right)}{7(x+1)}$

$=\log\dfrac{x(x+1)(x-1)}{7(x+1)}$

$=\log\dfrac{x(x-1)}{7}$

70. $\log x+\log\left(x^2-4\right)-\log 15-\log(x+2)$

$=\log x+\log\left(x^2-4\right)-\left(\log 15+\log(x+2)\right)$

$=\log\left(x\left(x^2-4\right)\right)-\log\left(15(x+2)\right)$

$=\log\dfrac{x\left(x^2-4\right)}{15(x+2)}$

$=\log\dfrac{x(x+2)(x-2)}{15(x+2)}$

$=\log\dfrac{x(x-2)}{15}$

71. $\log_5 13=\dfrac{\log 13}{\log 5}\approx 1.5937$

72. $\log_6 17 = \frac{\log 17}{\log 6} \approx 1.5812$

73. $\log_{14} 87.5 = \frac{\ln 87.5}{\ln 14} \approx 1.6944$

74. $\log_{16} 57.2 = \frac{\ln 57.2}{\ln 16} \approx 1.4595$

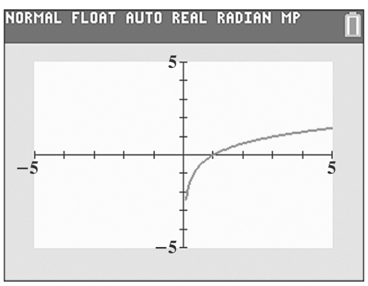
75. $\log_{0.1} 17 = \frac{\log 17}{\log 0.1} \approx -1.2304$

76. $\log_{0.3} 19 = \frac{\log 19}{\log 0.3} \approx -2.4456$

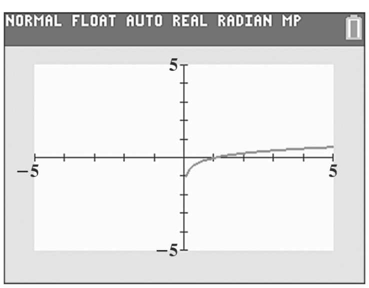
77. $\log_{\pi} 63 = \frac{\ln 63}{\ln \pi} \approx 3.6193$

78. $\log_{\pi} 400 = \frac{\ln 400}{\ln \pi} \approx 5.2340$

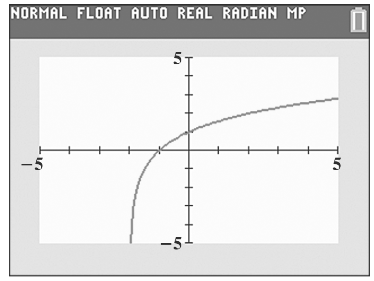
79. $y = \log_3 x = \frac{\log x}{\log 3}$



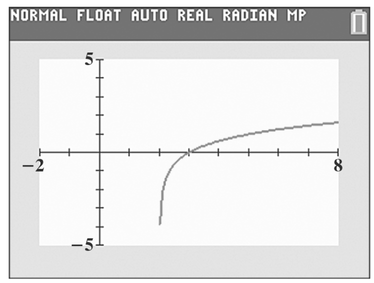
80. $y = \log_{15} x = \frac{\log x}{\log 15}$



81. $y = \log_2 (x+2) = \frac{\log(x+2)}{\log 2}$



82. $y = \log_3 (x-2) = \frac{\log(x-2)}{\log 3}$



83. $\log_b \frac{3}{2} = \log_b 3 - \log_b 2 = C - A$

84. $\log_b 6 = \log_b (2 \cdot 3)$
$= \log_b 2 + \log_b 3 = A + C$

85. $\log_b 8 = \log_b 2^3 = 3\log_b 2 = 3A$

86. $\log_b 81 = \log_b 3^4 = 4\log_b 3 = 4C$

87.
$$\begin{aligned}\log_b \sqrt{\frac{2}{27}} &= \log_b \left(\frac{2}{27}\right)^{\frac{1}{2}} \\ &= \frac{1}{2}\log_b \left(\frac{2}{3^3}\right) \\ &= \frac{1}{2}\left(\log_b 2 - \log_b 3^3\right) \\ &= \frac{1}{2}(\log_b 2 - 3\log_b 3) \\ &= \frac{1}{2}\log_b 2 - \frac{3}{2}\log_b 3 \\ &= \frac{1}{2}A - \frac{3}{2}C\end{aligned}$$

88. $\log_b \sqrt{\frac{3}{16}} = \log_b\left(\frac{\sqrt{3}}{4}\right)$
$= \log_b \sqrt{3} - \log_b 4$
$= \log_b 3^{\frac{1}{2}} - \log 2^2$
$= \frac{1}{2}\log_b 3 - 2\log 2$
$= \frac{1}{2}C - 2A$

89. false; $\ln e = 1$

90. false; $\ln e^e = 0$

91. false; $\log_4 (2x)^3 = 3\log_4 (2x)$

92. true; $\ln\left(8x^3\right) = \ln\left(2^3 x^3\right) = \ln(2x)^3 = 3\ln(2x)$

93. true; $x \log 10^x = x \cdot x = x^2$

94. false; $\ln(x \cdot 1) = \ln x + \ln 1$

95. true; $\ln(5x) + \ln 1 = \ln 5x + 0 = \ln 5x$

96. false; $\ln x + \ln(2x) = \ln(x \cdot 2x)) = \ln 2x^2$

97. false; $\log(x+3) - \log(2x) = \log\frac{x+3}{2x}$

98. false; $\log\frac{x+2}{x-1} = \log(x+2) - \log(x-1)$

99. true; quotient rule

100. true; product rule

101. true; $\log_3 7 = \frac{\log 7}{\log 3} = \frac{1}{\frac{\log 3}{\log 7}} = \frac{1}{\log_7 3}$

102. false; $e^x = \ln e^{e^x}$

103. a. $D = 10\log\frac{I}{I_0}$

b. $D_1 = 10\log\left(\frac{100I}{I_0}\right)$
$= 10\log\left(100I - I_0\right)$
$= 10\log 100 + 10\log I - 10\log I_0$
$= 10(2) + 10\log I - 10\log I_0$
$= 20 + 10\log\frac{I}{I_0}$

This is 20 more than the loudness level of the softer sound. This means that the 100 times louder sound will be 20 decibels louder.

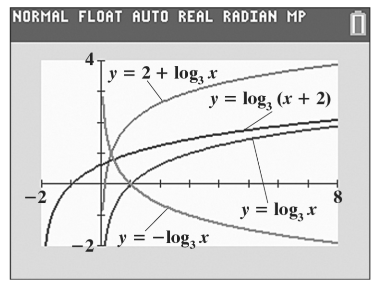
104. a. $t = \frac{1}{c}\ln\left(\frac{A}{A-N}\right)$

b. $t = \frac{1}{0.03}\left[\ln\frac{65}{65-30}\right]$
$t = \frac{1}{0.03}\ln\left(\frac{65}{35}\right)$
$t \approx 20.63$

It will take the chimpanzee a little more than 20.5 weeks to master 30 signs.

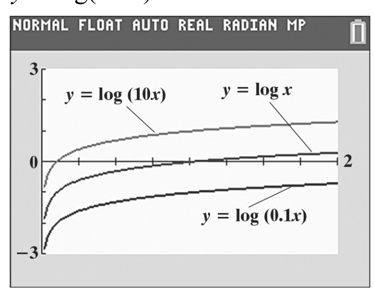
105. – 112. Answers will vary.

113. a. $y = \log_3 x = \frac{\ln x}{\ln 3}$



b. The graph of $y = 2 + \log_3 x$ is the graph of $y = \log_3 x$ shifted up two units.
The graph of $y = \log_3(x+2)$ is the graph of $y = \log_3 x$ shifted 2 units to the left.
The graph of $y = -\log_3 x$ is the graph of $y = \log_3 x$ reflected about the *x*–axis.

114. $y = \log x$
$y = \log(10x)$
$y = \log(0.1x)$



The graph of $y = \log(10x)$ is the graph of $y = \log x$ shifted up 1 unit.
The graph of $y = \log(0.1x)$ is the graph of $y = \log x$ shifted down 1 unit.

The product rule accounts for this relationship. Consider $y = \log(10x)$.

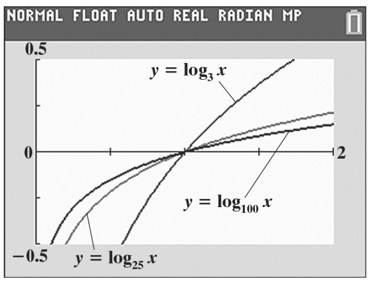
$y = \log(10x) = \log 10 + \log x = 1 + \log x$. Likewise consider $y = \log(0.1x)$.

$y = \log(0.1x) = \log 0.1 + \log x = -1 + \log x$.

115. $\log_3 x = \dfrac{\log x}{\log 3}$;

$\log_{25} x = \dfrac{\log x}{\log 25}$;

$\log_{100} x = \dfrac{\log x}{\log 100}$



a. top graph: $y = \log_{100} x$
bottom graph: $y = \log_3 x$

b. top graph: $y = \log_3 x$
bottom graph: $y = \log_{100} x$

c. If $y = \log_b x$ is graphed for two different values of *b*, the graph of the one with the larger base will be on top in the interval (0, 1) and the one with the smaller base will be on top in the interval $(1, \infty)$. Likewise, if $y = \log_b x$ is graphed for two different values of *b*, the graph of the one with the smaller base will be on the bottom in the interval (0, 1) and the one with the larger base will be on the bottom in the interval $(1, \infty)$.

116. – 120. Answers will vary.

121. makes sense

122. makes sense

123. makes sense

124. does not make sense; Explanations will vary.

Sample explanation: $\log_4 \sqrt{\dfrac{x}{y}} = \log_4 \left(\dfrac{x}{y}\right)^{\frac{1}{2}}$

$= \dfrac{1}{2}\log_4 \left(\dfrac{x}{y}\right)$

$= \dfrac{1}{2}(\log_4 x - \log_4 y)$

$= \dfrac{1}{2}\log_4 x - \dfrac{1}{2}\log_4 y$

125. true

126. false; Changes to make the statement true will vary. A sample change is:

$\dfrac{\log_7 49}{\log_7 7} = \dfrac{\log_7 49}{1} = \log_7 49 = 2$, but

$\log_7 49 - \log_7 7 = 2 - 1 = 1$.

127. false; Changes to make the statement true will vary. A sample change is: $\log_b (x^3 + y^3)$ cannot be simplified. If we were taking the logarithm of a product and not a sum, we would have been able to simplify as follows.

$\log_b (x^3 y^3) = \log_b x^3 + \log_b y^3$

$= 3\log_b x + 3\log_b y$

128. false; Changes to make the statement true will vary. A sample change is:

$\log_b (xy)^5 = 5\log_b (xy)$

$= 5(\log_b x + \log_b y)$

$= 5\log_b x + 5\log_b y$

129. $\log e = \log_{10} e = \dfrac{\ln e}{\ln 10} = \dfrac{1}{\ln 10}$

130. $\log_7 9 = \dfrac{\log 9}{\log 7} = \dfrac{\log 3^2}{\log 7} = \dfrac{2\log 3}{\log 7}$

$= \dfrac{2A}{B}$

131. $e^{\ln 8x^5 - \ln 2x^2} = e^{\ln\left(\frac{8x^5}{2x^2}\right)} = e^{\ln(4x^3)} = 4x^3$

132. $\dfrac{\log_b(x+h)-\log_b x}{h}$

$=\dfrac{\log_b \dfrac{x+h}{x}}{h}$

$=\dfrac{\log_b\left(1+\dfrac{h}{x}\right)}{h}$

$=\dfrac{1}{h}\log_b\left(1+\dfrac{h}{x}\right)$

$=\log_b\left(1+\dfrac{x}{h}\right)^{1/h}$

133. Let $\log_b M = R$ and $\log_b N = S$.

Then $\log_b M = R$ means $b^R = M$ and

$\log_b N = S$ means $b^S = N$.

Thus, $\dfrac{M}{N}=\dfrac{b^R}{b^S}$

$$\log_b \frac{M}{N} = \log_b b^{R-S}$$
$$\log_b \frac{M}{N} = R-S$$
$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

134. a. $(f\circ g)(x)=\dfrac{2}{\frac{1}{x}+1}=\dfrac{2x}{1+x}$

b. domain: $\{x \mid x\neq 0, x\neq -1\}$

or $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

135. It is not necessary to multiply out the polynomial to determine its degree. We can find the degree of the polynomial by adding the degrees of each of its factors. $f(x)=-2\underbrace{x^2}_{\text{degree 2}}\underbrace{(x-3)^2}_{\text{degree 2}}\underbrace{(x+5)}_{\text{degree 1}}$ has degree $2+2+1=5$.

$f(x)=-2x^2(x-3)^2(x+5)$ is of odd degree with a negative leading coefficient. Thus, the graph rises to the left and falls to the right.

136. $f(x)=\dfrac{4x^2}{x^2-9}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2-9}=\dfrac{4x^2}{x^2-9}=f(x)$

The y-axis symmetry.

$f(0)=\dfrac{4(0)^2}{0^2-9}=0$

The y-intercept is 0.

$4x^2=0$
$x=0$

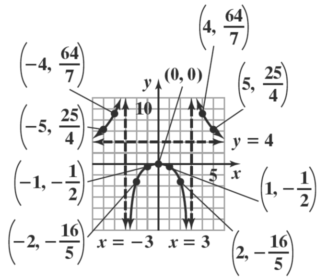
The x-intercept is 0.

vertical asymptotes:

$x^2-9=0$
$x=3, x=-3$

horizontal asymptote:

$y=\dfrac{4}{1}=4$



137.
$$a(x-2)=b(2x+3)$$
$$ax-2a=2bx+3b$$
$$ax-2bx=2a+3b$$
$$x(a-2b)=2a+3b$$
$$x=\frac{2a+3b}{a-2b}$$

138.
$$x(x-7)=3$$
$$x^2-7x=3$$
$$x^2-7x-3=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(-3)}}{2(1)}$$
$$x=\frac{7\pm\sqrt{61}}{2}$$

The solution set is $\left\{\dfrac{7\pm\sqrt{61}}{2}\right\}$.

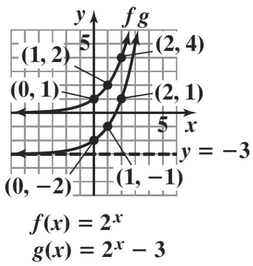
139.
$$\frac{x+2}{4x+3}=\frac{1}{x}$$
$$x(4x+3)\left(\frac{x+2}{4x+3}\right)=x(4x+3)\left(\frac{1}{x}\right)$$
$$x(x+2)=4x+3$$
$$x^2+2x=4x+3$$
$$x^2-2x-3=0$$
$$(x+1)(x-3)=0$$

$x+1=0$ or $x-3=0$
$x=-1 \qquad x=3$

The solution set is $\{-1,3\}$.

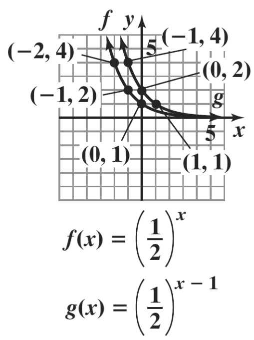
Mid-Chapter 3 Check Point

1.



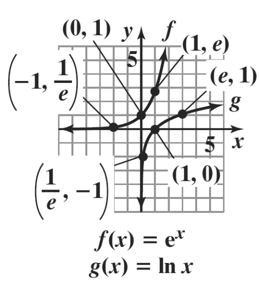
asymptote of f: $y = 0$
asymptote of g: $y = -3$
domain of f = domain of g = $(-\infty, \infty)$
range of f = $(0, \infty)$
range of g = $(-3, \infty)$

2.



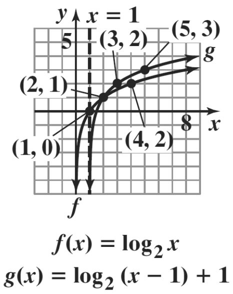
asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = range of g = $(0, \infty)$

3.



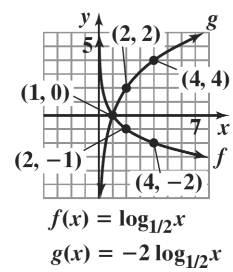
asymptote of f: $y = 0$
asymptote of g: $x = 0$
domain of f = range of g = $(-\infty, \infty)$
range of f = domain of g = $(0, \infty)$

4.



asymptote of f: $x = 0$
asymptote of g: $x = 1$
domain of f = $(0, \infty)$
domain of g = $(1, \infty)$
range of f = range of g = $(-\infty, \infty)$

5.



asymptote of f: $x = 0$
asymptote of g: $x = 0$
domain of f = domain of g = $(0, \infty)$
range of f = range of g = $(-\infty, \infty)$

6. $f(x) = \log_3(x+6)$

The argument of the logarithm must be positive:

$x + 6 > 0$
$x > -6$

domain: $\{x \mid x > -6\}$ or $(-6, \infty)$.

7. $f(x) = \log_3 x + 6$

The argument of the logarithm must be positive:

$x > 0$

domain: $\{x \mid x > 0\}$ or $(0, \infty)$.

8. $\log_3(x+6)^2$

The argument of the logarithm must be positive.
Now $(x+6)^2$ is always positive, except when $x = -6$

domain: $\{x \mid x \neq 0\}$ or $(-\infty, -6) \cup (-6, \infty)$.

9. $f(x) = 3^{x+6}$

domain: $\{x \mid x \text{ is a real number}\}$ or $(-\infty, \infty)$.

10. $\log_2 8 + \log_5 25 = \log_2 2^3 + \log_5 5^2 = 3 + 2 = 5$

11. $\log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2} = -2$

12. Let $\log_{100} 10 = y$

$$\begin{aligned} 100^y &= 10 \\ \left(10^2\right)^y &= 10^1 \\ 10^{2y} &= 10^1 \\ 2y &= 1 \\ y &= \frac{1}{2} \end{aligned}$$

13. $\log \sqrt[3]{10} = \log 10^{\frac{1}{3}} = \frac{1}{3}$

14. $\log_2 (\log_3 81) = \log_2 \left(\log_3 3^4\right) = \log_2 4 = \log_2 2^2 = 2$

15. $\log_3 \left(\log_2 \frac{1}{8}\right) = \log_3 \left(\log_2 \frac{1}{2^3}\right) = \log_3 \left(\log_2 2^{-3}\right) = \log_3 (-3)$ = not possible

This expression is impossible to evaluate because $\log_3 (-3)$ is undefined.

16. $6^{\log_6 5} = 5$

17. $\ln e^{\sqrt{7}} = \sqrt{7}$

18. $10^{\log 13} = 13$

19. $\log_{100} 0.1 = y$

$$\begin{aligned} 100^y &= 0.1 \\ \left(10^2\right)^y &= \frac{1}{10} \\ 10^{2y} &= 10^{-1} \\ 2y &= -1 \\ y &= -\frac{1}{2} \end{aligned}$$

20. $\log_\pi \pi^{\sqrt{\pi}} = \sqrt{\pi}$

21.
$$\begin{aligned} \log\left(\frac{\sqrt{xy}}{1000}\right) &= \log\left(\sqrt{xy}\right) - \log 1000 \\ &= \log(xy)^{\frac{1}{2}} - \log 10^3 \\ &= \frac{1}{2}\log(xy) - 3 \\ &= \frac{1}{2}(\log x + \log y) - 3 \\ &= \frac{1}{2}\log x + \frac{1}{2}\log y - 3 \end{aligned}$$

22. $\ln\left(e^{19}x^{20}\right) = \ln e^{19} + \ln x^{20} = 19 + 20\ln x$

23.
$$\begin{aligned} 8\log_7 x - \frac{1}{3}\log_7 y &= \log_7 x^8 - \log_7 y^{\frac{1}{3}} \\ &= \log_7 \left(\frac{x^8}{y^{\frac{1}{3}}}\right) \\ &= \log_7 \left(\frac{x^8}{\sqrt[3]{y}}\right) \end{aligned}$$

24.
$$\begin{aligned} 7\log_5 x + 2\log_5 x &= \log_5 x^7 + \log_5 x^2 \\ &= \log_5 \left(x^7 \cdot x^2\right) \\ &= \log_5 x^9 \end{aligned}$$

25.
$$\begin{aligned} &\frac{1}{2}\ln x - 3\ln y - \ln(z-2) \\ &= \ln x^{\frac{1}{2}} - \ln y^3 - \ln(z-2) \\ &= \ln\sqrt{x} - \left[\ln y^3 + \ln(z-2)\right] \\ &= \ln\sqrt{x} - \ln\left[y^3(z-2)\right] \\ &= \ln\left[\frac{\sqrt{x}}{y^3(z-2)}\right] \end{aligned}$$

26. Continuously: $A = 8000e^{0.08(3)} \approx 10{,}170$

Monthly: $A = 8000\left(1 + \frac{0.08}{12}\right)^{12\cdot 3} \approx 10{,}162$

$10{,}170 - 10{,}162 = 8$

Interest returned will be \$8 more if compounded continuously.

Section 3.4

Check Point Exercises

1. **a.**
$$\begin{aligned} 5^{3x-6} &= 125 \\ 5^{3x-6} &= 5^3 \\ 3x-6 &= 3 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$
The solution set is $\{3\}$.

b.
$$\begin{aligned} 8^{x+2} &= 4^{x-3} \\ \left(2^3\right)^{x+2} &= \left(2^2\right)^{x-3} \\ 2^{3x+6} &= 2^{2x-6} \\ 3x+6 &= 2x-6 \\ x &= -12 \end{aligned}$$
The solution set is $\{-12\}$.

2. **a.**
$$\begin{aligned} 5^x &= 134 \\ \ln 5^x &= \ln 134 \\ x\ln 5 &= \ln 134 \\ x &= \frac{\ln 134}{\ln 5} \approx 3.04 \end{aligned}$$
The solution set is $\left\{\frac{\ln 134}{\ln 5}\right\}$, approximately 3.04.

b.
$$\begin{aligned} 10^x &= 8000 \\ \log 10^x &= \log 8000 \\ x\log 10 &= \log 8000 \\ x &= \log 8000 \approx 3.90 \end{aligned}$$
The solution set is $\{\log 8000\}$, approximately 3.90.

3.
$$\begin{aligned} 7e^{2x}-5 &= 58 \\ 7e^{2x} &= 63 \\ e^{2x} &= 9 \\ \ln e^{2x} &= \ln 9 \\ 2x &= \ln 9 \\ x &= \frac{\ln 9}{2} \approx 1.10 \end{aligned}$$
The solution set is $\left\{\frac{\ln 9}{2}\right\}$, approximately 1.10.

4.
$$\begin{aligned} 3^{2x-1} &= 7^{x+1} \\ \ln 3^{2x-1} &= \ln 7^{x+1} \\ (2x-1)\ln 3 &= (x+1)\ln 7 \\ 2x\ln 3-\ln 3 &= x\ln 7+\ln 7 \\ 2x\ln 3-x\ln 7 &= \ln 3+\ln 7 \\ x(2\ln 3-\ln 7) &= \ln 3+\ln 7 \\ x &= \frac{\ln 3+\ln 7}{2\ln 3-\ln 7} \\ x &\approx 12.11 \end{aligned}$$
The solution set is $\left\{\frac{\ln 3+\ln 7}{2\ln 3-\ln 7}\right\}$, approximately 12.11.

5.
$$\begin{aligned} e^{2x}-8e^x+7 &= 0 \\ \left(e^x-7\right)\left(e^x-1\right) &= 0 \end{aligned}$$
$e^x-7=0$ or $e^x-1=0$

$e^x=7$ $\quad$ $e^x=1$

$\ln e^x=\ln 7$ $\quad$ $\ln e^x=\ln 1$

$x=\ln 7$ $\quad$ $x=0$

The solution set is $\{0, \ln 7\}$. The solutions are 0 and (approximately) 1.95.

6. **a.** $\log_2(x-4)=3$
$$\begin{aligned} 2^3 &= x-4 \\ 8 &= x-4 \\ 12 &= x \end{aligned}$$
Check:
$$\begin{aligned} \log_2(x-4) &= 3 \\ \log_2(12-4) &= 3 \\ \log_2 8 &= 3 \\ 3 &= 3 \end{aligned}$$
The solution set is $\{12\}$.

b.
$$\begin{aligned} 4\ln(3x) &= 8 \\ \ln(3x) &= 2 \\ e^{\ln(3x)} &= e^2 \\ 3x &= e^2 \\ x &= \frac{e^2}{3} \approx 2.46 \end{aligned}$$
Check
$$\begin{aligned} 4\ln(3x) &= 8 \\ 4\ln 3\left(\frac{e^2}{3}\right) &= 8 \\ 4\ln e^2 &= 8 \\ 4(2) &= 8 \\ 8 &= 8 \end{aligned}$$
The solution set is $\left\{\frac{e^2}{3}\right\}$, approximately 2.46.

7. $\log x + \log(x-3) = 1$

$$\log x(x-3) = 1$$
$$10^1 = x(x-3)$$
$$10 = x^2 - 3x$$
$$0 = x^2 - 3x - 10$$
$$0 = (x-5)(x+2)$$

$x-5=0$ or $x+2=0$

$x=5$ or $x=-2$

Check

Checking 5:

$$\log 5 + \log(5-3) = 1$$
$$\log 5 + \log 2 = 1$$
$$\log(5\cdot 2) = 1$$
$$\log 10 = 1$$
$$1 = 1$$

Checking –2:

$$\log x + \log(x-3) = 1$$
$$\log(-2) + \log(-2-3) = 1$$

Negative numbers do not have logarithms so –2 does not check.

The solution set is $\{5\}$.

8. $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

$$\ln(x-3) = \ln\frac{7x-23}{x+1}$$
$$x-3 = \frac{7x-23}{x+1}$$
$$(x-3)(x+1) = 7x-23$$
$$x^2 - 2x - 3 = 7x - 23$$
$$x^2 - 9x + 20 = 0$$
$$(x-4)(x-5) = 0$$

$x=4$ or $x=5$

Both values produce true statements.

The solution set is $\{4, 5\}$.

9. For a risk of 7%, let $R = 7$ in

$$R = 6e^{12.77x}$$
$$6e^{12.77x} = 7$$
$$e^{12.77x} = \frac{7}{6}$$
$$\ln e^{12.77x} = \ln\left(\frac{7}{6}\right)$$
$$12.77x = \ln\left(\frac{7}{6}\right)$$
$$x = \frac{\ln\left(\frac{7}{6}\right)}{12.77} \approx 0.01$$

For a blood alcohol concentration of 0.01, the risk of a car accident is 7%.

10. $A = P\left(1+\frac{r}{n}\right)^{nt}$

$$3600 = 1000\left(1+\frac{0.08}{4}\right)^{4t}$$
$$1000\left(1_-\frac{0.08}{4}\right)^{4t} = 3600$$
$$1000(1+0.02)^{4t} = 3600$$
$$1000(1.02)^{4t} = 3600$$
$$(1.02)^{4t} = \ln 3.6$$
$$4t\ln(1.02) = \ln 3.6$$
$$t = \frac{\ln 3.6}{4\ln 1.02}$$
$$\approx 16.2$$

After approximately 16.2 years, the \$1000 will grow to an accumulated value of \$3600.

11. $f(x) = 62 + 35\log(x-4)$

Solve the equation when $f(x) = 97$.

$$62 + 35\log(x-4) = 97$$
$$35\log(x-4) = 35$$
$$\log(x-4) = \frac{35}{35}$$
$$\log(x-4) = 1$$
$$x-4 = 10^1$$
$$x = 10+4$$
$$x = 14$$

At age 14, a girl will attain 97% of her adult height.

Concept and Vocabulary Check 3.4

1. $M = N$

2. $4x-1$

3. $\frac{\ln 20}{\ln 9}$

4. ln 6

5. 5^3

6. (x^2+x)

7. $\frac{7x-23}{x+1}$

8. false

9. true

10. false

11. true

Exercise Set 3.4

1. $2^x = 64$
$2^x = 2^6$
$x = 6$
The solution is 6, and the solution set is $\{6\}$.

2. $3^x = 81$
$3^x = 3^4$
$x = 4$
The solution set is $\{4\}$.

3. $5^x = 125$
$5^x = 5^3$
$x = 3$
The solution is 3, and the solution set is $\{3\}$.

4. $5^x = 625$
$5^x = 5^4$
$x = 4$
The solution set is $\{4\}$.

5. $2^{2x-1} = 32$
$2^{2x-1} = 2^5$
$2x - 1 = 5$
$2x = 6$
$x = 3$
The solution is 3, and the solution set is $\{3\}$.

6. $3^{2x+1} = 27$
$3^{2x+1} = 3^3$
$2x + 1 = 3$
$2x = 2$
$x = 1$
The solution set is $\{1\}$.

7. $4^{2x-1} = 64$
$4^{2x-1} = 4^3$
$2x - 1 = 3$
$2x = 4$
$x = 2$
The solution is 2, and the solution set is $\{2\}$.

8. $5^{3x-1} = 125$
$5^{3x-1} = 5^3$
$3x - 1 = 3$
$3x = 4$
$x = \frac{4}{3}$
The solution set is $\left\{\frac{4}{3}\right\}$.

9. $32^x = 8$
$\left(2^5\right)^x = 2^3$
$2^{5x} = 2^3$
$5x = 3$
$x = \frac{3}{5}$
The solution is $\frac{3}{5}$, and the solution set is $\left\{\frac{3}{5}\right\}$.

10. $4^x = 32$
$\left(2^2\right)^x = 2^5$
$2^{2x} = 2^5$
$2x = 5$
$x = \frac{5}{2}$
The solution set is $\left\{\frac{5}{2}\right\}$.

11. $9^x = 27$
$\left(3^2\right)^x = 3^3$
$3^{2x} = 3^3$
$2x = 3$
$x = \frac{3}{2}$
The solution is $\frac{3}{2}$, and the solution set is $\left\{\frac{3}{2}\right\}$.

12. $125^x = 625$
$\left(5^3\right)^x = 5^4$
$5^{3x} = 5^4$
$3x = 4$
$x = \frac{4}{3}$
The solution set is $\left\{\frac{4}{3}\right\}$.

13. $3^{1-x} = \frac{1}{27}$
$3^{1-x} = \frac{1}{3^3}$
$3^{1-x} = 3^{-3}$
$1 - x = -3$
$-x = -4$
$x = 4$
The solution set is $\{4\}$.

14.
$$5^{2-x}=\frac{1}{125}$$
$$5^{2-x}=\frac{1}{5^3}$$
$$5^{2-x}=5^{-3}$$
$$2-x=-3$$
$$-x=-5$$
$$x=5$$
The solution set is $\{5\}$.

15.
$$6^{\frac{x-3}{4}}=\sqrt{6}$$
$$6^{\frac{x-3}{4}}=6^{\frac{1}{2}}$$
$$\frac{x-3}{4}=\frac{1}{2}$$
$$2(x-3)=4(1)$$
$$2x-6=4$$
$$2x=10$$
$$x=5$$
The solution is 5, and the solution set is $\{5\}$.

16.
$$7^{\frac{x-2}{6}}=\sqrt{7}$$
$$7^{\frac{x-2}{6}}=7^{\frac{1}{2}}$$
$$\frac{x-2}{6}=\frac{1}{2}$$
$$2(x-2)=6(1)$$
$$2x-4=6$$
$$2x=10$$
$$x=5$$
The solution set is $\{5\}$.

17.
$$4^x=\frac{1}{\sqrt{2}}$$
$$\left(2^2\right)^x=\frac{1}{2^{\frac{1}{2}}}$$
$$2^{2x}=2^{-\frac{1}{2}}$$
$$2x=-\frac{1}{2}$$
$$x=\frac{1}{2}\left(-\frac{1}{2}\right)=-\frac{1}{4}$$
The solution is $-\frac{1}{4}$, and the solution set is $\left\{-\frac{1}{4}\right\}$.

18.
$$9^x=\frac{1}{\sqrt[3]{3}}$$
$$\left(3^2\right)^x=\frac{1}{3^{\frac{1}{3}}}$$
$$3^{2x}=3^{-\frac{1}{3}}$$
$$2x=-\frac{1}{3}$$
$$x=\frac{1}{2}\left(-\frac{1}{3}\right)=-\frac{1}{6}$$
The solution set is $\left\{-\frac{1}{6}\right\}$.

19.
$$8^{x+3}=16^{x-1}$$
$$\left(2^3\right)^{x+3}=\left(2^4\right)^{x-1}$$
$$2^{3x+9}=2^{4x-4}$$
$$3x+9=4x-4$$
$$13=x$$
The solution set is $\{13\}$.

20.
$$8^{1-x}=4^{x+2}$$
$$\left(2^3\right)^{1-x}=\left(2^2\right)^{x+2}$$
$$2^{3-3x}=2^{2x+4}$$
$$3-3x=2x+4$$
$$-5x=1$$
$$x=-\frac{1}{5}$$
The solution set is $\left\{-\frac{1}{5}\right\}$.

21.
$$e^{x+1}=\frac{1}{e}$$
$$e^{x+1}=e^{-1}$$
$$x+1=-1$$
$$x=-2$$
The solution set is $\{-2\}$.

22.
$$e^{x+4}=\frac{1}{e^{2x}}$$
$$e^{x+4}=e^{-2x}$$
$$x+4=-2x$$
$$3x=-4$$
$$x=-\frac{4}{3}$$
The solution set is $\left\{-\frac{4}{3}\right\}$.

23. $10^x = 3.91$
$\ln 10^x = \ln 3.91$
$x \ln 10 = \ln 3.91$
$x = \frac{\ln 3.91}{\ln 10} \approx 0.59$

24. $10^x = 8.07$
$\ln 10^x = \ln 8.07$
$x \ln 10 = \ln 8.07$
$x = \frac{\ln 8.07}{\ln 10} \approx 0.91$

25. $e^x = 5.7$
$\ln e^x = 5.7$
$x = \ln 5.7 \approx 1.74$

26. $e^x = 0.83$
$\ln e^x = \ln 0.83$
$x = \ln 0.83 \approx -0.19$

27. $5^x = 17$
$\ln 5^x = \ln 17$
$x \ln 5 = \ln 17$
$x = \frac{\ln 17}{\ln 5} \approx 1.76$

28. $19^x = 143$
$x \ln 19 = \ln 143$
$x = \frac{\ln 143}{\ln 19} \approx 1.69$

29. $5e^x = 23$
$e^x = \frac{23}{5}$
$\ln e^x = \ln \frac{23}{5}$
$x = \ln \frac{23}{5} \approx 1.53$

30. $9e^x = 107$
$e^x = \frac{107}{9}$
$\ln e^x = \ln \frac{107}{9}$
$x = \ln \frac{107}{9} \approx 2.48$

31. $3e^{5x} = 1977$
$e^{5x} = 659$
$\ln e^{5x} = \ln 659$
$x = \frac{\ln 659}{5} \approx 1.30$

32. $4e^{7x} = 10,273$
$e^{7x} = \frac{10,273}{4}$
$\ln e^{7x} = \ln\left(\frac{10,273}{4}\right)$
$7x = \ln\left(\frac{10,273}{4}\right)$
$x = \frac{1}{7}\ln\left(\frac{10,273}{4}\right) \approx 1.12$

33. $e^{1-5x} = 793$
$\ln e^{1-5x} = \ln 793$
$(1-5x)(\ln e) = \ln 793$
$1-5x = \ln 793$
$5x = 1 - \ln 793$
$x = \frac{1-\ln 793}{5} \approx -1.14$

34. $e^{1-8x} = 7957$
$\ln e^{1-8x} = \ln 7957$
$(1 - 8x) \ln e = \ln 7957$
$1 - 8x = \ln 7957$
$8x = 1 - \ln 7957$
$x = \frac{1-\ln 7957}{8} \approx -1.00$

35. $e^{5x-3} - 2 = 10,476$
$e^{5x-3} = 10,478$
$\ln e^{5x-3} = \ln 10,478$
$(5x-3)\ln e = \ln 10,478$
$5x - 3 = \ln 10,478$
$5x = \ln 10,478 + 3$
$x = \frac{\ln 10,478+3}{5} \approx 2.45$

36. $e^{4x-5} - 7 = 11,243$
$e^{4x-5} = 11,250$
$\ln e^{4x-5} = \ln 11,250$ $(4x - 5)\ln e = \ln 11,250$
$4x - 5 = \ln 11,250$
$x = \frac{\ln 11,250+5}{4} \approx 3.58$

37.
$$7^{x+2}=410$$
$$\ln 7^{x+2}=\ln 410$$
$$(x+2)\ln 7=\ln 410$$
$$x+2=\frac{\ln 410}{\ln 7}$$
$$x=\frac{\ln 410}{\ln 7}-2\approx 1.09$$

38.
$$5^{x-3}=137$$
$$\ln 5^{x-3}=\ln 137$$
$$(x-3)\ln 5=\ln 137$$
$$x-3=\frac{\ln 137}{\ln 5}$$
$$x=3+\frac{\ln 137}{\ln 5}\approx 6.06$$

39.
$$7^{0.3x}=813$$
$$\ln 7^{0.3x}=\ln 813$$
$$0.3x\ln 7=\ln 813$$
$$x=\frac{\ln 813}{0.3\ln 7}\approx 11.48$$

40.
$$3^{x/7}=0.2$$
$$\ln 3^{x/7}=\ln 0.2$$
$$\frac{x}{7}\ln 3=\ln 0.2$$
$$x\ln 3=7\ln 0.2$$
$$x=\frac{7\ln 0.2}{\ln 3}\approx -10.25$$

41.
$$5^{2x+3}=3^{x-1}$$
$$\ln 5^{2x+3}=\ln 3^{x-1}$$
$$(2x+3)\ln 5=(x-1)\ln 3$$
$$2x\ln 5+3\ln 5=x\ln 3-\ln 3$$
$$3\ln 5+\ln 3=x\ln 3-2x\ln 5$$
$$3\ln 5+\ln 3=x(\ln 3-2\ln 5)$$
$$\frac{3\ln 5+\ln 3}{\ln 3-2\ln 5}=x$$
$$-2.80\approx x$$

42.
$$7^{2x+1}=3^{x+2}$$
$$\ln 7^{2x+1}=\ln 3^{x+2}$$
$$(2x+1)\ln 7=(x+2)\ln 3$$
$$2x+1=(x+2)\frac{\ln 3}{\ln 7}$$
$$2x+1=x\frac{\ln 3}{\ln 7}+\frac{2\ln 3}{\ln 7}$$
$$2x-x\frac{\ln 3}{\ln 7}=\frac{2\ln 3}{\ln 7}-1$$
$$x\left(2-\frac{\ln 3}{\ln 7}\right)=\frac{2\ln 3}{\ln 7}-1$$
$$x=\frac{\frac{2\ln 3}{\ln 7}-1}{2-\frac{\ln 3}{\ln 7}}\approx 0.09$$

43.
$$e^{2x}-3e^{x}+2=0$$
$$\left(e^{x}-2\right)\left(e^{x}-1\right)=0$$

$e^{x}-2=0$ or $e^{x}-1=0$

$e^{x}=2$ | $e^{x}=1$

$\ln e^{x}=\ln 2$ | $\ln e^{x}=\ln 1$

$x=\ln 2$ | $x=0$

The solution set is {0, ln 2). The solutions are 0 and approximately 0.69.

44.
$$e^{2x}-2e^{x}-3=0$$
$$\left(e^{x}-3\right)\left(e^{x}+1\right)=0$$

$e^{x}-3=0$ or $e^{x}+1=0$

$e^{x}=3$ | $e^{x}=-1$

$\ln e^{x}=\ln 3$ | $\ln e^{x}=\ln(-1)$

$x=\ln 3$ | no solution

The solution set is {ln 3}. The solutions is approximately 1.10.

45.
$$e^{4x}+5e^{2x}-24=0$$
$$\left(e^{2x}+8\right)\left(e^{2x}-3\right)=0$$

$e^{2x}+8=0$ or $e^{2x}-3=0$

$e^{2x}=-8$ | $e^{2x}=3$

$\ln e^{2x}=\ln(-8)$ | $\ln e^{2x}=\ln 3$

$2x=\ln(-8)$ | $2x=\ln 3$

$\ln(-8)$ does not exist | $x=\frac{\ln 3}{2}$

$$x=\frac{\ln 3}{2}\approx 0.55$$

46. $e^{4x} - 3e^{2x} - 18 = 0$

$\left(e^{2x} - 6\right)\left(e^{2x} + 3\right) = 0$

$e^{2x} - 6 = 0$ or $e^{2x} + 3 = 0$

$e^{2x} = 6 \qquad e^{2x} = -3$

$\ln e^{2x} = \ln 6 \qquad \ln e^{2x} = \ln(-3)$

$2x = \ln 6 \qquad \ln(-3)$ does not exist.

$x = \frac{\ln 6}{2} \approx 0.90$

47. $3^{2x} + 3^x - 2 = 0$

$(3^x + 2)(3^x - 1) = 0$

$3^x + 2 = 0 \qquad 3^x - 1 = 0$

$3^x = -2 \qquad 3^x = 1$

$\log 3^x = \log(-2) \qquad \log 3^x = \log 1$

does not exist $\qquad \log 3 = 0$

$x = \frac{0}{\log 3}$

$x = 0$

The solution set is $\{0\}$.

48. $2^{2x} + 2^x - 12 = 0$

$(2^x + 4)(2^x - 3) = 0$

$2^x + 4 = 0 \qquad 2^x - 3 = 0$

$2^x = -4 \qquad 2^x = 3$

$\ln 2^x = \ln(-4) \qquad \ln 2^x = \ln 3$

does not exist $\qquad x \ln 2 = \ln 3$

$x = \frac{\ln 3}{\ln 2}$

$x \approx 1.58$

49. $\log_3 x = 4$

$3^4 = x$

$81 = x$

50. $\log_5 x = 3$

$5^3 = x$

$125 = x$

51. $\ln x = 2$

$e^2 = x$

$7.39 \approx x$

52. $\ln x = 3$

$e^3 = x$

$20.09 \approx x$

53. $\log_4(x + 5) = 3$

$4^3 = x + 5$

$59 = x$

54. $\log_5(x - 7) = 2$

$5^2 = x - 7$

$32 = x$

55. $\log_2(x + 25) = 4$

$2^4 = x + 25$

$16 = x + 25$

$-9 = x$

56. $\log_2(x + 50) = 5$

$2^5 = x + 50$

$32 = x + 50$

$-18 = x$

57. $\log_3(x + 4) = -3$

$3^{-3} = x + 4$

$\frac{1}{27} = x + 4$

$\frac{1}{27} - 4 = x$

$-\frac{107}{27} = x$

$-3.96 \approx x$

58. $\log_7(x + 2) = -2$

$7^{-2} = x + 2$

$\frac{1}{49} = x + 2$

$\frac{1}{49} - 2 = x$

$-\frac{97}{49} = x$

$-1.98 \approx x$

59. $\log_4(3x + 2) = 3$

$4^3 = 3x + 2$

$64 = 3x + 2$

$62 = 3x$

$\frac{62}{3} = x$

$20.67 \approx x$

60. $\log_2(4x + 1) = 5$

$2^5 = 4x + 1$

$32 = 4x + 1$

$31 = 4x$

$\frac{31}{4} = x$

$7.75 = x$

61. $5\ln(2x) = 20$
$\ln(2x) = 4$
$e^{\ln(2x)} = e^4$
$2x = e^4$
$x = \dfrac{e^4}{2} \approx 27.30$

62. $6\ln(2x) = 30$
$\ln(2x) = 5$
$e^{\ln(2x)} = e^5$
$2x = e^5$
$x = \dfrac{e^5}{2} \approx 74.21$

63. $6 + 2\ln x = 5$
$2\ln x = -1$
$\ln x = -\dfrac{1}{2}$
$e^{\ln x} = e^{-1/2}$
$x = e^{-1/2} \approx 0.61$

64. $7 + 3\ln x = 6$
$3\ln x = -1$
$\ln x = -\dfrac{1}{3}$
$e^{\ln x} = e^{-1/3}$
$x = e^{-1/3} \approx 0.72$

65. $\ln\sqrt{x+3} = 1$
$e^{\ln\sqrt{x+3}} = e^1$
$\sqrt{x+3} = e$
$x + 3 = e^2$
$x = e^2 - 3 \approx 4.39$

66. $\ln\sqrt{x+4} = 1$
$e^{\ln\sqrt{x+4}} = e^1$
$\sqrt{x+4} = e$
$x + 4 = e^2$
$x = e^2 - 4 \approx 3.39.$

67. $\log_5 x + \log_5(4x-1) = 1$
$\log_5(4x^2 - x) = 1$
$4x^2 - x = 5$
$4x^2 - x - 5 = 0$
$(4x-5)(x+1) = 0$
$x = \dfrac{5}{4}$ or $x = -1$

$x = -1$ does not check because $\log_5(-1)$ does not exist.

The solution set is $\left\{\dfrac{5}{4}\right\}$.

68. $\log_6(x+5) + \log_6 x = 2$
$\log_6 x(x+5) = 2$
$x(x+5) = 6^2$
$x^2 + 5x = 36$
$x^2 + 5x - 36 = 0$
$(x+9)(x-4) = 0$
$x = -9$ or $x = 4$
$x = -9$ does not check because $\log_6(-9+5)$ does not exist.
The solution set is $\{4\}$.

69. $\log_3(x+6) + \log_3(x+4) = 1$
$\log_3[(x+6)(x+4)] = 1$
$(x+6)(x+4) = 3^1$
$x^2 + 10x + 24 = 3$
$x^2 + 10x + 21 = 0$
$(x+7)(x+3) = 0$
$x = -7$ or $x = -3$
$x = -7$ does not check because $\log_3(-7+6)$ does not exist. The solution set is $\{-3\}$.

70. $\log_6(x+3) + \log_3(x+4) = 1$
$\log_6[(x+3)(x+4)] = 1$
$(x+3)(x+4) = 6^1$
$x^2 + 7x + 12 = 6$
$x^2 + 7x + 6 = 0$
$(x+6)(x+1) = 0$
$x = -6$ or $x = -1$
$x = -6$ does not check because $\log_6(-7+3)$ does not exist. The solution set is $\{-1\}$.

71. $\log_2(x+2) - \log_2(x-5) = 3$
$\log_2\left(\dfrac{x+2}{x-5}\right) = 3$
$\dfrac{x+2}{x-5} = 2^3$
$\dfrac{x+2}{x-5} = 8$
$x + 2 = 8(x-5)$
$x + 2 = 8x - 40$
$7x = 42$
$x = 6$

72. $\log_4(x+2)-\log_4(x-1)=1$

$$\log_4\left(\frac{x+2}{x-1}\right)=1$$
$$\frac{x+2}{x-1}=4^1$$
$$\frac{x+2}{x-1}=4$$
$$x+2=4(x-1)$$
$$x+2=4x-4$$
$$6=3x$$
$$2=x$$

73. $2\log_3(x+4)=\log_3 9+2$

$$2\log_3(x+4)=2+2$$
$$2\log_3(x+4)=4$$
$$\log_3(x+4)=2$$
$$3^2=x+4$$
$$9=x+4$$
$$5=x$$

74. $3\log_2(x-1)=5-\log_2 4$

$$3\log_2(x-1)=5-2$$
$$3\log_2(x-1)=3$$
$$\log_2(x-1)=1$$
$$2^1=x-1$$
$$3=x$$

75. $\log_2(x-6)+\log_2(x-4)-\log_2 x=2$

$$\log_2\frac{(x-6)(x-4)}{x}=2$$
$$\frac{(x-6)(x-4)}{x}=2^2$$
$$x^2-10x+24=4x$$
$$x^2-14x+24=0$$
$$(x-12)(x-2)=0$$

$x-12=0$ or $x-2=0$
$x=12$ $\quad$ $x=2$

The solution set is {12} since $\log_2(2-6)=\log_2(-4)$ is not possible.

76. $\log_2(x-3)+\log_2 x-\log_2(x+2)=2$

$$\log_2\frac{(x-3)x}{(x+2)}=2$$
$$2^2=\frac{x^2-3x}{x+2}$$
$$4(x+2)=x^2-3x$$
$$4x+8=x^2-3x$$
$$0=x^2-7x-8$$
$$0=(x+1)(x-8)$$

$x+1=0$ or $x-8=0$
$x=-1$ $\quad$ $x=8$

$\log_2(-1-3)=\log_2(-4)$ does not exist, so the solution set is {8}

77. $\log(x+4)=\log x+\log 4$

$$\log(x+4)=\log 4x$$
$$x+4=4x$$
$$4=3x$$
$$x=\frac{4}{3}$$

78. $\log(5x+1)=\log(2x+3)+\log 2$

$$\log(5x+1)=\log(4x+6)$$
$$5x+1=4x+6$$
$$x=5$$

79. $\log(3x-3)=\log(x+1)+\log 4$

$$\log(3x-3)=\log(4x+4)$$
$$3x-3=4x+4$$
$$-7=x$$

This value is rejected. The solution set is $\{\ \}$.

80. $\log(2x-1)=\log(x+3)+\log 3$

$$\log(2x-1)=\log(3x+9)$$
$$2x-1=3x+9$$
$$-10=x$$

This value is rejected. The solution set is $\{\ \}$.

81. $2\log x=\log 25$

$$\log x^2=\log 25$$
$$x^2=25$$
$$x=\pm 5$$

–5 is rejected. The solution set is {5}.

82. $3\log x=\log 125$

$$\log x^3=\log 125$$
$$x^3=125$$
$$x=5$$

83. $\log(x+4)-\log 2=\log(5x+1)$

$$\log\frac{x+4}{2}=\log(5x+1)$$
$$\frac{x+4}{2}=5x+1$$
$$x+4=10x+2$$
$$-9x=-2$$
$$x=\frac{2}{9}$$
$$x\approx 0.22$$

84. $\log(x+7)-\log 3=\log(7x+1)$

$$\log\frac{x+7}{3}=\log(7x+1)$$
$$\frac{x+7}{3}=7x+1$$
$$x+7=21x+3$$
$$-20x=-4$$
$$x=\frac{1}{5}$$
$$x\approx 0.2$$

85. $2\log x-\log 7=\log 112$

$$\log x^2-\log 7=\log 112$$
$$\log\frac{x^2}{7}=\log 112$$
$$\frac{x^2}{7}=112$$
$$x^2=784$$
$$x=\pm 28$$

–28 is rejected. The solution set is {28}.

86. $\log(x-2)+\log 5=\log 100$

$$\log(5x-10)=\log 100$$
$$5x-10=100$$
$$5x=110$$
$$x=22$$

87. $\log x+\log(x+3)=\log 10$

$$\log(x^2+3x)=\log 10$$
$$x^2+3x=10$$
$$x^2+3x-10=0$$
$$(x+5)(x-2)=0$$
$$x=-5 \text{ or } x=2$$

–5 is rejected. The solution set is {2}.

88. $\log(x+3)+\log(x-2)=\log 14$

$$\log(x^2+x-6)=\log 14$$
$$x^2+x-6=14$$
$$x^2+x-20=0$$
$$(x+5)(x-4)=0$$
$$x=-5 \text{ or } x=4$$

–5 is rejected. The solution set is {4}.

89. $\ln(x-4)+\ln(x+1)=\ln(x-8)$

$$\ln(x^2-3x-4)=\ln(x-8)$$
$$x^2-3x-4=x-8$$
$$x^2-4x+4=0$$
$$(x-2)(x-2)=0$$
$$x=2$$

2 is rejected. The solution set is { }.

90. $\log_2(x-1)-\log_2(x+3)=\log_2\left(\frac{1}{x}\right)$

$$\log_2\frac{x-1}{x+3}=\log_2\left(\frac{1}{x}\right)$$
$$\frac{x-1}{x+3}=\frac{1}{x}$$
$$x^2-x=x+3$$
$$x^2-2x-3=0$$
$$(x+1)(x-3)=0$$
$$x=-1 \text{ or } x=3$$

–1 is rejected. The solution set is {3}.

91. $\ln(x-2)-\ln(x+3)=\ln(x-1)-\ln(x+7)$

$$\ln\frac{x-2}{x+3}=\ln\frac{x-1}{x+7}$$
$$\frac{x-2}{x+3}=\frac{x-1}{x+7}$$
$$(x-2)(x+7)=(x+3)(x-1)$$
$$x^2+5x-14=x^2+2x-3$$
$$3x=11$$
$$x=\frac{11}{3}$$
$$x\approx 3.67$$

92. $\ln(x-5)-\ln(x+4)=\ln(x-1)-\ln(x+2)$

$$\ln\frac{x-5}{x+4}=\ln\frac{x-1}{x+2}$$
$$\frac{x-5}{x+4}=\frac{x-1}{x+2}$$
$$(x-5)(x+2)=(x+4)(x-1)$$
$$x^2-3x-10=x^2+3x-4$$
$$-6x=6$$
$$x=-1$$

–1 is rejected. The solution set is { }.

93. $5^{2x}\cdot 5^{4x}=125$

$$5^{2x+4x}=5^3$$
$$5^{6x}=5^3$$
$$6x=3$$
$$x=\frac{1}{2}$$

94. $3^{x+2}\cdot 3^x=81$

$$3^{(x+2)+x}=3^4$$
$$3^{2x+2}=3^4$$
$$2x+2=4$$
$$2x=2$$
$$x=1$$

95. $2|\ln x| - 6 = 0$

$$2|\ln x| = 6$$
$$|\ln x| = 3$$

$\ln x = 3$ or $\ln x = -3$

$x = e^3$ $\quad x = e^{-3}$

$x \approx 20.09$ $\quad x \approx 0.05$

96. $3|\log x| - 6 = 0$

$$3|\log x| = 6$$
$$|\log x| = 2$$

$\log x = 2$ or $\log x = -2$

$x = 10^2$ $\quad x = 10^{-2}$

$x = 100$ $\quad x = 0.01$

97. $3^{x^2} = 45$

$$\ln 3^{x^2} = \ln 45$$
$$x^2 \ln 3 = \ln 45$$
$$x^2 = \frac{\ln 45}{\ln 3}$$
$$x = \pm\sqrt{\frac{\ln 45}{\ln 3}} \approx \pm 1.86$$

98. $5^{x^2} = 50$

$$\ln 5^{x^2} = \ln 50$$
$$x^2 \ln 5 = \ln 50$$
$$x^2 = \frac{\ln 50}{\ln 5}$$
$$x = \pm\sqrt{\frac{\ln 50}{\ln 5}} \approx \pm 1.56$$

99. $\ln(2x+1) + \ln(x-3) - 2\ln x = 0$

$$\ln(2x+1) + \ln(x-3) - \ln x^2 = 0$$
$$\ln\frac{(2x+1)(x-3)}{x^2} = 0$$
$$\frac{(2x+1)(x-3)}{x^2} = e^0$$
$$\frac{2x^2 - 5x - 3}{x^2} = 1$$
$$2x^2 - 5x - 3 = x^2$$
$$x^2 - 5x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-3)}}{2(1)}$$
$$x = \frac{5 \pm \sqrt{37}}{2}$$
$$x = \frac{5 + \sqrt{37}}{2} \approx 5.54$$
$$x = \frac{5 - \sqrt{37}}{2} \approx -0.54 \text{ (rejected)}$$

The solution set is $\left\{\frac{5 + \sqrt{37}}{2}\right\}$.

100. $\ln 3 - \ln(x+5) - \ln x = 0$

$$\ln\frac{3}{x(x+5)} = 0$$
$$e^0 = \frac{3}{x(x+5)}$$
$$1 = \frac{3}{x(x+5)}$$
$$x(x+5) = 3$$
$$x^2 + 5x = 3$$
$$x^2 + 5x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} x^2 + 5x - 3 = 0$$
$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-3)}}{2(1)}$$
$$x = \frac{-5 \pm \sqrt{37}}{2}$$
$$x = \frac{-5 + \sqrt{37}}{2} \approx 0.54$$
$$x = \frac{-5 - \sqrt{37}}{2} \approx -5.54 \text{ (rejected)}$$

The solution set is $\left\{\frac{-5 + \sqrt{37}}{2}\right\}$.

101. $5^{x^2-12} = 25^{2x}$

$$5^{x^2-12} = \left(5^2\right)^{2x}$$
$$5^{x^2-12} = 5^{4x}$$
$$x^2 - 12 = 4x$$
$$x^2 - 4x - 12 = 0$$
$$(x-6)(x+2) = 0$$

Apply the zero product property:

$x - 6 = 0$ or $x + 2 = 0$

$x = 6$ $\quad x = -2$

The solutions are -2 and 6, and the solution set is $\{-2, 6\}$.

102.
$$3^{x^2-12} = 9^{2x}$$
$$3^{x^2-12} = \left(3^2\right)^{2x}$$
$$3^{x^2-12} = 3^{4x}$$
$$x^2 - 12 = 4x$$
$$x^2 - 4x - 12 = 0$$
$$(x-6)(x+2) = 0$$
Apply the zero product property:
$x - 6 = 0$ or $x + 2 = 0$
$x = 6$ $\quad x = -2$
The solutions are -2 and 6, and the solution set is $\{-2, 6\}$.

103. a. 2010 is 0 years after 2010.
$$A = 37.3e^{0.0095t}$$
$$A = 37.3e^{0.0095(0)} = 37.3$$
The population of California was 37.3 million in 2010.

b.
$$A = 37.3e^{0.0095t}$$
$$40 = 37.3e^{0.0095t}$$
$$\frac{40}{37.3} = e^{0.0095t}$$
$$\ln\frac{40}{37.3} = \ln e^{0.0095t}$$
$$0.0095t = \ln\frac{40}{37.3}$$
$$t = \frac{\ln\frac{40}{37.3}}{0.0095} \approx 7$$
The population of California will reach 40 million about 7 years after 2010, or 2017.

104. a. 2010 is 0 years after 2010.
$$A = 25.1e^{0.0187t}$$
$$A = 25.1e^{0.0187(0)} = 25.1$$
The population of Texas was 25.1 million in 2010.

b.
$$A = 25.1e^{0.0187t}$$
$$28 = 25.1e^{0.0187t}$$
$$\frac{28}{25.1} = e^{0.0187t}$$
$$\ln\frac{28}{25.1} = \ln e^{0.0187t}$$
$$0.0187t = \ln\frac{28}{25.1}$$
$$t = \frac{\ln\frac{28}{25.1}}{0.0187} \approx 6$$
The population of Texas will reach 27 million about 6 years after 2010, or 2016.

105. $f(x) = 20(0.975)^x$
$$1 = 20(0.975)^x$$
$$\frac{1}{20} = 0.975^x$$
$$\ln\frac{1}{20} = \ln 0.975^x$$
$$\ln\frac{1}{20} = x\ln 0.975$$
$$x = \frac{\ln\frac{1}{20}}{\ln 0.975}$$
$$x \approx 118$$
There is 1% of surface sunlight at 118 feet. This is represented by the point (118,1).

106. $f(x) = 20(0.975)^x$
$$3 = 20(0.975)^x$$
$$\frac{3}{20} = 0.975^x$$
$$\ln\frac{3}{20} = \ln 0.975^x$$
$$\ln\frac{3}{20} = x\ln 0.975$$
$$x = \frac{\ln\frac{3}{20}}{\ln 0.975}$$
$$x \approx 75$$
There is 3% of surface sunlight at 75 feet. This is represented by the point (75,3).

107. $20{,}000 = 12{,}500\left(1 + \frac{0.0575}{4}\right)^{4t}$
$$12{,}500(1.014375)^{4t} = 20{,}000$$
$$(1.014375)^{4t} = 1.6$$
$$\ln(1.014375)^{4t} = \ln 1.6$$
$$4t\ln(1.014375) = \ln 1.6$$
$$t = \frac{\ln 1.6}{4\ln 1.014375} \approx 8.2$$
8.2 years

108.
$$15{,}000 = 7250\left(1 + \frac{0.065}{12}\right)^{12t}$$
$$7250(1.005416667)^{12t} = 15{,}000$$
$$(1.005416667)^{12t} = \frac{60}{29}$$
$$\ln(1.005416667)^{12t} = \ln\left(\frac{60}{29}\right)$$
$$12t\ln(1.00541667) = \ln\left(\frac{60}{29}\right)$$
$$t = \frac{\ln\left(\frac{60}{29}\right)}{12\ln 1.00541667} \approx 11.2 \text{ years}$$

109. $$1400 = 1000\left(1+\frac{0.168}{360}\right)^{360\cdot t}$$
$$\frac{1400}{1000} = \left(1+\frac{0.168}{360}\right)^{360\cdot t}$$
$$\ln\left(1+\frac{0.168}{360}\right)^{360\cdot t} = \ln\left(\frac{1400}{1000}\right)$$
$$360\cdot t\ln\left(1+\frac{0.168}{360}\right) = \ln 1.4$$
$$t = \frac{\ln 1.4}{360\ln\left(1+\frac{0.168}{360}\right)}$$
$$t \approx 2.0$$

110. $$9000 = 5000\left(1+\frac{0.147}{360}\right)^{360\cdot t}$$
$$\frac{9000}{5000} = \left(1+\frac{0.147}{360}\right)^{360\cdot t}$$
$$\ln\left(1+\frac{0.147}{360}\right)^{360\cdot t} = \ln\left(\frac{9000}{5000}\right)$$
$$360\cdot t\ln\left(1+\frac{0.147}{360}\right) = \ln 1.8$$
$$t = \frac{\ln 1.8}{360\ln\left(1+\frac{0.147}{360}\right)}$$
$$t \approx 4.0$$

111. accumulated amount = 2(8000) = 16,000
$$16,000 = 8000e^{0.08t}$$
$$e^{0.08t} = 2$$
$$\ln e^{0.08t} = \ln 2$$
$$0.08t = \ln 2$$
$$t = \frac{\ln 2}{0.08}$$
$$t \approx 8.7$$
The amount would double in 8.7 years.

112. $$12,000 = 8000e^{0.203t}$$
$$e^{0.203t} = \frac{12,000}{8000}$$
$$\ln e^{0.203t} = \ln 1.5$$
$$0.203t = \ln 1.5$$
$$t = \frac{\ln 1.5}{0.203}$$
$$t \approx 2.0$$
About 2 years.

113. accumulated amount = 3(2350) = 7050
$$7050 = 2350e^{0.157t}$$
$$e^{0.157t} = \frac{7050}{2350}$$
$$\ln e^{0.157t} = \ln 3$$
$$0.157t = \ln 3$$
$$t = \frac{\ln 3}{0.157}$$
$$t \approx 7.0$$
About 7 years.

114. $$25,000 = 17,425e^{0.0425t}$$
$$e^{0.0425t} = \frac{1000}{697}$$
$$\ln e^{0.0425t} = \ln\left(\frac{1000}{697}\right)$$
$$0.0425t = \ln\left(\frac{1000}{697}\right)$$
$$t = \frac{\ln\left(\frac{1000}{697}\right)}{0.0425} \approx 8.5 \text{ years}$$

115. a. 2009 is 3 years after 2006.
$$f(x) = 1.2\ln x + 15.7$$
$$f(3) = 1.2\ln 3 + 15.7 \approx 17.0$$
According to the function, 17.0% of the of the U.S. gross domestic product went toward healthcare in 2009. This underestimates the value shown in the graph by 0.3%.

b. $$f(x) = 1.2\ln x + 15.7$$
$$18.5 = 1.2\ln x + 15.7$$
$$2.8 = 1.2\ln x$$
$$\frac{2.8}{1.2} = \frac{1.2\ln x}{1.2}$$
$$\frac{2.8}{1.2} = \ln x$$
$$x = e^{\frac{2.8}{1.2}}$$
$$x \approx 10$$
If the trend continues, 18.5% of the U.S. gross domestic product will go toward healthcare 10 years after 2006, or 2016.

116. a. 2008 is 2 years after 2006.
$$f(x) = 1.2\ln x + 15.7$$
$$f(2) = 1.2\ln 2 + 15.7 \approx 16.5$$
According to the function, 16.5% of the of the U.S. gross domestic product went toward healthcare in 2008. This overestimates the value shown in the graph by 0.3%.

b. $f(x) = 1.2\ln x + 15.7$

$$18.6 = 1.2\ln x + 15.7$$
$$2.9 = 1.2\ln x$$
$$\frac{2.9}{1.2} = \frac{1.2\ln x}{1.2}$$
$$\frac{2.9}{1.2} = \ln x$$
$$x = e^{\frac{2.9}{1.2}}$$
$$x \approx 11$$

If the trend continues, 18.6% of the U.S. gross domestic product will go toward healthcare 11 years after 2006, or 2017.

117. $P(x) = 95 - 30\log_2 x$

$$40 = 95 - 30\log_2 x$$
$$30\log_2 x = 45$$
$$\log_2 x = 1.5$$
$$x = 2^{1.5} \approx 2.8$$

Only half the students recall the important features of the lecture after 2.8 days.
This is represented by the point (2.8, 50).

118. $P(x) = 95 - 30\log_2 x$

$$0 = 95 - 30\log_2 x$$
$$30\log_2 x = 95$$
$$\log_2 x = \frac{95}{30}$$
$$2^{\frac{95}{30}} = x$$
$$9.0 \approx x$$

After 9 days, all students have forgotten the important features of the classroom lecture.
This is represented by the point (9.0, 0).

119. a. $\text{pH} = -\log x$

$$5.6 = -\log x$$
$$-5.6 = \log x$$
$$x = 10^{-5.6}$$

The hydrogen ion concentration is $10^{-5.6}$ mole per liter.

b. $\text{pH} = -\log x$

$$2.4 = -\log x$$
$$-2.4 = \log x$$
$$x = 10^{-2.4}$$

The hydrogen ion concentration is $10^{-2.4}$ mole per liter.

c. $\dfrac{10^{-2.4}}{10^{-5.6}} = 10^{-2.4-(-5.6)} = 10^{3.2}$

The concentration of the acidic rainfall in part (b) is $10^{3.2}$ times greater than the normal rainfall in part (a).

120. a. $\text{pH} = -\log x$

$$2.3 = -\log x$$
$$-2.3 = \log x$$
$$x = 10^{-2.3}$$

The hydrogen ion concentration is $10^{-2.3}$ mole per liter.

b. $\text{pH} = -\log x$

$$1 = -\log x$$
$$-1 = \log x$$
$$x = 10^{-1}$$

The hydrogen ion concentration is 10^{-1} mole per liter.

c. $\dfrac{10^{-1}}{10^{-2.3}} = 10^{-1-(-2.3)} = 10^{1.3}$

The concentration of the acidic stomach in part (b) is $10^{1.3}$ times greater than the lemon juice in part (a).

121. – 124. Answers will vary.

125. The intersection point is (2, 8).

Verify: $x = 2$

$$2^{x+1} = 8$$
$$2^{2+1} = 2$$
$$2^3 = 8$$
$$8 = 8$$

The solution set is $\{2\}$.

126. The intersection point is (1, 9).

Verify $x = 1$:

$$3^{x+1} = 9$$
$$3^{1+1} = 9$$
$$3^2 = 9$$
$$9 = 9$$

The solution set is $\{1\}$.

127. The intersection point is (4, 2).

Verify: $x = 4$

$$\log_3(4 \cdot 4 - 7) = 2$$
$$\log_3 9 = 2$$
$$2 = 2$$

The solution set is $\{4\}$.

128. The intersection point is $\left(\frac{11}{3}, 2\right)$.

Verify: $x = \dfrac{11}{3}$

$$\log_3\left(3 \cdot \frac{11}{3} - 2\right) = 2$$
$$\log_3(11\text{-}2) = 2$$
$$\log_3 9 = 2$$
$$2 = 2$$

The solution set is $\left\{\frac{11}{3}\right\}$.

129. The intersection point is (2, 1).

Verify: $x = 2$

$$\begin{aligned}\log(2+3)+\log 2 &= 1\\ \log 5+\log 2 &= 1\\ \log(5\cdot 2) &= 1\\ \log 10 &= 1\\ 1 &= 1\end{aligned}$$

The solution set is $\{2\}$.

130. The intersection point is (20, 2).

Verify $x = 20$:

$$\begin{aligned}\log(x-15)+\log x &= 2\\ \log(20-15)+\log 20 &= 2\\ \log 5+\log 20 &= 2\\ \log 100 &= 2\\ 100 &= 10^2\\ 100 &= 100\end{aligned}$$

The solution set is $\{20\}$.

131. There are 2 points of intersection, approximately (–1.391606, 0.21678798) and (1.6855579, 6.3711158).

Verify $x \approx -1.391606$

$3^x = 2x + 3$

$3^{-1.391606} \approx 2(-1.391606) + 3$

$0.2167879803 \approx 0.216788$

Verify $x \approx 1.6855579$

$3^x = 2x + 3$

$3^{1.6855579} \approx 2(1.6855579) + 3$

$6.37111582 \approx 6.371158$

The solution set is {–1.391606, 1.6855579}.

132. There are 2 points of intersection, approximately (−1.291641, 0.12507831) and (1.2793139, 7.8379416).

Verify: $x \approx -1.291641$

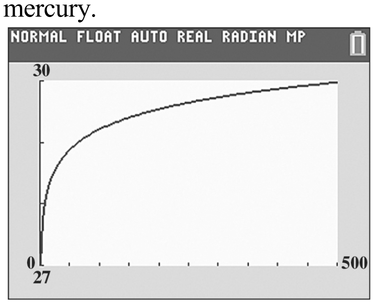
$$\begin{aligned}5^x &= 3x+4\\ 5^{-1.291641} &= 3(-1.291641)+4\\ 0.1250782178 &\approx 0.125077\end{aligned}$$

Verify: $x \approx 1.2793139$

$$\begin{aligned}5^{1.2793139} &= 3(1.2793139)+4\\ 7.837941942 &\approx 7.8379417\end{aligned}$$

The solution set is {–1.291641, 1,2793139}.

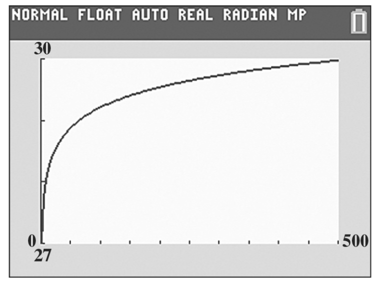
133. As the distance from the eye increases, barometric air pressure increases, leveling off at about 30 inches of mercury.



134.

$$\begin{aligned}29 &= 0.48\ln(x+1)+27\\ 0.48\ln(x+1) &= 2\\ \ln(x+1) &= \frac{1}{0.24}\\ e^{\ln(x+1)} &= e^{1/0.24}\\ x+1 &= e^{1/0.24}\\ x &= e^{1/0.24}-1 \approx 63.5\end{aligned}$$

The barometric air pressure is 29 inches of mercury at a distance of about 63.5 miles from the eye of a hurricane.



The point of intersection is approximately (63.5, 29).

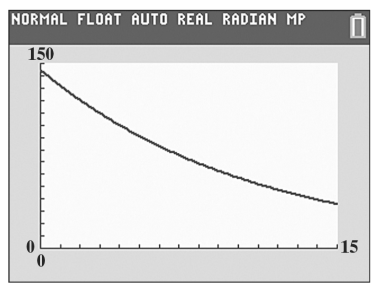
135. $P(t) = 145e^{-0.092t}$

When $P = 70$, $t \approx 7.9$, so it will take about 7.9 minutes.

Verify:

$70 = 45e^{-0.092(7.9)}$

$70 \approx 70.10076749$

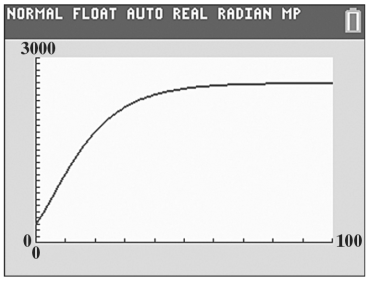


The runner's pulse will be 70 beats per minute after about 7.9 minutes.

Verifying algebraically:

$$\begin{aligned}P(7.9) &= 145e^{-0.092(7.9)}\\ &= 145e^{-07268} \approx 70\end{aligned}$$

136. $W(t) = 2600\left(1 - 0.51e^{-0.075t}\right)^3$



An adult female elephant weighing 1800 kilograms is about 20 years old.

137. does not make sense; Explanations will vary. Sample explanation: $2^x = 15$ requires logarithms. $2^x = 16$ can be solved by rewriting 16 as 2^4.

$$2^x = 15$$
$$\ln 2^x = \ln 15$$
$$x \ln 2 = \ln 15$$
$$x = \frac{\ln 15}{\ln 2}$$

$$2^x = 16$$
$$2^x = 2^4$$
$$x = 4$$

138. does not make sense; Explanations will vary. Sample explanation: The first equation is solved by rewriting it in exponential form. The second equation is solved by using the one-to-one property of logarithms.

139. makes sense

140. makes sense

141. false; Changes to make the statement true will vary. A sample change is: If $\log(x+3) = 2,$ then $10^2 = x + 3.$

142. false; Changes to make the statement true will vary. A sample change is: If $\log(7x+3) - \log(2x+5) = 4,$ then $\log\left(\frac{7x+3}{2x+5}\right) = 4,$ and $10^4 = \frac{7x+3}{2x+5}.$

143. true

144. false; Changes to make the statement true will vary. A sample change is: $x^{10} = 5.71$ is not an exponential equation, because there is not a variable in an exponent.

145. Account paying 3% interest:

$$A = 4000\left(1 + \frac{0.03}{1}\right)^{1 \cdot t}$$

Account paying 5% interest:

$$A = 2000\left(1 + \frac{0.05}{1}\right)^{1 \cdot t}$$

The two accounts will have the same balance when

$$4000(1.03)^t = 2000(1.05)^t$$
$$(1.03)^t = 0.5(1.05)^t$$
$$\left(\frac{1.03}{1.05}\right)^t = 0.5$$
$$\ln\left(\frac{1.03}{1.05}\right)^t = \ln 0.5$$
$$t \ln\left(\frac{1.03}{1.05}\right) = \ln 0.5$$
$$t = \frac{\ln 0.5}{\ln\left(\frac{1.03}{1.05}\right)} \approx 36$$

The accounts will have the same balance in about 36 years.

146.

$$(\ln x)^2 = \ln x^2$$
$$(\ln x)^2 = 2 \ln x$$
$$(\ln x)^2 - 2 \ln x = 0$$
$$\ln x(\ln x - 2) = 0$$
$$\ln x = 2 \quad \text{or} \quad \ln x = 0$$
$$e^{\ln x} = e^2 \qquad x = 1$$
$$x = e^2$$

The solution set is $\{1, e^2\}$.

147. $(\log x)(2\log x + 1) = 6$

$$2(\log x)^2 + \log x - 6 = 0$$
$$(2\log x - 3)(\log x + 2) = 0$$
$$2\log x - 3 = 0 \text{ or } \log x + 2 = 0$$
$$2\log x = 3 \qquad \log x = -2$$
$$\log x = \frac{3}{2} \qquad x = 10^{-2}$$
$$x = 10^{3/2} \qquad x = \frac{1}{100}$$
$$x = 10\sqrt{10}$$

The solution set is $\left\{\frac{1}{100}, 10\sqrt{10}\right\}$.

Check by direct substitution:

$$\text{Check: } x = 10\sqrt{10} = 10^{3/2}$$
$$(\log x)(2\log x + 1) = 6$$
$$\left(\log 10^{3/2}\right)(2\log 10^{3/2} + 1) = 6$$
$$\left(\frac{3}{2}\right)\left(2\cdot\frac{3}{2} + 1\right) = 6$$
$$\left(\frac{3}{2}\right)(3+1) = 6$$
$$\left(\frac{3}{2}\right)(4) = 6$$
$$6 = 6$$

148. $\ln(\ln x) = 0$
$$e^{\ln(\ln x)} = e^0$$
$$\ln x = 1$$
$$e^{\ln x} = e^1$$
$$x = e$$
The solution set is $\{e\}$.

149. Answers will vary.

150. a. domain: $\{x \mid -4 \le x < \infty\}$ or $[-4, \infty)$.

b. range: $\{y \mid -\infty < y \le 3\}$ or $(-\infty, 3]$.

c. The x-intercepts are –4 and 4.

d. The y-intercepts is 2.

e. f is increasing on the interval $(-4, -2)$.

f. f is decreasing on the interval $(-2, \infty)$.

g. f has a relative maximum at $x = -2$.

h. The relative maximum of f is 3.

i. $f(-3) = 2$

151. $f(x) = -4x^2 - 16x + 3$

a. $a = -4$. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{16}{-8} = -2$

$f(-2) = -4(-2)^2 - 16(-2) + 3 = 19$

The minimum point is 19 at $x = -2$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 19]$

152.

4	1	–9	26	–24
		4	–20	24
	1	–5	6	0

$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$
$$x = 2 \quad \text{or} \quad x = 3$$
The solution set is $\{2, 3, 4\}$.

153. $A = 10e^{-0.003t}$

a. 2006: $A = 10e^{-0.003(0)} = 10$ million

2007: $A = 10e^{-0.003(1)} \approx 9.97$ million

2008: $A = 10e^{-0.003(2)} \approx 9.94$ million

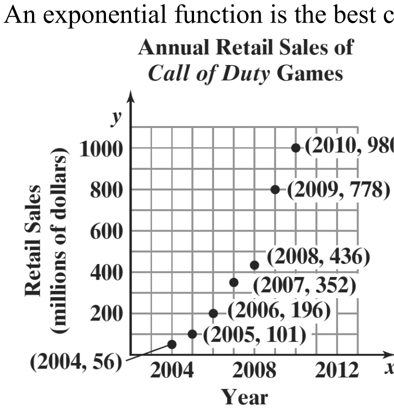
2009: $A = 10e^{-0.003(3)} \approx 9.91$ million

b. The population is decreasing.

154. a. $e^{\ln 3} = 3$

b.
$$e^{\ln 3} = 3$$
$$\left(e^{\ln 3}\right)^x = 3^x$$
$$e^{(\ln 3)x} = 3^x$$

155. An exponential function is the best choice.



Section 3.5

Check Point Exercises

1. **a.** $A_0 = 807$. Since 2011 is 11 years after 2000, when $t = 11$, $A = 1052$.

$$A = A_0 e^{kt}$$
$$1052 = 807e^{k(11)}$$
$$\frac{1052}{807} = e^{11k}$$
$$\ln\left(\frac{1052}{807}\right) = \ln e^{11k}$$
$$\ln\left(\frac{1052}{807}\right) = 11k$$
$$k = \frac{\ln\left(\frac{1052}{807}\right)}{11} \approx 0.024$$

Thus, the growth function is $A = 807e^{0.024t}$.

b.

$$A = 807e^{0.024t}$$
$$2000 = 807e^{0.024t}$$
$$\frac{2000}{807} = e^{0.024t}$$
$$\ln\left(\frac{2000}{807}\right) = \ln e^{0.024t}$$
$$\ln\left(\frac{2000}{807}\right) = 0.024t$$
$$t = \frac{\ln\left(\frac{2000}{807}\right)}{0.024} \approx 38$$

Africa's population will reach 2000 million approximately 38 years after 2000, or 2038.

2. **a.** In the exponential decay model $A = A_0 e^{kt}$, substitute $\frac{A_0}{2}$ for A since the amount present after 28 years is half the original amount.

$$\frac{A_0}{2} = A_0 e^{k \cdot 28}$$
$$e^{28k} = \frac{1}{2}$$
$$\ln e^{28k} = \ln\frac{1}{2}$$
$$28k = \ln\frac{1}{2}$$
$$k = \frac{\ln\frac{1}{2}}{28} \approx -0.0248$$

So the exponential decay model is $A = A_0 e^{-0.0248t}$

b. Substitute 60 for A_0 and 10 for A in the model from part (a) and solve for t.

$$10 = 60e^{-0.0248t}$$
$$e^{-0.0248t} = \frac{1}{6}$$
$$\ln e^{-0.0248t} = \ln\frac{1}{6}$$
$$-0.0248t = \ln\frac{1}{6}$$
$$t = \frac{\ln\frac{1}{6}}{-0.0248} \approx 72$$

The strontium-90 will decay to a level of 10 grams about 72 years after the accident.

3. **a.** The time prior to learning trials corresponds to $t = 0$.

$$f(0) = \frac{0.8}{1+e^{-0.2(0)}} = 0.4$$

The proportion of correct responses prior to learning trials was 0.4.

b. Substitute 10 for t in the model:

$$f(10) = \frac{0.8}{1+e^{-0.2(10)}} \approx 0.7$$

The proportion of correct responses after 10 learning trials was 0.7.

c. In the logistic growth model, $f(t) = \frac{c}{1+ae^{-bt}}$, the constant c represents the limiting size that $f(t)$ can attain. The limiting size of the proportion of correct responses as continued learning trials take place is 0.8.

4. **a.**

$$T = C + (T_o - C)e^{kt}$$
$$80 = 30 + (100-30)e^{k5}$$
$$80 = 30 + 70e^{5k}$$
$$50 = 70e^{5k}$$
$$\frac{5}{7} = e^{5k}$$
$$\ln\frac{5}{7} = \ln e^{5k}$$
$$\ln\frac{5}{7} = 5k$$
$$\frac{\ln\frac{5}{7}}{5} = k$$
$$-0.0673 \approx k$$
$$T = 30 + 70e^{-0.0673t}$$

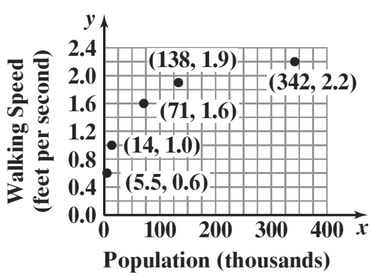
b. $T = 30 + 70e^{-0.0673(20)} \approx 48^{\circ}$
After 20 minutes, the temperature will be 48°.

c. $35 = 30 + 70e^{-0.0673t}$

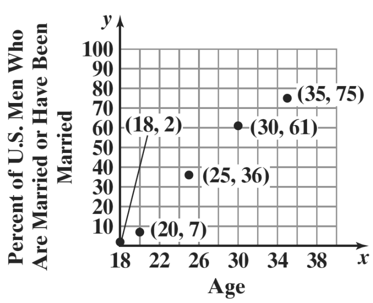
$$5 = 70e^{-0.0673t}$$
$$\frac{1}{14} = e^{-0.0673t}$$
$$\ln\frac{1}{14} = \ln e^{-0.0673t}$$
$$\ln\frac{1}{14} = -0.0673t$$
$$\frac{\ln\frac{1}{14}}{-0.0673} = t$$
$$39 \approx t$$

The temperature will reach 35° after 39 min.

5. A logarithmic function would be a good choice for modeling the data.



6. An exponential function would be a good choice for modeling the data although model choices may vary.



7. a. 1970 is 21 years after 1949.

$$f(x) = 0.074x + 2.294$$
$$f(21) = 0.074(21) + 2.294$$
$$\approx 3.8$$

$$g(x) = 2.577(1.017)^x$$
$$g(21) = 2.577(1.017)^{21}$$
$$\approx 3.7$$

The exponential function *g* serves as a better model for 1970.

b. 2050 is 101 years after 1949.

$$f(x) = 0.074x + 2.294$$
$$f(101) = 0.074(101) + 2.294$$
$$\approx 9.8$$

$$g(x) = 2.577(1.017)^x$$
$$g(101) = 2.577(1.017)^{101}$$
$$\approx 14.1$$

The linear function *f* serves as a better model for 2050.

8. $y = 4(7.8)^x$
$= 4e^{(\ln 7.8)x}$

Rounded to three decimal places:

$y = 4e^{(\ln 7.8)x}$
$= 4e^{2.054x}$

Concept and Vocabulary Check 3.5

1. > 0; < 0

2. A_0; A

3. A; c

4. logarithmic

5. exponential

6. linear

7. $\ln 5$

Exercise Set 3.5

1. Since 2010 is 0 years after 2010, find A when $t = 0$:

$$A = 127.3e^{-0.006t}$$
$$A = 127.3e^{-0.006(0)}$$
$$A = 127.3e^{0}$$
$$A = 127.3(1)$$
$$A = 127.3$$

In 2010, the population of Japan was 127.3 million.

2. Since 2010 is 0 years after 2010, find A when $t = 0$:

$$A = 31.5e^{0.019t}$$
$$A = 31.5e^{0.019(0)}$$
$$A = 31.5e^{0}$$
$$A = 31.5(1)$$
$$A = 31.5$$

In 2010, the population of Iraq was 31.5 million.

3. Since $k = 0.019$, Iraq has the greatest growth rate at 1.9% per year.

4. Since k is negative for both Japan and Russia, they have decreasing populations. The population of Japan is dropping at a rate of 0.6% per year. The population of Russia is dropping at a rate of 0.5% per year.

5. Substitute $A = 1377$ into the model for India and solve for t:

$$1377 = 1173.1e^{0.008t}$$
$$\frac{1377}{1173.1} = e^{0.008t}$$
$$\ln\frac{1377}{1173.1} = \ln e^{0.008t}$$
$$\ln\frac{1377}{1173.1} = 0.008t$$
$$t = \frac{\ln\frac{1377}{1173.1}}{0.008} \approx 20$$

The population of India will be 1377 million approximately 20 years after 2010, or 2030.

6. Substitute $A = 1491$ into the model for India and solve for t:

$$1491 = 1173.1e^{0.008t}$$
$$\frac{1491}{1173.1} = e^{0.008t}$$
$$\ln\frac{1491}{1173.1} = \ln e^{0.008t}$$
$$\ln\frac{1491}{1173.1} = 0.008t$$
$$t = \frac{\ln\frac{1491}{1173.1}}{0.008} \approx 30$$

The population of India will be 1491 million approximately 30 years after 2010, or 2040.

7. **a.** $A_0 = 6.04$. Since 2050 is 50 years after 2000, when $t = 50$, $A = 10$.

$$A = A_0e^{kt}$$
$$10 = 6.04e^{k(50)}$$
$$\frac{10}{6.04} = e^{50k}$$
$$\ln\left(\frac{10}{6.04}\right) = \ln e^{50k}$$
$$\ln\left(\frac{10}{6.04}\right) = 50k$$
$$k = \frac{\ln\left(\frac{10}{6.04}\right)}{50} \approx 0.01$$

Thus, the growth function is $A = 6.04e^{0.01t}$.

b.

$$9 = 6.04e^{0.01t}$$
$$\frac{9}{6.04} = e^{0.01t}$$
$$\ln\left(\frac{9}{6.04}\right) = \ln e^{0.01t}$$
$$\ln\left(\frac{9}{6.04}\right) = 0.01t$$
$$t = \frac{\ln\left(\frac{9}{6.04}\right)}{0.01} \approx 40$$

Now, 2000 + 40 = 2040, so the population will be 9 million is approximately the year 2040.

8. **a.** $A_0 = 3.2$. Since 2050 is 50 years after 2000, when $t = 50$, $A = 12$.

$$A = A_0e^{kt}$$
$$12 = 3.2e^{k(50)}$$
$$\frac{12}{3.2} = e^{50k}$$
$$\ln\left(\frac{12}{3.2}\right) = \ln e^{50k}$$
$$\ln\left(\frac{12}{3.2}\right) = 50k$$
$$k = \frac{\ln\left(\frac{12}{3.2}\right)}{50} \approx 0.026$$

Thus, the growth function is $A = 3.2e^{0.026t}$.

b. $3.2e^{0.026t} = 9$

$e^{0.026t} = \frac{9}{3.2}$

$\ln e^{0.026t} = \ln\left(\frac{9}{3.2}\right)$

$0.026t = \left(\frac{9}{3.2}\right)$

$t = \frac{\ln\left(\frac{9}{3.2}\right)}{0.026} \approx 40$

Now, 2000 + 40 = 2040, so the population will be 9 million is approximately the year 2040.

9. $P(x) = 99.9e^{0.0095t}$

$P(40) = 99.9e^{0.0095(40)}$

$P(40) = 99.9e^{0.0095(40)} \approx 146.1$

The population is projected to be 146.1 million in 2050.

10. $P(x) = 184.4e^{0.0149t}$

$P(40) = 184.4e^{0.0149(40)}$

$P(40) = 184.4e^{0.0149(40)} \approx 334.7$

The population is projected to be 334.7 million in 2050.

11. $P(x) = 44.2e^{kt}$

$62.9 = 44.2e^{40k}$

$\frac{62.9}{44.2} = e^{40k}$

$\ln\left(\frac{62.9}{44.2}\right) = \ln e^{40k}$

$\ln\left(\frac{62.9}{44.2}\right) = 40k$

$\frac{\ln\left(\frac{62.9}{44.2}\right)}{40} = k$

$k \approx 0.0088$

The growth rate is 0.0088.

12. $P(x) = 21.3e^{kt}$

$42.7 = 21.3e^{40k}$

$\frac{42.7}{21.3} = e^{40k}$

$\ln\left(\frac{42.7}{21.3}\right) = \ln e^{40k}$

$\ln\left(\frac{42.7}{21.3}\right) = 40k$

$\frac{\ln\left(\frac{42.7}{21.3}\right)}{40} = k$

$k \approx 0.0174$

The growth rate is 0.0174.

13. $P(x) = 82.3e^{kt}$

$70.5 = 82.3e^{40k}$

$\frac{70.5}{82.3} = e^{40k}$

$\ln\left(\frac{70.5}{82.3}\right) = \ln e^{40k}$

$\ln\left(\frac{70.5}{82.3}\right) = 40k$

$\frac{\ln\left(\frac{70.5}{82.3}\right)}{40} = k$

$k \approx -0.0039$

The growth rate is –0.0039.

14. $P(x) = 7.1e^{kt}$

$5.4 = 7.1e^{40k}$

$\frac{5.4}{7.1} = e^{40k}$

$\ln\left(\frac{5.4}{7.1}\right) = \ln e^{40k}$

$\ln\left(\frac{5.4}{7.1}\right) = 40k$

$\frac{\ln\left(\frac{5.4}{7.1}\right)}{40} = k$

$k \approx -0.0068$

The growth rate is –0.0068.

15. $A = 16e^{-0.000121t}$

$A = 16e^{-0.000121(5715)}$

$A = 16e^{-0.691515}$

$A \approx 8.01$

Approximately 8 grams of carbon-14 will be present in 5715 years.

16. $A = 16e^{-0.000121t}$
$A = 16e^{-0.000121(11430)}$
$A = 16e^{-1.38303}$
$A \approx 4.01$
Approximately 4 grams of carbon-14 will be present in 11,430 years.

17. After 10 seconds, there will be $16 \cdot \frac{1}{2} = 8$ grams present. After 20 seconds, there will be $8 \cdot \frac{1}{2} = 4$ grams present. After 30 seconds, there will be $4 \cdot \frac{1}{2} = 2$ grams present. After 40 seconds, there will be $2 \cdot \frac{1}{2} = 1$ grams present. After 50 seconds, there will be $1 \cdot \frac{1}{2} = \frac{1}{2}$ gram present.

18. After 25,000 years, there will be $16 \cdot \frac{1}{2} = 8$ grams present. After 50,000 years, there will be $8 \cdot \frac{1}{2} = 4$ grams present. After 75,000 years, there will be $4 \cdot \frac{1}{2} = 2$ grams present. After 100,000 years, there will be $2 \cdot \frac{1}{2} = 1$ gram present. After 125,000 years, there will be $1 \cdot \frac{1}{2} = \frac{1}{2}$ gram present.

19.
$$A = A_0 e^{-0.000121t}$$
$$15 = 100e^{-0.000121t}$$
$$\frac{15}{100} = e^{-0.000121t}$$
$$\ln 0.15 = \ln e^{-0.000121t}$$
$$\ln 0.15 = -0.000121t$$
$$t = \frac{\ln 0.15}{-0.000121} \approx 15,679$$
The paintings are approximately 15,679 years old.

20.
$$A = A_0 e^{-0.000121t}$$
$$88 = 100e^{-0.000121t}$$
$$\frac{88}{100} = e^{-0.000121t}$$
$$\ln 0.88 = \ln e^{-0.000121t}$$
$$\ln 0.88 = -0.000121t$$
$$t = \frac{\ln 0.88}{-0.000121} \approx 1056$$
In 1989, the skeletons were approximately 1056 years old.

21.
$$0.5 = e^{kt}$$
$$0.5 = e^{-0.055t}$$
$$\ln 0.5 = \ln e^{-0.055t}$$
$$\ln 0.5 = -0.055t$$
$$\frac{\ln 0.5}{-0.055} = t$$
$$t \approx 12.6$$
The half-life is 12.6 years.

22.
$$0.5 = e^{kt}$$
$$0.5 = e^{-0.063t}$$
$$\ln 0.5 = \ln e^{-0.063t}$$
$$\ln 0.5 = -0.063t$$
$$\frac{\ln 0.5}{-0.063} = t$$
$$t \approx 11.0$$
The half-life is 11.0 years.

23.
$$0.5 = e^{kt}$$
$$0.5 = e^{1620k}$$
$$\ln 0.5 = \ln e^{1620k}$$
$$\ln 0.5 = 1620k$$
$$\frac{\ln 0.5}{1620} = k$$
$$k \approx -0.000428$$
The decay rate is 0.0428% per year.

24.
$$0.5 = e^{kt}$$
$$0.5 = e^{4560k}$$
$$\ln 0.5 = \ln e^{4560k}$$
$$\ln 0.5 = 4560k$$
$$\frac{\ln 0.5}{4560} = k$$
$$k \approx -0.000152$$
The decay rate is 0.0152% per year.

25.
$$0.5 = e^{kt}$$
$$0.5 = e^{17.5k}$$
$$\ln 0.5 = \ln e^{17.5k}$$
$$\ln 0.5 = 17.5k$$
$$\frac{\ln 0.5}{17.5} = k$$
$$k \approx -0.039608$$
The decay rate is 3.9608% per day.

26.
$$0.5 = e^{kt}$$
$$0.5 = e^{113k}$$
$$\ln 0.5 = \ln e^{113k}$$
$$\ln 0.5 = 113k$$
$$\frac{\ln 0.5}{113} = k$$
$$k \approx -0.006134$$
The decay rate is 0.6134% per hour.

27. **a.**

$$\frac{1}{2} = 1e^{k1.31}$$
$$\ln\frac{1}{2} = \ln e^{1.31k}$$
$$\ln\frac{1}{2} = 1.31k$$
$$k = \frac{\ln\frac{1}{2}}{1.31} \approx -0.52912$$

The exponential model is given by $A = A_0 e^{-0.52912t}$.

b.

$$A = A_0 e^{-0.52912t}$$
$$0.945A_0 = A_0 e^{-0.52912t}$$
$$0.945 = e^{-0.52912t}$$
$$\ln 0.945 = \ln e^{-0.52912t}$$
$$\ln 0.945 = -0.52912t$$
$$t = \frac{\ln 0.945}{-0.52912} \approx 0.1069$$

The age of the dinosaur ones is approximately 0.1069 billion or 106,900,000 years old.

28. First find the decay equation.

$$0.5 = e^{kt}$$
$$0.5 = e^{7340k}$$
$$\ln 0.5 = \ln e^{7340k}$$
$$\ln 0.5 = 7340k$$
$$\frac{\ln 0.5}{7340} = k$$
$$k \approx -0.000094$$
$$A = e^{-0.000094t}$$

Next use the decay equation answer question.

$$A = e^{-0.000094t}$$
$$0.2 = e^{-0.000094t}$$
$$\ln 0.2 = \ln e^{-0.000094t}$$
$$\ln 0.2 = -0.000094t$$
$$\frac{\ln 0.2}{-0.000094} = t$$
$$t \approx 17121.7$$

It will take 17121.7 years.

For greater accuracy, use $k = \frac{\ln 0.5}{7340}$.

$A = e^{\frac{\ln 0.5}{7340}\cdot t}$ gives 17043.0 years

29. First find the decay equation.

$$0.5 = e^{kt}$$
$$0.5 = e^{22k}$$
$$\ln 0.5 = \ln e^{22k}$$
$$\ln 0.5 = 22k$$
$$\frac{\ln 0.5}{22} = k$$
$$k \approx -0.031507$$
$$A = e^{-0.031507t}$$

Next use the decay equation answer question.

$$A = e^{-0.031507t}$$
$$0.8 = e^{-0.031507t}$$
$$\ln 0.8 = \ln e^{-0.031507t}$$
$$\ln 0.8 = -0.031507t$$
$$\frac{\ln 0.8}{-0.031507} = t$$
$$t \approx 7.1$$

It will take 7.1 years.

30. First find the decay equation.

$$0.5 = e^{kt}$$
$$0.5 = e^{12k}$$
$$\ln 0.5 = \ln e^{12k}$$
$$\ln 0.5 = 12k$$
$$\frac{\ln 0.5}{12} = k$$
$$k \approx -0.057762$$
$$A = e^{-0.057762t}$$

Next use the decay equation answer question.

$$A = e^{-0.057762t}$$
$$0.7 = e^{-0.057762t}$$
$$\ln 0.7 = \ln e^{-0.057762t}$$
$$\ln 0.7 = -0.057762t$$
$$\frac{\ln 0.7}{-0.057762} = t$$
$$t \approx 6.2$$

It will take 6.2 hours.

31. First find the decay equation.

$$0.5 = e^{kt}$$
$$0.5 = e^{36k}$$
$$\ln 0.5 = \ln e^{36k}$$
$$\ln 0.5 = 36k$$
$$\frac{\ln 0.5}{36} = k$$
$$k \approx -0.019254$$
$$A = e^{-0.019254t}$$

Next use the decay equation answer question.

$$A = e^{-0.019254t}$$
$$0.9 = e^{-0.019254t}$$
$$\ln 0.9 = \ln e^{-0.019254t}$$
$$\ln 0.9 = -0.019254t$$
$$\frac{\ln 0.9}{-0.019254} = t$$
$$t \approx 5.5$$

It will take 5.5 hours.

32. $$A = A_0 e^{kt}$$
$$1000 = 1400e^{k5}$$
$$\frac{1000}{1400} = e^{5k}$$
$$\ln\frac{5}{7} = 5k$$
$$k = \frac{\ln\frac{5}{7}}{5} \approx -0.0673$$

The exponential model is given by $A = A_0 e^{-0.0673t}$.

$$100 = 1000e^{-0.0673t}$$
$$\frac{100}{1000} = e^{-0.0673t}$$
$$\ln\frac{1}{10} = \ln e^{-0.0673t}$$
$$\ln\frac{1}{10} = -0.0673t$$
$$t = \frac{\ln\frac{1}{10}}{-0.0673} \approx 34.2$$

The population will drop below 100 birds approximately 34 years from now. (This is 39 years from the time the population was 1400.)

33. $$2A_0 = A_0 e^{kt}$$
$$2 = e^{kt}$$
$$\ln 2 = \ln e^{kt}$$
$$\ln 2 = kt$$
$$t = \frac{\ln 2}{k}$$

The population will double in $t = \frac{\ln 2}{k}$ years.

34. $$A = A_0 e^{kt}$$
$$3A_0 = A_0 e^{kt}$$
$$3 = e^{kt}$$
$$\ln 3 = \ln e^{kt}$$
$$\ln 3 = kt$$
$$t = \frac{\ln 3}{k}$$

The population will triple in $t = \frac{\ln 3}{k}$ years.

35. $A = 4.3e^{0.01t}$

a. $k = 0.01$, so New Zealand's growth rate is 1%.

b. $$A = 4.3e^{0.01t}$$
$$2 \cdot 4.3 = 4.3e^{0.01t}$$
$$2 = e^{0.01t}$$
$$\ln 2 = \ln e^{0.01t}$$
$$\ln 2 = 0.01t$$
$$t = \frac{\ln 2}{0.01} \approx 69$$

New Zealand's population will double in approximately 69 years.

36. $A = 112.5e^{0.012t}$

a. $k = 0.012$, so Mexico's growth rate is 1.2%.

b. $$A = 112.5e^{0.012t}$$
$$2 \cdot 112.5 = 112.5e^{0.012t}$$
$$2 = e^{0.012t}$$
$$\ln 2 = \ln e^{0.012t}$$
$$\ln 2 = 0.012t$$
$$t = \frac{\ln 2}{0.012} \approx 58$$

Mexico's population will double in approximately 58 years.

37. a. When the epidemic began, $t = 0$.

$$f(0) = \frac{100,000}{1+5000e^{0}} \approx 20$$

Twenty people became ill when the epidemic began.

b. $$f(4) = \frac{100,000}{1+5,000e^{-4}} \approx 1080$$

About 1080 people were ill at the end of the fourth week.

c. In the logistic growth model,

$$f(t) = \frac{c}{1+ae^{-bt}},$$

the constant c represents the limiting size that $f(t)$ can attain. The limiting size of the population that becomes ill is 100,000 people.

38. $$f(x) = \frac{12.57}{1+4.11e^{-0.026(x)}}$$
$$f(51) = \frac{12.57}{1+4.11e^{-0.026(51)}} \approx 6.0$$

The function models the data quite well.

39. $f(x)=\dfrac{12.57}{1+4.11e^{-0.026(x)}}$

$f(61)=\dfrac{12.57}{1+4.11e^{-0.026(61)}}\approx 6.8$

The function models the data quite well.

40.

$$\begin{aligned}
f(x)&=\frac{12.57}{1+4.11e^{-0.026(x)}}\\
7&=\frac{12.57}{1+4.11e^{-0.026(x)}}\\
7\left(1+4.11e^{-0.026(x)}\right)&=12.57\\
7+28.77e^{-0.026(x)}&=12.57\\
28.77e^{-0.026(x)}&=5.57\\
e^{-0.026(x)}&=\frac{5.57}{28.77}\\
\ln e^{-0.026(x)}&=\ln\frac{5.57}{28.77}\\
-0.026x&=\ln\frac{5.57}{28.77}\\
x&=\frac{\ln\frac{5.57}{28.77}}{-0.026}\\
x&\approx 63
\end{aligned}$$

The world population will reach 7 billion 63 years after 1949, or 2012.

41.

$$\begin{aligned}
f(x)&=\frac{12.57}{1+4.11e^{-0.026(x)}}\\
8&=\frac{12.57}{1+4.11e^{-0.026(x)}}\\
8\left(1+4.11e^{-0.026(x)}\right)&=12.57\\
8+32.88e^{-0.026(x)}&=12.57\\
32.88e^{-0.026(x)}&=4.57\\
e^{-0.026(x)}&=\frac{4.57}{32.88}\\
\ln e^{-0.026(x)}&=\ln\frac{4.57}{32.88}\\
-0.026x&=\ln\frac{4.57}{32.88}\\
x&=\frac{\ln\frac{4.57}{32.88}}{-0.026}\\
x&\approx 76
\end{aligned}$$

The world population will reach 8 billion 76 years after 1949, or 2025.

42. $f(x)=\dfrac{12.57}{1+4.11e^{-0.026(x)}}$

As x increases, the exponent of e will decrease. This will make $e^{-0.026(x)}$ become very close to 0 and make the denominator become very close to 1. Thus, the limiting size of this function is 12.57 billion.

43. $P(20)=\dfrac{90}{1+271e^{-0.122(20)}}\approx 3.7$

The probability that a 20-year-old has some coronary heart disease is about 3.7%.

44. $P(80)=\dfrac{90}{1+271e^{-0.122(80)}}\approx 88.6$

The probability that an 80-year-old has some coronary heart disease is about 88.6%.

45.

$$\begin{aligned}
0.5&=\frac{0.9}{1+271e^{-0.122t}}\\
0.5\left(1+271e^{-0.122t}\right)&=0.9\\
1+271e^{-0.122t}&=1.8\\
271e^{-0.122t}&=0.8\\
e^{-0.122t}&=\frac{0.8}{271}\\
\ln e^{-0.122t}&=\ln\frac{0.8}{271}\\
-0.122t&=\ln\frac{0.8}{271}\\
t&=\frac{\ln\frac{0.8}{271}}{-0.122}\approx 48
\end{aligned}$$

The probability of some coronary heart disease is 50% at about age 48.

46.

$$\begin{aligned}
70&=\frac{90}{1+271e^{-0.122x}}\\
70(1+271e^{-0.122x})&=90\\
1+271e^{-0.122x}&=\frac{90}{70}\\
271e^{-0.122x}&=\frac{2}{7}\\
e^{-0.122x}&=\frac{2}{1897}\\
-0.122x&=\ln\frac{2}{1897}\\
x&=\frac{\ln\frac{2}{1897}}{-0.122}\\
x&\approx 56
\end{aligned}$$

The probability of some coronary heart disease is 70% at about age 56.

47. a.

$$55 = 45 + (70 - 45)e^{k10}$$
$$10 = 25e^{10k}$$
$$\frac{2}{5} = e^{10k}$$
$$\ln\frac{2}{5} = \ln e^{10k}$$
$$\ln\frac{2}{5} = 10k$$
$$\frac{\ln\frac{2}{5}}{10} = k$$
$$-0.0916 \approx k$$
$$T = 45 + 25e^{-0.0916t}$$

b. $T = 45 + 25e^{-0.0916(15)} \approx 51^{\circ}$
After 15 minutes, the temperature will be 51°.

c.

$$50 = 45 + 25e^{-0.0916t}$$
$$5 = 25e^{-0.0916t}$$
$$\frac{1}{5} = e^{-0.0916t}$$
$$\ln\frac{1}{5} = \ln e^{-0.0916t}$$
$$\ln\frac{1}{5} = -0.0916t$$
$$\frac{\ln\frac{1}{5}}{-0.0916} = t$$
$$18 \approx t$$

The temperature will reach 50° after 18 min.

48. a.

$$T = C + (T_o - C)e^{kt}$$
$$300 = 70 + (450 - 70)e^{k5}$$
$$230 = 380e^{5k}$$
$$\frac{23}{38} = e^{5k}$$
$$\ln\frac{23}{38} = \ln e^{5k}$$
$$\ln\frac{23}{38} = 5k$$
$$\frac{\ln\frac{23}{38}}{5} = k$$
$$-0.1004 \approx k$$
$$T = 70 + 380e^{-0.1004t}$$

b. $T = 70 + 380ke^{-0.1004(20)} \approx 121^{\circ}$
After 20 minutes, the temperature will be 121°.

c.

$$140 = 70 + 380e^{-0.1004t}$$
$$70 = 380e^{-0.1004t}$$
$$\frac{7}{38} = e^{-0.1004t}$$
$$\ln\frac{7}{38} = \ln e^{-0.1004t}$$
$$\ln\frac{7}{38} = -0.1004t$$
$$\frac{\ln\frac{7}{38}}{-0.1004} = t$$
$$17 \approx t$$

The temperature will reach 140° after 17 min.

49.

$$T = C + (T_o - C)e^{kt}$$
$$38 = 75 + (28 - 75)e^{k10}$$
$$-37 = -47e^{10k}$$
$$\frac{-37}{-47} = e^{10k}$$
$$\ln\frac{37}{47} = \ln e^{10k}$$
$$\ln\frac{37}{47} = 10k$$
$$\frac{\ln\frac{37}{47}}{10} = k$$
$$-0.0239 \approx k$$
$$T = 75 - 47e^{-0.0239t}$$

$$50 = 75 - 47e^{-0.0239t}$$
$$-25 = -47e^{-0.0239t}$$
$$\frac{-25}{-47} = e^{-0.0239t}$$
$$\ln\frac{25}{47} = \ln e^{-0.0239t}$$
$$\ln\frac{25}{47} = -0.0239t$$
$$\frac{\ln\frac{17}{47}}{-0.0239} = t$$
$$26 = t$$

The temperature will reach 50° after 26 min.

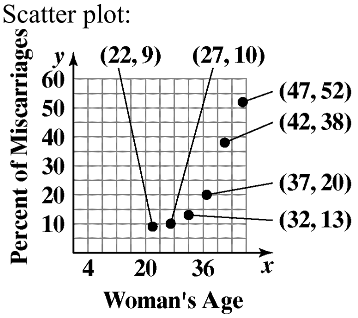
50.

$$T = C + (T_o - C)e^{kt}$$
$$30 = 65 + (24 - 65)e^{k10}$$
$$-35 = -41e^{10k}$$
$$\frac{35}{41} = e^{10k}$$
$$\ln\frac{35}{41} = \ln e^{10k}$$
$$\ln\frac{35}{41} = 10k$$
$$\frac{\ln\frac{35}{41}}{10} = k$$
$$-0.0158 \approx k$$
$$T = 65 - 41e^{-0.0158t}$$

$$45 = 65 - 41e^{-0.0158t}$$
$$-20 = -41e^{-0.0158t}$$
$$\frac{20}{41} = e^{-0.0158t}$$
$$\ln\frac{20}{41} = \ln e^{-0.0158t}$$
$$\ln\frac{20}{41} = -0.0158t$$
$$\frac{\ln\frac{20}{41}}{-0.0158} = t$$
$$45 \approx t$$

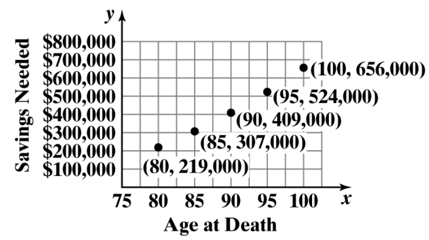
The temperature will reach 45° after 45 min.

51. **a.** Scatter plot:



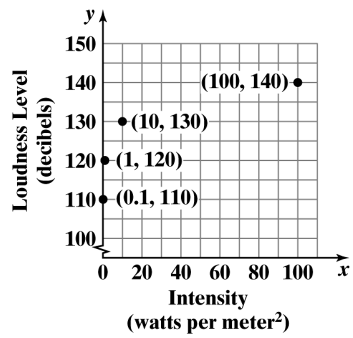
b. An exponential function appears to be the best choice for modeling the data.

52. **a.** Scatter plot:



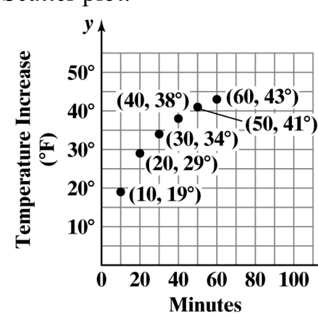
b. An exponential function appears to be the best choice for modeling the data.

53. **a.** Scatter plot:



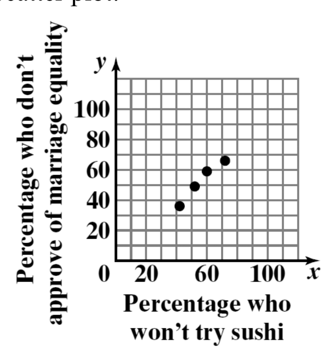
b. A logarithmic function appears to be the best choice for modeling the data.

54. **a.** Scatter plot:



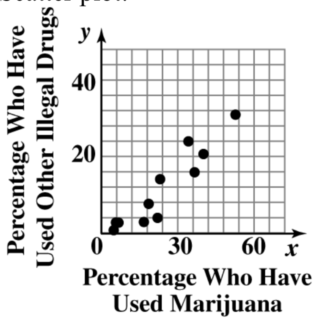
b. A logarithmic function appears to be the best choice for modeling the data.

55. **a.** Scatter plot:



b. A logarithmic function appears to be the best choice for modeling the data.

56. a. Scatter plot:



b. A linear function appears to be the best choice for modeling the data.

57. $y = 100(4.6)^x$ is equivalent to
$y = 100e^{(\ln 4.6)x}$;
Using $\ln 4.6 \approx 1.526$,
$y = 100e^{1.526x}$.

58. $y = 1000(7.3)^x$ is equivalent to
$y = 1000e^{(\ln 7.3)x}$;
Using $\ln 7.3 \approx 1.988$,
$y = 1000e^{1.988x}$.

59. $y = 2.5(0.7)^x$ is equivalent to
$y = 2.5e^{(\ln 0.7)x}$;
Using $\ln 0.7 \approx -0.357$,
$y = 2.5e^{-0.357x}$.

60. $y = 4.5(0.6)^x$ is equivalent to
$y = 4.5e^{(\ln 0.6)x}$;
Using $\ln 0.6 \approx -0.511$,
$y = 4.5e^{-0.511x}$.

61. – 69. Answers will vary.

70. a. The exponential model is $y = 201.2(1.011)^x$. Since $r \approx 0.999$ is very close to 1, the model fits the data well.

b.
$$y = 201.2(1.011)^x$$
$$y = 201.2e^{(\ln 1.011)x}$$
$$y = 201.2e^{0.0109x}$$
Since $k = .0109$, the population of the United States is increasing by about 1% each year.

71. The logarithmic model is $y = 191.9 + 24.569 \ln x$. Since $r = 0.870$ is fairly close to 1, the model fits the data okay, but not great.

72. The linear model is $y = 2.657x + 197.923$. Since $r \approx 0.997$ is close to 1, the model fits the data well.

73. The power regression model is $y = 195.05x^{0.100}$. Since $r = 0.896$, the model fits the data fairly well.

74. Using r, the model of best fit is the exponential model $y = 201.2(1.011)^x$.
The model of second best fit is the linear model $y = 2.657x + 197.923$.
Using the exponential model:
$$335 = 201.2(1.011)^x$$
$$\frac{335}{201.2} = (1.011)^x$$
$$\ln\left(\frac{335}{201.2}\right) = \ln(1.011)^x$$
$$\ln\left(\frac{335}{201.2}\right) = x\ln(1.011)$$
$$x = \frac{\ln\left(\frac{335}{201.2}\right)}{\ln(1.011)} \approx 47$$
$1969 + 47 = 2016$
Using the linear model:
$$y = 2.657x + 197.923$$
$$335 = 2.657x + 197.923$$
$$137.077 = 2.657x$$
$$x = \frac{137.077}{2.657} \approx 52$$
$1969 + 52 = 2021$

According to the exponential model, the U.S. population will reach 335 million around the year 2016. According to the linear model, the U.S. population will reach 335 million around the year 2021. Both results are reasonably close to the result found in Example 1 (2020).
Explanations will vary.

75. a. Exponential Regression:
$y = 3.46(1.02)^x$; $r \approx 0.994$
Logarithmic Regression:
$y = 14.752 \ln x - 26.512$; $r \approx 0.673$
Linear Regression:
$y = 0.557x - 10.972$; $r \approx 0.947$
The exponential model has an r value closer to 1. Thus, the better model is $y = 3.46(1.02)^x$.

b. $y = 3.46(1.02)^x$
$y = 3.46e^{(\ln 1.02)x}$
$y = 3.46e^{0.02x}$
The 65-and-over population is increasing by approximately 2% each year.

76. Models and predictions will vary. Sample models are provided
Exercise 47: $y = 1.402(1.078)^x$
Exercise 48: $y = 2896.7(1.056)^x$
Exercise 49: $y = 120 + 4.343\ln x$
Exercise 50: $y = -11.629 + 13.424\ln x$
Exercise 51: $y = 0.063x - 124.16$
Exercise 52: $y = 0.742x - 1449.669$

77. does not make sense; Explanations will vary. Sample explanation: Since the car's value is decreasing (depreciating), the growth rate is negative.

78. does not make sense; Explanations will vary. Sample explanation: This is not necessarily so. Growth rate measures how fast a population is growing relative to that population. It does not indicate how the size of a population compares to the size of another population.

79. makes sense

80. makes sense

81. true

82. true

83. true

84. true

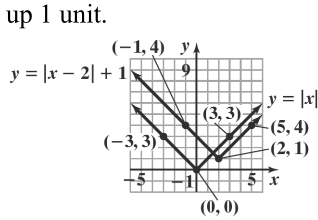
85. Use data to find k.
$$827 = 70 + (85.6 - 70)e^{k30}$$
$$12.7 = 15.6e^{30k}$$
$$\frac{12.7}{15.6} = e^{30k}$$
$$\ln\frac{12.7}{15.6} = \ln e^{30k}$$
$$\ln\frac{12.7}{15.6} = 30k$$
$$\frac{\ln\frac{12.7}{15.6}}{30} = k$$
$$-0.0069 \approx k$$

Use k to write equation.
$$85.6 = 70 + (98.6 - 70)e^{-0.0069t}$$
$$15.6 = 28.6e^{-0.0069t}$$
$$\frac{15.6}{28.6} = e^{-0.0069t}$$
$$\ln\frac{15.6}{28.6} = \ln e^{-0.0069t}$$
$$\ln\frac{15.6}{28.6} = -0.0069$$
$$\frac{\ln\frac{15.6}{28.6}}{-0.0069} = t$$
$$88 \approx t$$
The death occurred at 88 minutes before 9:30, or 8:02 am.

86. Answers will vary.

87. Let x = the computer's price before the reduction.
$$x - 0.60x = 440$$
$$0.40x = 440$$
$$x = \frac{440}{0.40}$$
$$x = 1100$$
Before the reduction the computer's price was \$1100.

88. The graph of $y = |x|$ is shifted 2 unit right and shifted up 1 unit.



89. $3x - y + 5 = 0$
$$-y = -3x - 5$$
$$y = 3x + 5$$
The slope of the line $y = 3x + 5$ is 3.
$y - y_1 = m(x - x_1)$
$y + 4 = 3(x - 1)$ point-slope
$y + 4 = 3x - 3$
$y = 3x - 7$ slope-intercept

90.
$$\frac{5\pi}{4} = 2\pi x$$
$$\frac{5\pi}{4\cdot 2\pi} = \frac{2\pi x}{2\pi}$$
$$\frac{5}{8} = x$$

The solution set is $\left\{\frac{5}{8}\right\}$.

91.
$$\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{17\pi - 12\pi}{6} = \frac{5\pi}{6}$$

92.
$$-\frac{\pi}{12} + 2\pi = -\frac{\pi}{12} + \frac{24\pi}{12} = \frac{-\pi + 24\pi}{12} = \frac{23\pi}{12}$$

Chapter 3 Review Exercises

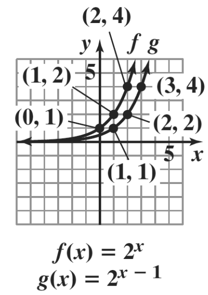
1. This is the graph of $f(x) = 4^x$ reflected about the y-axis, so the function is $g(x) = 4^{-x}$.

2. This is the graph of $f(x) = 4^x$ reflected about the x-axis and about the y-axis, so the function is $h(x) = -4^{-x}$.

3. This is the graph of $f(x) = 4^x$ reflected about the x-axis and about the y-axis then shifted upward 3 units, so the function is $r(x) = -4^{-x} + 3$.

4. This is the graph of $f(x) = 4^x$.

5. The graph of $g(x)$ shifts the graph of $f(x)$ one unit to the right.



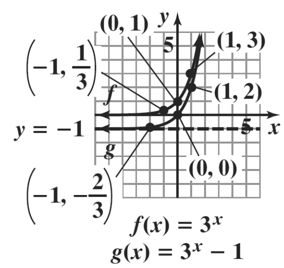
asymptote of f: $y = 0$

asymptote of g: $y = 0$

domain of f = domain of g = $(-\infty, \infty)$

range of f = range of g = $(0, \infty)$

6. The graph of $g(x)$ shifts the graph of $f(x)$ one unit down.



asymptote of f: $y = 0$

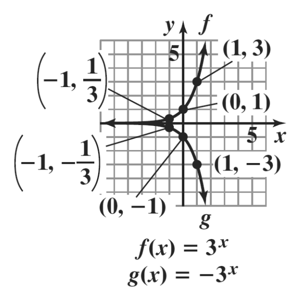
asymptote of g: $y = -1$

domain of f = domain of g = $(-\infty, \infty)$

range of f = $(0, \infty)$

range of g = $(-1, \infty)$

7. The graph of $g(x)$ reflects the graph of $f(x)$ about the y – axis.



asymptote of f: $y = 0$

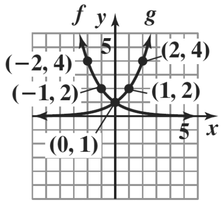
asymptote of g: $y = 0$

domain of f = domain of g = $(-\infty, \infty)$

range of f = $(0, \infty)$

range of g = $(-\infty, 0)$

8. The graph of $g(x)$ reflects the graph of $f(x)$ about the x-axis.



$f(x) = \left(\frac{1}{2}\right)^x$

$g(x) = \left(\frac{1}{2}\right)^{-x}$

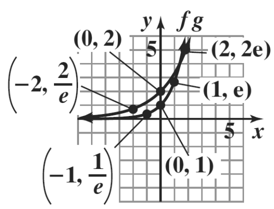
asymptote of f: $y = 0$

asymptote of g: $y = 0$

domain of f = domain of $g = (-\infty, \infty)$

range of f = range of $g = (0, \infty)$

9. The graph of $g(x)$ vertically stretches the graph of $f(x)$ by a factor of 2.



$f(x) = e^x$
$g(x) = 2e^{x/2}$

asymptote of f: $y = 0$

asymptote of g: $y = 0$

domain of f = domain of $g = (-\infty, \infty)$

range of f = range of $g = (0, \infty)$

10. 5.5% compounded semiannually:

$$A = 5000\left(1 + \frac{0.055}{2}\right)^{2\cdot 5} \approx 6558.26$$

5.25% compounded monthly:

$$A = 5000\left(1 + \frac{0.0525}{12}\right)^{12\cdot 5} \approx 6497.16$$

5.5% compounded semiannually yields the greater return.

11. 7% compounded monthly:

$$A = 14,000\left(1 + \frac{0.07}{12}\right)^{12\cdot 10} \approx 28,135.26$$

6.85% compounded continuously:

$$A = 14,000e^{0.0685(10)} \approx 27,772.81$$

7% compounded monthly yields the greater return.

12. **a.** When first taken out of the microwave, the temperature of the coffee was 200°.

b. After 20 minutes, the temperature of the coffee was about 120°.

$T = 70 + 130e^{-0.04855(20)} \approx 119.23$

Using a calculator, the temperature is about 119°.

c. The coffee will cool to about 70°; The temperature of the room is 70°.

13. $49^{1/2} = 7$

14. $4^3 = x$

15. $3^y = 81$

16. $\log_6 216 = 3$

17. $\log_b 625 = 4$

18. $\log_{13} 874 = y$

19. $\log_4 64 = 3$ because $4^3 = 64$.

20. $\log_5 \frac{1}{25} = -2$ because $5^{-2} = \frac{1}{25}$.

21. $\log_3 (-9)$ is undefined and cannot be evaluated since $\log_b x$ is defined only for $x > 0$.

22. $\log_{16} 4 = \frac{1}{2}$ because $16^{1/2} = \sqrt{16} = 4$.

23. Because $\log_b b = 1$, we conclude $\log_{17} 17 = 1$.

24. Because $\log_b b^x = x$, we conclude $\log_3 3^8 = 8$.

25. Because $\ln e^x = x$, we conclude $\ln e^5 = 5$.

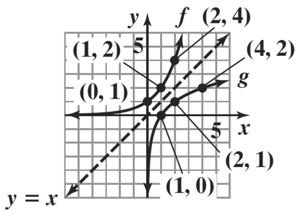
26. $\log_3 \frac{1}{\sqrt{3}} = \log_3 \frac{1}{3^{\frac{1}{2}}} = \log_3 3^{-\frac{1}{2}} = -\frac{1}{2}$

27. $\ln \frac{1}{e^2} = \ln e^{-2} = -2$

28. $\log\frac{1}{1000} = \log\frac{1}{10^3} = \log 10^{-3} = -3$

29. Because $\log_b = 1$,
we conclude $\log_8 8 = 1$.
So, $\log_3(\log_8 8) = \log_3 1$.
Because $\log_b 1 = 0$
we conclude $\log_3 1 = 0$.
Therefore, $\log_3(\log_8 8) = 0$.

30.

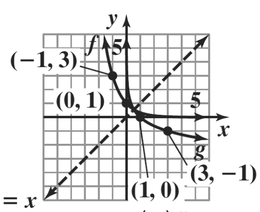


$f(x) = 2^x$
$g(x) = \log_2 x$

domain of f = range of g = $(-\infty, \infty)$

range of f = domain of g = $(0, \infty)$

31.



$f(x) = \left(\frac{1}{3}\right)^x$

$g(x) = \log_{1/3} x$

domain of f = range of g = $(-\infty, \infty)$

range of f = domain of g = $(0, \infty)$

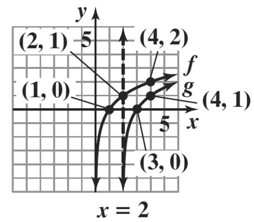
32. This is the graph of $f(x) = \log x$ reflected about the y-axis, so the function is $g(x) = \log(-x)$.

33. This is the graph of $f(x) = \log x$ shifted left 2 units, reflected about the y-axis, then shifted upward one unit, so the function is $r(x) = 1 + \log(2 - x)$.

34. This is the graph of $f(x) = \log x$ shifted left 2 units then reflected about the y-axis, so the function is $h(x) = \log(2 - x)$.

35. This is the graph of $f(x) = \log x$.

36.

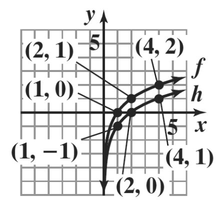


$f(x) = \log_2 x$
$g(x) = \log_2(x - 2)$

x-intercept: (3, 0)
vertical asymptote: $x = 2$
domain: $(2, \infty)$

range: $(-\infty, \infty)$

37.

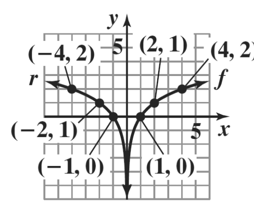


$f(x) = \log_2 x$
$h(x) = -1 + \log_2 x$

x-intercept: (2, 0)
vertical asymptote: $x = 0$
domain: $(0, \infty)$

range: $(-\infty, \infty)$

38.

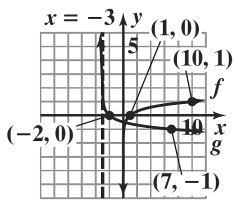


$f(x) = \log_2 x$
$r(x) = \log_2(-x)$

x-intercept: (–1, 0)
vertical asymptote: $x = 0$
domain: $(-\infty, 0)$

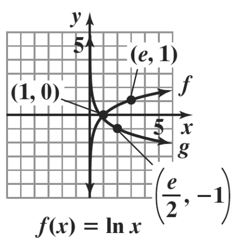
range: $(-\infty, \infty)$

39.



asymptote of f: $x = 0$
asymptote of g: $x = -3$
domain of f = $(0, \infty)$
domain of g = $(-3, \infty)$
range of f = range of g = $(-\infty, \infty)$

40.



asymptote of f: $x = 0$
asymptote of g: $x = 0$
domain of f = domain of g = $(0, \infty)$
range of f = range of g = $(-\infty, \infty)$

41. The domain of *f* consists of all *x* for which $x + 5 > 0$.
Solving this inequality for *x*, we obtain $x > -5$.
Thus the domain of *f* is $(-5, \infty)$

42. The domain of *f* consists of all *x* for which $3 - x > 0$.
Solving this inequality for *x*, we obtain $x < 3$.
Thus, the domain of *f* is $(-\infty, 3)$.

43. The domain of *f* consists of all *x* for which
$(x-1)^2 > 0$.
Solving this inequality for *x*, we obtain
$x < 1$ or $x > 1$.
Thus, the domain of *f* is $(-\infty, 1) \cup (1, \infty)$.

44. Because $\ln e^x = x$, we conclude $\ln e^{6x} = 6x$.

45. Because $e^{\ln x} = x$, we conclude $e^{\ln \sqrt{x}} = \sqrt{x}$.

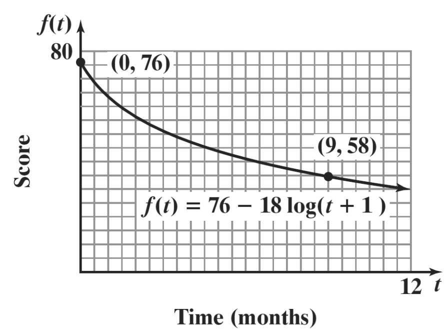
46. Because $10^{\log x} = x$, we conclude $10^{\log 4x^2} = 4x^2$.

47. $R = \log \dfrac{1000 I_0}{I_0} = \log 1000 = 3$
The Richter scale magnitude is 3.0.

48. **a.** $f(0) = 76 - 18\log(0+1) = 76$
When first given, the average score was 76.

b.
$f(2) = 76 - 18\log(2+1) \approx 67$
$f(4) = 76 - 18\log(4+1) \approx 63$
$f(6) = 76 - 18\log(6+1) \approx 61$
$f(8) = 76 - 18\log(8+1) \approx 59$
$f(12) = 76 - 18\log(12+1) \approx 56$
After 2, 4, 6, 8, and 12 months, the average scores are about 67, 63, 61, 59, and 56, respectively.

c.



Retention decreases as time passes.

49. $t = \dfrac{1}{0.06} \ln\left(\dfrac{12}{12-5}\right) \approx 8.98$
It will take about 9 weeks.

50. $\log_6 (36x^3)$
$= \log_6 36 + \log_6 x^3$
$= \log_6 36 + 3\log_6 x$
$= 2 + 3\log_6 x$

51. $\log_4 \dfrac{\sqrt{x}}{64} = \log_4 x^{1/2} - \log_4 64$
$= \dfrac{1}{2}\log_4 x - 3$

52. $\log_2 \dfrac{xy^2}{64} = \log_2 xy^2 - \log_2 64$
$= \log_2 x + \log_2 y^2 - \log_2 64$
$= \log_2 x + 2\log_2 y - 6$

53. $\ln\sqrt[3]{\frac{x}{e}}$

$= \ln\left(\frac{x}{e}\right)^{1/3}$

$= \frac{1}{3}[\ln x - \ln e]$

$= \frac{1}{3}\ln x - \frac{1}{3}\ln e$

$= \frac{1}{3}\ln x - \frac{1}{3}$

54. $\log_b 7 + \log_b 3$

$= \log_b(7 \cdot 3)$

$= \log_b 21$

55. $\log 3 - 3\log x$

$= \log 3 - \log x^3$

$= \log\frac{3}{x^3}$

56. $3\ln x + 4\ln y$

$= \ln x^3 + \ln y^4$

$= \ln\left(x^3 y^4\right)$

57. $\frac{1}{2}\ln x - \ln y$

$= \ln x^{1/2} - \ln y$

$= \ln\frac{\sqrt{x}}{y}$

58. $\log_6 72,348 = \frac{\log 72,348}{\log 6} \approx 6.2448$

59. $\log_4 0.863 = \frac{\ln 0.863}{\ln 4} \approx -0.1063$

60. true; $(\ln x)(\ln 1) = (\ln x)(0) = 0$

61. false; $\log(x+9) - \log(x+1) = \log\frac{(x+9)}{(x+1)}$

62. false; $\log_2 x^4 = 4\log_2 x$

63. true; $\ln e^x = x\ln e$

64. $2^{4x-2} = 64$

$2^{4x-2} = 2^6$

$4x - 2 = 6$

$4x = 8$

$x = 2$

65. $125^x = 25$

$\left(5^3\right)^x = 5^2$

$5^{3x} = 5^2$

$3x = 2$

$x = \frac{2}{3}$

66. $10^x = 7000$

$\log 10^x = \log 7000$

$x\log 10 = \log 7000$

$x = \log 7000$

$x \approx 3.85$

67. $9^{x+2} = 27^{-x}$

$\left(3^2\right)^{x+2} = \left(3^3\right)^{-x}$

$3^{2x+4} = 3^{-3x}$

$2x + 4 = -3x$

$5x = -4$

$x = -\frac{4}{5}$

68. $8^x = 12,143$

$\ln 8^x = \ln 12,143$

$x\ln 8 = \ln 12,143$

$x = \frac{\ln 12,143}{\ln 8} \approx 4.52$

69. $9e^{5x} = 1269$

$e^{5x} = 141$

$\ln e^{5x} = \ln 141$

$5x = \ln 141$

$x = \frac{\ln 141}{5}$

$x = \frac{1}{5}\ln 141 \approx 0.99$

70. $e^{12-5x} - 7 = 123$

$e^{12-5x} = 130$

$\ln e^{12-5x} = \ln 130$

$12 - 5x = \ln 130$

$5x = 12 - \ln 130$

$x = \frac{12 - \ln 130}{5} \approx 1.43$

71. $5^{4x+2} = 37,500$

$\ln 5^{4x+2} = \ln 37,500$

$(4x+2)\ln 5 = \ln 37,500$

$4x\ln 5 + 2\ln 5 = \ln 37,500$

$4x\ln 5 = \ln 37,500 - 2\ln 5$

$x = \dfrac{\ln 37,500 - 2\ln 5}{4\ln 5} \approx 1.14$

72. $3^{x+4} = 7^{2x-1}$

$\ln 3^{x+4} = \ln 7^{2x-1}$

$(x+4)\ln 3 = (2x-1)\ln 7$

$x\ln 3 + 4\ln 3 = 2x\ln 7 - \ln 7$

$x\ln 3 - 2x\ln 7 = -4\ln 3 - \ln 7$

$x(\ln 3 - 2\ln 7) = -4\ln 3 - \ln 7$

$x = \dfrac{-4\ln 3 - \ln 7}{\ln 3 - 2\ln 7}$

$x = \dfrac{4\ln 3 + \ln 7}{2\ln 7 - \ln 3}$

$x \approx 2.27$

73. $e^{2x} - e^x - 6 = 0$

$(e^x - 3)(e^x + 2) = 0$

$e^x - 3 = 0$ or $e^x + 2 = 0$

$e^x = 3$ $\quad e^x = -2$

$\ln e^x = \ln 3$ $\quad \ln e^x - \ln(-2)$

$x = \ln 3$ $\quad x = \ln(-2)$

$x = \ln 3 \approx 1.099$ $\quad \ln(-2)$ does not exist.

The solution set is $\{\ln 3\}$, approximately 1.10.

74. $\log_4(3x-5) = 3$

$3x - 5 = 4^3$

$3x - 5 = 64$

$3x = 69$

$x = 23$

The solutions set is {23}.

75. $3 + 4\ln(2x) = 15$

$4\ln(2x) = 12$

$\ln(2x) = 3$

$2x = e^3$

$x = \dfrac{e^3}{2}$

$x \approx 10.04$

The solutions set is $\left\{\dfrac{e^3}{2}\right\}$.

76. $\log_2(x+3) + \log_2(x-3) = 4$

$\log_2(x+3)(x-3) = 4$

$\log_2(x^2 - 9) = 4$

$x^2 - 9 = 2^4$

$x^2 - 9 = 16$

$x^2 = 25$

$x = \pm 5$

$x = -5$ does not check because $\log_2(-5+3)$ does not exist.

The solution set is {5}.

77. $\log_3(x-1) - \log_3(x+2) = 2$

$\log_3 \dfrac{x-1}{x+2} = 2$

$\dfrac{x-1}{x-2} = 3^2$

$\dfrac{x-1}{x+2} = 9$

$x - 1 = 9(x + 2)$

$x - 1 = 9x + 18$

$8x = -19$

$x = -\dfrac{19}{8}$

$x = -\dfrac{19}{8}$ does not check because $\log_3\left(-\dfrac{19}{8} - 1\right)$ does not exist.

The solution set is $\varnothing$.

78. $\ln(x+4) - \ln(x+1) = \ln x$

$\ln \dfrac{x+4}{x+1} = \ln x$

$\dfrac{x+4}{x+1} = x$

$x(x+1) = x + 4$

$x^2 + x = x + 4$

$x^2 = 4$

$x = \pm 2$

$x = -2$ does not check and must be rejected.

The solution set is {2}.

79. $\log_4(2x+1) = \log_4(x-3) + \log_4(x+5)$

$\log_4(2x+1) = \log_4(x-3) + \log_4(x+5)$

$\log_4(2x+1) = \log_4(x^2 + 2x - 15)$

$2x + 1 = x^2 + 2x - 15$

$16 = x^2$

$x^2 = 16$

$x = \pm 4$

$x = -4$ does not check and must be rejected.

The solution set is {4}.

80. $P(x) = 14.7e^{-0.21x}$

$$4.6 = 14.7e^{-0.21x}$$
$$\frac{4.6}{14.7} = e^{-0.21x}$$
$$\ln\frac{4.6}{14.7} = \ln e^{-0.21x}$$
$$\ln\frac{4.6}{14.7} = -0.21x$$
$$t = \frac{\ln\frac{4.6}{14.7}}{-0.21} \approx 5.5$$

The peak of Mt. Everest is about 5.5 miles above sea level.

81. $f(t) = 33.4(1.66)^t$

$$421 = 33.4(1.66)^t$$
$$\frac{421}{33.4} = (1.66)^t$$
$$\ln\frac{421}{33.4} = \ln(1.66)^t$$
$$\ln\frac{421}{33.4} = t\ln 1.66$$
$$t = \frac{\ln\frac{421}{33.4}}{\ln 1.66} \approx 5$$

The model projects that 421 million PC and tablet sales were sold approximately 5 years after the year 2009 in the year 2014.

82. a. $W(x) = 11\ln x + 49$

$$W(51) = 11\ln 51 + 49$$
$$\approx 92$$

The model value is the same as the percent displayed by the graph.

b. $B(x) = 16\ln x + 23$

$$90 = 16\ln x + 23$$
$$67 = 16\ln x$$
$$4.1875 = \ln x$$
$$e^{4.1875} = e^{\ln x}$$
$$x = e^{4.1875} \approx 66$$

66 years after 1961, or 2027, approximately 90% of black adults will have completed high school.

83. $20,000 = 12,500\left(1 + \frac{0.065}{4}\right)^{4t}$

$$12,500(1.01625)^{4t} = 20,000$$
$$(1.01625)^{4t} = 1.6$$
$$\ln(1.01625)^{4t} = \ln 1.6$$
$$4t\ln 1.01625 = \ln 1.6$$
$$t = \frac{\ln 1.6}{4\ln 1.01625} \approx 7.3$$

It will take about 7.3 years.

84. $3 \cdot 50,000 = 50,000e^{0.075t}$

$$50,000e^{0.075t} = 150,000$$
$$e^{0.075} = 3$$
$$\ln e^{0.075t} = \ln 3$$
$$0.075t = \ln 3$$
$$t = \frac{\ln 3}{0.075} \approx 14.6$$

It will take about 14.6 years.

85. When an investment value triples, $A = 3P$.

$$3P = Pe^{5r}$$
$$e^{5r} = 3$$
$$\ln e^{5r} = \ln 3$$
$$5r = \ln 3$$
$$r = \frac{\ln 3}{5} \approx 0.2197$$

The interest rate would need to be about 22%

86. a.

$$50.5 = 35.3e^{k10}$$
$$\frac{50.5}{35.3} = e^{10k}$$
$$\ln\frac{50.5}{35.3} = \ln e^{10k}$$
$$\ln\frac{50.5}{35.3} = 10k$$
$$\frac{\ln\frac{50.5}{35.3}}{10} = k$$
$$0.036 \approx k$$
$$A = 35.3e^{0.036t}$$

b. $A = 35.3e^{0.036(15)} \approx 60.6$

In 2015, the population will be about 60.6 million.

c.

$$70 = 35.3e^{0.036t}$$
$$\frac{70}{35.3} = e^{0.036t}$$
$$\ln\frac{70}{35.3} = \ln e^{0.036t}$$
$$\ln\frac{70}{35.3} = 0.036t$$
$$\frac{\ln\frac{70}{35.3}}{0.036} = t$$
$$19 \approx t$$

The population will reach 70 million about 19 years after 2000, in 2019.

87. Use the half-life of 140 days to find k.

$$A = A_0e^{kt}$$
$$\frac{1}{2} = e^{k\cdot 140}$$
$$\frac{1}{2} = e^{140k}$$
$$\ln\frac{1}{2} = \ln e^{140k}$$
$$\ln\frac{1}{2} = 140k$$
$$\frac{\ln\frac{1}{2}}{140} = k$$
$$k \approx -0.004951$$

Use $A = A_0e^{kt}$ to find t.

$$A = A_0e^{-0.004951t}$$
$$0.2 = e^{-0.004951t}$$
$$\ln 0.2 = \ln e^{-0.004951t}$$
$$\ln 0.2 = -0.004951t$$
$$t = \frac{\ln 0.2}{-0.004951}$$
$$t \approx 325$$

It will take about 325 days for the substance to decay to 20% of its original amount.

88. **a.** $f(0) = \dfrac{500,000}{1+2499e^{-0.92(0)}} = 200$

200 people became ill when the epidemic began.

b. $f(6) = \dfrac{500,000}{1+2499e^{-0.92(6)}} = 45,411$

45,410 were ill after 6 weeks.

c. 500,000 people

89. **a.**

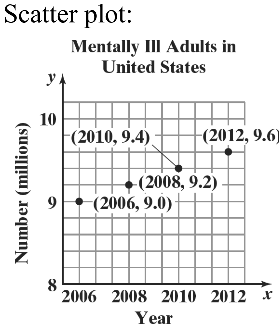
$$T = C + (T_o - C)e^{kt}$$
$$150 = 65 + (185-65)e^{k2}$$
$$90 = 120e^{2k}$$
$$\frac{90}{120} = e^{2k}$$
$$\ln\frac{3}{4} = \ln e^{2k}$$
$$\ln\frac{3}{4} = 2k$$
$$\frac{\ln\frac{3}{4}}{2} = k$$
$$-0.1438 \approx k$$
$$T = 65 + 120e^{-0.1438t}$$

b.

$$105 = 65 + 120e^{-0.1438t}$$
$$40 = 120e^{-0.1438t}$$
$$\frac{1}{3} = e^{-0.1438t}$$
$$\ln\frac{1}{3} = \ln e^{-0.1438t}$$
$$\ln\frac{1}{3} = -0.1438t$$
$$\frac{\ln\frac{1}{3}}{-0.1438} = t$$
$$7.6 \approx t$$

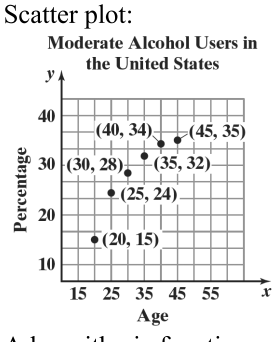
The temperature will reach 105° after 8 min.

90. **a.** Scatter plot:



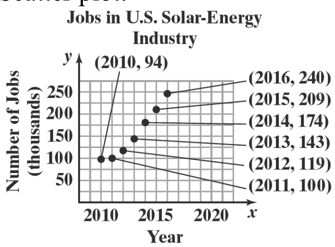
b. A linear function appears to be the best choice for modeling the data.

91. **a.** Scatter plot:



b. A logarithmic function appears to be the best choice for modeling the data.

92. a. Scatter plot:



b. An exponential function appears to be the better choice for modeling the data.

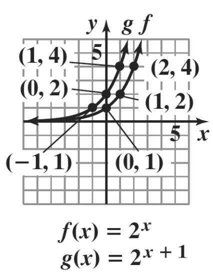
93. $y = 73(2.6)^x$
$y = 73e^{(\ln 2.6)x}$
$y = 73e^{0.956x}$

94. $y = 6.5(0.43)^x$
$y = 6.5e^{(\ln 0.43)x}$
$y = 6.5e^{-0.844x}$

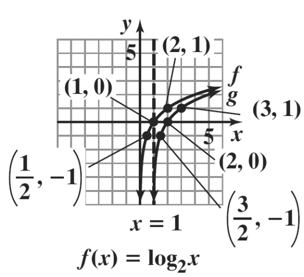
95. Answers will vary.

Chapter 3 Test

1.



2.



3. $5^3 = 125$

4. $\log_{36} 6 = \frac{1}{2}$

5. The domain of *f* consists of all *x* for which $3 - x > 0$. Solving this inequality for *x*, we obtain $x < 3$.
Thus, the domain of *f* is $(-\infty, 3)$.

6. $\log_4(64x^5) = \log_4 64 + \log_4 x^5$
$= 3 + 5\log_4 x$

7. $\log_3 \frac{\sqrt[3]{x}}{81} = \log_3 x^{\frac{1}{3}} - \log_3 81$
$= \frac{1}{3}\log_3 x - 4$

8. $6\log x + 2\log y = \log x^6 + \log y^2$
$= \log(x^6 y^2)$

9. $\ln 7 - 3\ln x = \ln 7 - \ln x^3$
$= \ln \frac{7}{x^3}$

10. $\log_{15} 71 = \frac{\log 71}{\log 15} \approx 1.5741$

11. $3^{x-2} = 9^{x+4}$
$3^{x-2} = (3^2)^{x+4}$
$3^{x-2} = 3^{2x+8}$
$x - 2 = 2x + 8$
$-x = 10$
$x = -10$

12. $5^x = 1.4$
$\ln 5^x = \ln 1.4$
$x\ln 5 = \ln 1.4$
$x = \frac{\ln 1.4}{\ln 5} \approx 0.2091$

13. $400e^{0.005x} = 1600$
$e^{0.005x} = 4$
$\ln e^{0.005x} = \ln 4$
$0.005x = \ln 4$
$x = \frac{\ln 4}{0.005} \approx 277.2589$

14. $e^{2x}-6e^{x}+5=0$

$\left(e^{x}-5\right)\left(e^{x}-1\right)=0$

$e^{x}-5=0$ or $e^{x}-1=0$

$e^{x}=5$ $\quad$ $e^{x}=1$

$\ln e^{x}=\ln 5$ $\quad$ $\ln e^{x}=\ln 1$

$x=\ln 5$ $\quad$ $x=\ln 1$

$x\approx 1.6094$ $\quad$ $x=0$

The solution set is $\{0,\ln 5\}$; $\ln\approx 1.6094$.

15. $\log_6(4x-1)=3$

$$\begin{aligned}4x-1&=6^3\\4x-1&=216\\4x&=217\\x&=\frac{217}{4}=54.25\end{aligned}$$

16. $2\ln 3x=8$

$$\begin{aligned}\ln 3x&=4\\3x&=e^4\\x&=\frac{e^4}{3}\approx 18.1994\end{aligned}$$

17. $\log x+\log(x+15)=2$

$$\begin{aligned}\log\left(x^2+15x\right)&=2\\x^2+15x&=10^2\\x^2+15x-100&=0\\(x+20)(x-5)&=0\end{aligned}$$

$x+20=0$ or $x-5=0$

$x=-20$ $\quad$ $x=5$

$x=-20$ does not check because $\log(-20)$ does not exist.

The solution set is $\{5\}$.

18. $\ln(x-4)-\ln(x+1)=\ln 6$

$$\begin{aligned}\ln\frac{x-4}{x+1}&=\ln 6\\\frac{x-4}{x+1}&=6\\6(x+1)&=x-4\\6x+6&=x-4\\5x&=-10\\x&=-2\end{aligned}$$

$x=-2$ does not check and must be rejected.

The solution set is $\varnothing$.

19. $D=10\log\frac{10^{12}I_0}{I_0}$

$$\begin{aligned}&=10\log 10^{12}\\&=10\cdot 12\\&=120\end{aligned}$$

The loudness of the sound is 120 decibels.

20. Since $\ln e^{x}=x$, $\ln e^{5x}=5x$.

21. $\log_b b=1$ because $b^1=b$.

22. $\log_6 1=0$ because $6^0=1$.

23. 6.5% compounded semiannually:

$$A=3,000\left(1+\frac{0.065}{2}\right)^{2(10)}\approx \$5,687.51$$

6% compounded continuously:

$$A=3,000e^{0.06(10)}\approx \$5,466.36$$

6.5% compounded semiannually yields about $221 more than 6% compounded continuously.

24.

$$\begin{aligned}8000&=4000\left(1+\frac{0.05}{4}\right)^{4t}\\\frac{8000}{4000}&=(1+0.0125)^{4t}\\2&=(1.0125)^{4t}\\\ln 2&=\ln(1.0125)^{4t}\\\ln 2&=4t\ln(1.0125)\\\frac{\ln 2}{4\ln(1.0125)}&=\frac{4t\ln(1.0125)}{4\ln(1.0125)}\\t&=\frac{\ln 2}{4\ln(1.0125)}\approx 13.9\end{aligned}$$

It will take approximately 13.9 years for the money to grow to $8000.

25.

$$\begin{aligned}2&=1e^{r10}\\2&=e^{10r}\\\ln 2&=\ln e^{10r}\\\ln 2&=10r\\r&=\frac{\ln 2}{10}\approx 0.069\end{aligned}$$

The money will double in 10 years with an interest rate of approximately 6.9%.

26. **a.** $A = 82.3e^{-0.004(x)}$

$A = 82.3e^{-0.004(0)} \approx 82.3$

In 2010, the population of Germany was 82.3 million.

b. The population of Germany is decreasing. We can tell because the model has a negative $k = -0.004$.

c. $79.1 = 82.3e^{-0.002t}$

$\frac{79.1}{82.3} = e^{-0.004t}$

$\ln\frac{79.1}{82.3} = \ln e^{-0.004t}$

$\ln\frac{79.1}{82.3} = -0.004t$

$t = \frac{\ln\frac{79.1}{82.3}}{-0.004} \approx 10$

The population of Germany will be 79.1 million approximately 10 years after 2010 in the year 2020.

27. In 2010, $t = 0$ and $A_0 = 4121$
In 2050, $t = 2050 - 2010 = 40$ and $A = 5231$.

$5231 = 4121e^{k40}$

$\frac{5231}{4121} = e^{40k}$

$\ln\frac{5231}{4121} = \ln e^{40k}$

$\ln\frac{5231}{4121} = 40k$

$\frac{\ln\frac{5231}{4121}}{40} = k$

$0.006 \approx k$

The exponential growth function is $A = 4121e^{0.006t}$.

28. First find the decay equation.

$0.5 = e^{kt}$

$0.5 = e^{7.2k}$

$\ln 0.5 = \ln e^{7.2k}$

$\ln 0.5 = 7.2k$

$\frac{\ln 0.5}{7.2} = k$

$k \approx -0.096270$

$A = e^{-0.096270t}$

Next use the decay equation answer question.

$A = e^{-0.096270t}$

$0.3 = e^{-0.096270t}$

$\ln 0.3 = \ln e^{-0.096270t}$

$\ln 0.3 = -0.096270t$

$\frac{\ln 0.3}{-0.096270} = t$

$t \approx 12.5$

It will take 12.5 days.

29. **a.** $f(0) = \frac{140}{1+9e^{-0.165(0)}} = 14$

Fourteen elk were initially introduced to the habitat.

b. $f(10) = \frac{140}{1+9e^{-0.165(10)}} \approx 51$

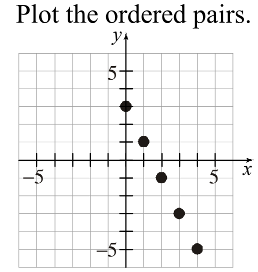
After 10 years, about 51 elk are expected.

c. In the logistic growth model,

$f(t) = \frac{c}{1+ae^{-bt}}$,

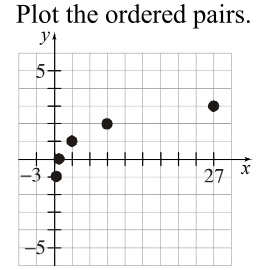
the constant *c* represents the limiting size that *f*(*t*) can attain. The limiting size of the elk population is 140 elk.

30. Plot the ordered pairs.



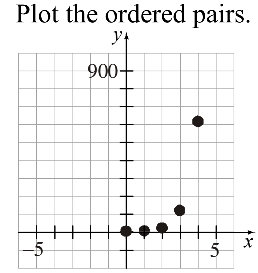
The values appear to belong to a linear function.

31. Plot the ordered pairs.



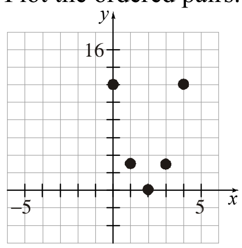
The values appear to belong to a logarithmic function.

32. Plot the ordered pairs.



The values appear to belong to an exponential function.

33. Plot the ordered pairs.



The values appear to belong to a quadratic function.

34. $y = 96(0.38)^x$

$y = 96e^{(\ln 0.38)x}$

$y = 96e^{-0.968x}$

Cumulative Review Exercises (Chapters P–3)

1. $|3x - 4| = 2$

$3x - 4 = 2$ or $3x - 4 = -2$

$3x = 6$ $\quad 3x = 2$

$x = 2$ $\quad x = \frac{2}{3}$

The solution set is $\left\{\frac{2}{3}, 2\right\}$.

2. $x^2 + 2x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

$$x = \frac{-2 \pm 4i}{2}$$

$$x = -1 \pm 2i$$

The solution set is $\{-1 \pm 2i\}$.

3. $x^4 + x^3 - 3x^2 - x + 2 = 0$

p: ±1, ±2

q: ±1

$\frac{p}{q}$: ±1, ±2

–2	1	1	–3	–1	2
		–2	2	2	–2
	1	–1	–1	1	0

$$(x+2)(x^3 - x^2 - x + 1) = 0$$
$$(x+2)[x^2(x-1) - (x-1)] = 0$$
$$(x+2)(x^2 - 1)(x-1) = 0$$
$$(x+2)(x+1)(x-1)(x-1) = 0$$
$$(x+2)(x+1)(x-1)^2 = 0$$

$x + 2 = 0$ or $x + 1 = 0$ or $x - 1 = 0$

$x = -2$ $\quad x = -1$ $\quad x = 1$

The solution set is $\{-2, -1, 1\}$.

4. $e^{5x} - 32 = 96$

$$e^{5x} = 128$$
$$\ln e^{5x} = \ln 128$$
$$5x = \ln 128$$
$$x = \frac{\ln 128}{5} \approx 0.9704$$

The solution set is $\left\{\frac{\ln 128}{5}\right\}$, approximately 0.9704.

5. $\log_2(x+5) + \log_2(x-1) = 4$

$$\log_2[(x+5)(x-1)] = 4$$
$$(x+5)(x-1) = 2^4$$
$$x^2 + 4x - 5 = 16$$
$$x^2 + 4x - 21 = 0$$
$$(x+7)(x-3) = 0$$

$x + 7 = 0$ or $x - 3 = 0$

$x = -7$ $\quad x = 3$

$x = -7$ does not check because $\log_2(-7+5)$ does not exist.

The solution set is $\{3\}$.

6. $\ln(x+4) + \ln(x+1) = 2\ln(x+3)$

$$\ln((x+4)(x+1)) = \ln(x+3)^2$$
$$(x+4)(x+1) = (x+3)^2$$
$$x^2 + 5x + 4 = x^2 + 6x + 9$$
$$5x + 4 = 6x + 9$$
$$-x = 5$$
$$x = -5$$

$x = -5$ does not check and must be rejected.

The solution set is $\varnothing$.

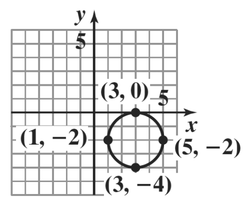
7. $14 - 5x \geq -6$
$-5x \geq -20$
$x \leq 4$

The solution set is $(-\infty, 4]$.

8. $|2x - 4| \leq 2$
$2x - 4 \leq 2$ and $2x - 4 \geq -2$
$2x \leq 6$ $\quad$ $2x \geq 2$
$x \leq 3$ and $x \geq 1$

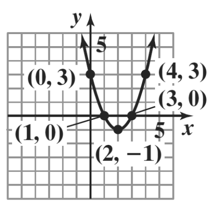
The solution set is $[1, 3]$.

9. Circle with center: (3, –2) and radius of 2



$(x - 3)^2 + (y + 2)^2 = 4$

10. Parabola with vertex: (2, –1)



$f(x) = (x - 2)^2 - 1$

11. x-intercepts:

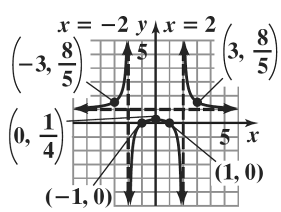
$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

The x-intercepts are $(1, 0)$ and $(-1, 0)$.

vertical asymptotes:

$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

The vertical asymptotes are $x = 2$ and $x = -2$.
Horizontal asymptote: $y = 1$

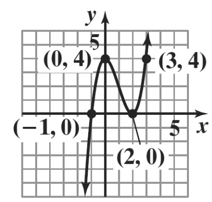


$f(x) = \dfrac{x^2 - 1}{x^2 - 4}$

12. x-intercepts:

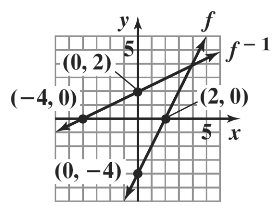
$x - 2 = 0$ or $x + 1 = 0$
$x = 2$ or $x = -1$

The x-intercepts are $(2, 0)$ and $(-1, 0)$.



$f(x) = (x - 2)^2(x + 1)$

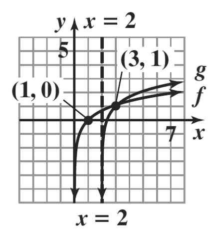
13.



$f(x) = 2x - 4$

$f^{-1}(x) = \dfrac{x + 4}{2}$

14.



$f(x) = \ln x$

$g(x) = \ln(x - 2) + 1$

15. $m = \dfrac{3 - (-3)}{1 - 3} = \dfrac{6}{-2} = -3$

Using (1, 3) point-slope form:

$y - 3 = -3(x - 1)$

slope-intercept form:

$y - 3 = -3(x - 1)$
$y - 3 = -3x + 3$
$y = -3x + 6$

16. $(f \circ g)(x) = f(x + 2)$
$= (x + 2)^2$
$= x^2 + 4x + 4$

$(g \circ f)(x) = g\left(x^2\right)$
$= x^2 + 2$

17. y varies inversely as the square of x is expressed as

$y = \frac{k}{x^2}$.

The hours, H, vary inversely as the square of the number of cups of coffee, C can be expressed

as $H = \frac{k}{C^2}$.

Use the given values to find k.

$$H = \frac{k}{C^2}$$
$$8 = \frac{k}{2^2}$$
$$32 = k$$

Substitute the value of k into the equation.

$$H = \frac{k}{C^2}$$
$$H = \frac{32}{C^2}$$

Use the equation to find H when $C = 4$.

$$H = \frac{32}{C^2}$$
$$H = \frac{32}{4^2}$$
$$H = 2$$

If 4 cups of coffee are consumed you should expect to sleep 2 hours.

18. $s(t) = -16t^2 + 64t + 5$

The ball reaches its maximum height at

$t = \frac{-b}{2a} = \frac{-(64)}{2(-16)} = 2$ seconds.

The maximum height is $s(2)$.

$s(2) = -16(2)^2 + 64(2) + 5 = 69$ feet.

19. $s(t) = -16t^2 + 64t + 5$

Let $s(t) = 0$:

$0 = -16t^2 + 64t + 5$

Use the quadratic formula to solve.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-(64) \pm \sqrt{(64)^2 - 4(-16)(5)}}{2(-16)}$$
$$t \approx 4.1, \quad t \approx -0.1$$

The negative value is rejected.
The ball hits the ground after about 4.1 seconds.

20.
$$40x + 10(1.5x) = 660$$
$$40x + 15x = 660$$
$$55x = 660$$
$$x = 12$$

Your normal hourly salary is \$12 per hour.