Chapter 5
Analytic Trigonometry

Section 5.1

Check Point Exercises

1. $\csc x \tan x = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$

$= \frac{1}{\cos x}$

$= \sec x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

2. $\cos x \cot x + \sin x = \cos x \cdot \frac{\cos x}{\sin x} + \sin x$

$= \frac{\cos^2 x}{\sin x} + \sin x$

$= \frac{\cos^2 x}{\sin x} + \sin x \cdot \frac{\sin x}{\sin x}$

$= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$= \frac{\cos^2 x + \sin^2 x}{\sin x}$

$= \frac{1}{\sin x}$

$= \csc x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

3. $\sin x - \sin x \cos^2 x = \sin x\left(1 - \cos^2 x\right)$

$= \sin x \cdot \sin^2 x$

$= \sin^3 x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

4. $\frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= \csc\theta + \cot\theta$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

5. $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$

$= \frac{\sin x(\sin x)}{(1+\cos x)\sin x} + \frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$

$= \frac{\sin^2 x}{(1+\cos x)\sin x} + \frac{1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$

$= \frac{\sin^2 x+\cos^2 x+2\cos x+1}{(1+\cos x)\sin x}$

$= \frac{1+1+2\cos x}{(1+\cos x)\sin x}$

$= \frac{2+2\cos x}{(1+\cos x)\sin x}$

$= \frac{2\left(1+\cos x\right)}{(1+\cos x)\sin x}$

$= \frac{2}{\sin x}$

$= 2\csc x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

6. $\frac{\cos x}{1+\sin x} = \frac{\cos x}{(1+\sin x)} \cdot \frac{1-\sin x}{1-\sin x}$

$= \frac{\cos x(1-\sin x)}{1-\sin^2 x}$

$= \frac{\cos x(1-\sin x)}{\cos^2 x}$

$= \frac{1-\sin x}{\cos x}$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

7. $\dfrac{\sec x+\csc(-x)}{\sec x\csc x}$
$=\dfrac{\sec x-\csc x}{\sec x\csc x}$
$=\dfrac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}\frac{1}{\sin x}}$
$=\dfrac{\frac{\sin x}{\cos x\sin x}-\frac{\cos x}{\cos x\sin x}}{\frac{1}{\cos x\sin x}}$
$=\dfrac{\frac{\sin x-\cos x}{\cos x\sin x}}{\frac{1}{\cos x\sin x}}$
$=\dfrac{\sin x-\cos x}{\cos x\sin x}\dfrac{\cos x\sin x}{1}$
$=\sin x-\cos x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

8. Left side:

$\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}$
$=\dfrac{1(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}+\dfrac{1(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$
$=\dfrac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$
$=\dfrac{2}{(1+\sin\theta)(1-\sin\theta)}$
$=\dfrac{2}{1-\sin^2\theta}$

Right side:

$2+2\tan^2\theta=2+2\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)$
$=\dfrac{2\cos^2\theta}{\cos^2\theta}+\dfrac{2\sin^2\theta}{\cos^2\theta}$
$=\dfrac{2\cos^2\theta+2\sin^2\theta}{\cos^2\theta}$
$=\dfrac{2}{\cos^2\theta}=\dfrac{2}{1-\sin^2\theta}$

The identity is verified because both sides are equal to $\dfrac{2}{1-\sin^2\theta}$.

Concept and Vocabulary Check 5.1

1. complicated; other

2. sines; cosines

3. false

4. $(\csc x-1)(\csc x+1)$

5. identical/the same

Exercise Set 5.1

1. $\sin x\sec x=\sin x\cdot\dfrac{1}{\cos x}$
$=\dfrac{\sin x}{\cos x}$
$=\tan x$

2. $2\cos x\csc x=\cos x\cdot\dfrac{1}{\sin x}$
$=\dfrac{\cos x}{\sin x}$
$=\cot x$

3. $\tan(-x)\cdot\cos x=-\tan x\cdot\cos x$
$=-\dfrac{\sin x}{\cos x}\cdot\cos x$
$=-\sin x$

4. $\cot(-x)\sin x=-\cot x\sin x$
$=-\dfrac{\cos x}{\sin x}\cdot\sin x$
$=-\cos x$

5. $\tan x\csc x\cos x=\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\sin x}\cos x$
$=1$

6. $\cot x\sec x\sin x=\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\cos x}\cdot\sin x$
$=1$

7. $\sec x-\sec x\sin^2 x=\sec x(1-\sin^2 x)$
$=\dfrac{1}{\cos x}\cdot\cos^2 x$
$=\cos x$

8. $\csc x-\csc x\cos^2 x=\csc x(1-\cos^2 x)$
$=\dfrac{1}{\sin x}\cdot\sin x^2$
$=\sin x$

9. $\cos^2 x - \sin^2 x = \left(1-\sin^2 x\right) - \sin^2 x$
$= 1 - \sin^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$

10. $\cos^2 x - \sin^2 x = \cos^2 x - (1-\cos^2 x)$
$= \cos^2 x - 1 + \cos^2 x$
$= 2\cos^2 x - 1$

11. $\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta$
$= \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$
$= \frac{1-\sin^2\theta}{\sin\theta}$
$= \frac{\cos^2\theta}{\sin\theta}$
$= \frac{\cos\theta}{\sin\theta}\cdot\cos\theta$
$= \cot\theta\cos\theta$

12. $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$
$= \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$
$= \frac{\sin^2\theta}{\cos\theta\sin\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta}$
$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$
$= \frac{1}{\cos\theta\sin\theta}$
$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$
$= \sec\theta\csc\theta$

13. $\frac{\tan\theta\cot\theta}{\csc\theta} = \frac{\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$
$= \frac{1}{\frac{1}{\sin\theta}}$
$= 1 \div \frac{1}{\sin\theta}$
$= 1\cdot\frac{\sin\theta}{1}$
$= \sin\theta$

14. $\frac{\cos\theta\sec\theta}{\cot\theta} = \frac{\frac{\cos\theta}{1}\cdot\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$
$= \frac{1}{\frac{\cos\theta}{\sin\theta}}$
$= 1 \div \frac{\cos\theta}{\sin\theta}$
$= 1\cdot\frac{\sin\theta}{\cos\theta}$
$= \tan\theta$

15. $\sin^2\theta(1+\cot^2\theta) = \sin^2\theta(\csc^2\theta)$
$= \sin^2\theta\cdot\frac{1}{\sin^2\theta}$
$= 1$

16. $\cos^2\theta(1+\tan^2\theta) = \cos^2\theta(\sec^2\theta)$
$= \cos^2\theta\cdot\frac{1}{\cos^2\theta}$
$= 1$

17. $\frac{1-\cos^2 t}{\cos t} = \frac{\sin^2 t}{\cos t}$
$= \sin t\cdot\frac{\sin t}{\cos t}$
$= \sin t\tan t$

18. $\frac{1-\sin^2 t}{\sin t} = \frac{\cos^2 t}{\sin t}$
$= \cos t\cdot\frac{\cos t}{\sin t}$
$= \cos t\cot t$

19. $\frac{\csc^2 t}{\cot t} = \frac{\frac{1}{\sin^2 t}}{\frac{\cos t}{\sin t}}$
$= \frac{1}{\sin^2 t} \div \frac{\cos t}{\sin t}$
$= \frac{1}{\sin^2 t}\cdot\frac{\sin t}{\cos t}$
$= \frac{1}{\sin t}\cdot\frac{1}{\cos t}$
$= \csc t\sec t$

20. $$\frac{\sec^2 t}{\tan t} = \frac{\frac{1}{\cos^2 t}}{\frac{\sin t}{\cos t}}$$
$$= \frac{1}{\cos^2 t} \div \frac{\sin t}{\cos t}$$
$$= \frac{1}{\cos^2 t} \cdot \frac{\cos t}{\sin t}$$
$$= \frac{1}{\cos t} \cdot \frac{1}{\sin t}$$
$$= \sec t \csc t$$

21. $$\frac{\tan^2 t}{\sec t} = \frac{\sec^2 t - 1}{\sec t}$$
$$= \frac{\sec^2 t}{\sec t} - \frac{1}{\sec t}$$
$$= \sec t - \cos t$$

22. $$\frac{\cot^2 t}{\csc t} = \frac{\csc^2 t - 1}{\csc t}$$
$$= \frac{\csc^2 t}{\csc t} - \frac{1}{\csc t}$$
$$= \csc t - \sin t$$

23. $$\frac{1-\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \csc\theta - \cot\theta$$

24. $$\frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta - \tan\theta$$

25. $$\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = \frac{\sin t}{\frac{1}{\sin t}} + \frac{\cos t}{\frac{1}{\cos t}}$$
$$= \sin t \div \frac{1}{\sin t} + \cos t \div \frac{1}{\cos t}$$
$$= \sin t \cdot \frac{\sin t}{1} + \cos t \cdot \frac{\cos t}{1}$$
$$= \sin^2 t + \cos^2 t$$
$$= 1$$

26. $$\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \frac{\sin t}{\frac{\sin t}{\cos t}} + \frac{\cos t}{\frac{\cos t}{\sin t}}$$
$$= \sin t \div \frac{\sin t}{\cos t} + \cos t \div \frac{\cos t}{\sin t}$$
$$= \sin t \cdot \frac{\cos t}{\sin t} + \cos t \cdot \frac{\sin t}{\cos t}$$
$$= \cos t + \sin t$$
$$= \sin t + \cos t$$

27. $$\tan t + \frac{\cos t}{1+\sin t}$$
$$= \frac{\sin t}{\cos t} + \frac{\cos t}{1+\sin t}$$
$$= \frac{\sin t}{\cos t} \cdot \frac{1+\sin t}{1+\sin t} + \frac{\cos t}{1+\sin t} \cdot \frac{\cos t}{\cos t}$$
$$= \frac{\sin t + \sin^2 t}{\cos t(1+\sin t)} + \frac{\cos^2 t}{\cos t(1+\sin t)}$$
$$\frac{\sin t + \sin^2 t + \cos^2 t}{\cos t(1+\sin t)}$$
$$= \frac{1+\sin t}{\cos t(1+\sin t)}$$
$$= \frac{1}{\cos t}$$
$$= \sec t$$

28. $$\cot t + \frac{\sin t}{1+\cos t} = \frac{\cos t}{\sin t} + \frac{\sin t}{1+\cos t}$$
$$= \frac{\cos t}{\sin t} \cdot \frac{1+\cos t}{1+\cos t} + \frac{\sin t}{1+\cos t} \cdot \frac{\sin t}{\sin t}$$
$$= \frac{\cos t + \cos^2 t}{\sin t(1+\cos t)} + \frac{\sin^2 t}{\sin t(1+\cos t)}$$
$$= \frac{\cos t + \cos^2 t + \sin^2 t}{\sin t(1+\cos t)}$$
$$= \frac{\cos t + 1}{\sin t(1+\cos t)}$$
$$= \frac{1}{\sin t}$$
$$= \csc t$$

29. $$1 - \frac{\sin^2 x}{1+\cos x} = 1 - \frac{\sin^2 x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}$$
$$= 1 - \frac{\sin^2 x(1-\cos x)}{1-\cos^2 x}$$
$$= 1 - \frac{\sin^2 x(1-\cos x)}{\sin^2 x}$$
$$= 1 - 1 + \cos x$$
$$= \cos x$$

30. $$1 - \frac{\cos^2 x}{1+\sin x} = 1 - \frac{\cos^2}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}$$
$$= 1 - \frac{\cos^2 x(1-\sin x)}{1-\sin^2 x}$$
$$= 1 - \frac{\cos^2 x(1-\sin x)}{\cos^2 x}$$
$$= 1 - 1 + \sin x$$
$$= \sin x$$

31. $$\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{\cos x(1+\sin x)}{1-\sin^2 x}+\frac{1-\sin x}{\cos x}$$
$$==\frac{\cos x(1+\sin x)}{\cos^2 x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{1+\sin x}{\cos x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{2}{\cos x}$$
$$=2\cdot\frac{1}{\cos x}$$
$$=2\sec x$$

32. $$\frac{\sin x}{\cos x+1}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x}{\cos x+1}\cdot\frac{\cos x-1}{\cos x-1}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x(\cos x-1)}{\cos^2 x-1}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x(\cos x-1)}{-\sin^2 x}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x(1-\cos x)}{\sin^2 x}+\frac{\cos x-1}{\sin x}$$
$$=\frac{1-\cos x}{\sin x}+\frac{\cos x-1}{\sin x}$$
$$=\frac{0}{\sin x}$$
$$=0$$

33. $$\sec^2 x\csc^2 x=(1+\tan^2 x)\csc^2 x$$
$$=\csc^2 x+\tan^2 x\csc^2 x$$
$$=\csc^2 x+\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x}$$
$$=\csc^2 x+\frac{1}{\cos^2 x}$$
$$=\csc^2 x+\sec^2 x$$
$$=\sec^2 x+\csc^2 x$$

34. $$\csc^2 x\sec x=(1+\cot^2 x)\sec x$$
$$=\sec x+\cot^2 x\sec x$$
$$=\sec x+\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos x}$$
$$=\sec x+\frac{\cos x}{\sin^2 x}$$
$$=\sec x+\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}$$
$$=\sec x+\csc x\cot x$$

35. $$\frac{\sec x-\csc x}{\sec x+\csc x}=\frac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}+\frac{1}{\sin x}}$$
$$=\frac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}+\frac{1}{\sin x}}\cdot\frac{\sin x}{\sin x}$$
$$=\frac{\frac{\sin x}{\cos x}-1}{\frac{\sin x}{\cos x}+1}$$
$$=\frac{\tan x-1}{\tan x+1}$$

36. $$\frac{\csc x-\sec x}{\csc x+\sec x}=\frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}$$
$$=\frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}\cdot\frac{\cos x}{\cos x}$$
$$=\frac{\frac{\cos x}{\sin x}-1}{\frac{\cos x}{\sin x}+1}$$
$$=\frac{\cot x-1}{\cot x+1}$$

37. $$\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\frac{(\sin x+\cos x)(\sin x-\cos x)}{\sin x+\cos x}$$
$$=\sin x-\cos x$$

38. $$\frac{\tan^2 x-\cot^2 x}{\tan x+\cot x}=\frac{(\tan x-\cot x)(\tan x+\cot x)}{\tan x+\cot x}$$
$$=\tan x-\cot x$$

39. $$\tan^2 2x+\sin^2 2x+\cos^2 2x=\tan^2 2x+1$$
$$=\sec^2 2x$$

40. $$\cot^2 2x+\cos^2 2x+\sin^2 2x=\cot^2 2x+1$$
$$=\csc^2 2x$$

41. $$\frac{\tan 2\theta+\cot 2\theta}{\csc 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\sin 2\theta}}$$

$$=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\sin 2\theta}}$$

$$=\frac{\frac{\sin^2 2\theta+\cos 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\sin 2\theta}}$$

$$=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\sin 2\theta}$$

$$=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\sin 2\theta}{1}$$

$$=\frac{1}{\cos 2\theta}=\sec 2\theta$$

42. $$\frac{\tan 2\theta+\cot 2\theta}{\sec 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$

$$=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\cos 2\theta}}$$

$$=\frac{\frac{\sin^2 2\theta}{\cos 2\theta\sin 2\theta}+\frac{\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$

$$=\frac{\frac{\sin^2 2\theta+\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$

$$=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\cos 2\theta}$$

$$=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\cos 2\theta}{1}$$

$$=\frac{1}{\sin 2\theta}$$

$$=\csc 2\theta$$

43. $$\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}}{1-\frac{\sin x}{\cos y}\cdot\frac{\sin y}{\cos y}}\cdot\frac{\cos x\cos y}{\cos x\cos y}$$

$$=\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$$

44. $$\frac{\cot x+\cot y}{1-\cot x\cot y}=\frac{\frac{\cos x}{\sin x}+\frac{\cos y}{\sin y}}{1-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}}$$
$$=\frac{\frac{\cos x}{\sin x}+\frac{\cos y}{\sin y}}{1-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}}\cdot\frac{\sin x\sin y}{\sin x\sin y}$$
$$=\frac{\frac{\cos x}{\sin x}\cdot\frac{\sin x\sin y}{1}+\frac{\cos y}{\sin y}\cdot\frac{\sin x\sin y}{1}}{\sin x\sin y-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}\cdot\frac{\sin x\sin y}{1}}$$
$$=\frac{\cos x\sin y+\sin x\cos y}{\sin x\sin y-\cos x\cos y}$$

45. Left side:
$$(\sec x-\tan x)^2=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$$
$$=\left(\frac{1-\sin x}{\cos x}\right)^2$$
$$=\frac{(1-\sin x)^2}{\cos^2 x}$$
Right side:
$$\frac{1-\sin x}{1+\sin x}=\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$
$$=\frac{(1-\sin x)^2}{1-\sin^2 x}$$
$$=\frac{(1-\sin x)^2}{\cos^2 x}$$
The identity is verified because both sides are equal to $\frac{(1-\sin x)^2}{\cos^2 x}$.

46. Left side: $(\csc x-\cot x)^2=\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)^2=\left(\frac{1-\cos x}{\sin x}\right)^2=\frac{(1-\cos x)^2}{\sin^2 x}$

Right side: $\frac{1-\cos x}{1+\cos x}=\frac{1-\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}=\frac{(1-\cos x)^2}{1-\cos^2 x}=\frac{(1-\cos x)^2}{\sin^2 x}$

The identity is verified because both sides are equal to $\frac{(1-\cos x)^2}{\sin^2 x}$.

47. $$\frac{\tan t}{\sec t-1}=\frac{\tan t}{\sec t-1}\cdot\frac{\sec t+1}{\sec t+1}$$
$$=\frac{\tan t(\sec t+1)}{\sec^2 t-1}$$
$$=\frac{\tan t(\sec t+1)}{\tan^2 t}$$
$$=\frac{\sec t+1}{\tan t}$$

48. $$\begin{aligned}\frac{\cot t}{\csc t+1} &= \frac{\cot t}{\csc t+1}\cdot\frac{\csc t-1}{\csc t-1}\\ &= \frac{\cot t(\csc t-1)}{\csc^2 t-1}\\ &= \frac{\cot t(\csc t-1)}{\cot^2 t}\\ &= \frac{\csc t-1}{\cot t}\end{aligned}$$

49. Left side:

$$\begin{aligned}\frac{1+\cos t}{1-\cos t} &= \frac{1+\cos t}{1-\cos t}\cdot\frac{1+\cos t}{1+\cos t}\\ &= \frac{(1+\cos t)^2}{1-\cos^2 t}\\ &= \frac{(1+\cos t)^2}{\sin^2 t}\end{aligned}$$

Right side:

$$\begin{aligned}(\csc t+\cot t)^2 &= \left(\frac{1}{\sin t}+\frac{\cos t}{\sin t}\right)^2\\ &= \left(\frac{1+\cos t}{\sin t}\right)^2\\ &= \frac{(1+\cos t)^2}{\sin^2 t}\end{aligned}$$

The identity is verified because both sides are equal to $\frac{(1+\cos t)^2}{\sin^2 t}$.

50. Left side:

$$\begin{aligned}\frac{\cos^2 t+4\cos t+4}{\cos t+2} &= \frac{(\cos t+2)(\cos t+2)}{\cos t+2}\\ &= \cos t+2\end{aligned}$$

Right side:

$$\begin{aligned}\frac{2\sec t+1}{\sec t} &= \frac{2\sec t}{\sec t}+\frac{1}{\sec t}\\ &= 2+\cos t\\ &= \cos t+2\end{aligned}$$

The identity is verified because both sides are equal to $\cos t+2$.

51. $$\begin{aligned}\cos^4 t-\sin^4 t &= \left(\cos^2 t-\sin^2 t\right)\left(\cos^2 t+\sin^2 t\right)\\ &= \left(\cos^2 t-\sin^2 t\right)\cdot 1\\ &= 1-\sin^2 t-\sin^2 t\\ &= 1-2\sin^2 t\end{aligned}$$

52. $$\begin{aligned}\sin^4 t-\cos^4 t &= \left(\sin^2 t-\cos^2 t\right)\left(\sin^2 t+\cos^2 t\right)\\ &= \left(\sin^2 t-\cos^2 t\right)\cdot 1\\ &= 1-\cos^2 t-\cos^2 t\\ &= 1-2\cos^2 t\end{aligned}$$

53. $\dfrac{\sin\theta-\cos\theta}{\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta}$

$=\dfrac{(\sin\theta-\cos\theta)\cos\theta}{\cos\theta\sin\theta}+\dfrac{(\cos\theta-\sin\theta)\sin\theta}{\cos\theta\sin\theta}$

$=\dfrac{\sin\theta\cos\theta-\cos^2\theta+\sin\theta\cos\theta-\sin^2\theta}{\sin\theta\cos\theta}$

$=\dfrac{2\sin\theta\cos\theta-\left(\cos^2\theta+\sin^2\theta\right)}{\sin\theta\cos\theta}$

$=\dfrac{2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$

$=\dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}-\dfrac{1}{\sin\theta\cos\theta}$

$=2-\dfrac{1}{\sin\theta}\cdot\dfrac{1}{\cos\theta}$

$=2-\csc\theta\sec\theta$

$=2-\sec\theta\csc\theta$

54. $\dfrac{\sin\theta}{1-\cot\theta}-\dfrac{\cos\theta}{\tan\theta-1}$

$=\dfrac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-1}$

$=\dfrac{\sin\theta}{\dfrac{\sin\theta}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}}$

$=\dfrac{\sin\theta}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta-\cos\theta}{\cos\theta}}$

$=\sin\theta\div\dfrac{\sin\theta-\cos\theta}{\sin\theta}-\cos\theta\div\dfrac{\sin\theta-\cos\theta}{\cos\theta}$

$=\sin\theta\cdot\dfrac{\sin\theta}{\sin\theta-\cos\theta}-\cos\theta\cdot\dfrac{\cos\theta}{\sin\theta-\cos\theta}$

$=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}-\dfrac{\cos^2\theta}{\sin\theta-\cos\theta}$

$=\dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

$\dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$

$=\sin\theta+\cos\theta$

55. $\left(\tan^2\theta+1\right)\left(\cos^2\theta+1\right)$

$=\tan^2\theta\cos^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\sin^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\sin^2\theta+\cos^2\theta+\tan^2\theta+1$

$=1+\tan^2\theta+1$

$=\tan^2\theta+2$

56. $(\cot^2\theta+1)(\sin^2+1)$
$=\cot^2\theta\sin^2\theta+\cot^2\theta+\sin^2\theta+1$
$=\dfrac{\cos^2\theta}{\sin^2\theta}\sin^2\theta+\cot^2\theta+\sin^2\theta+1$
$=\cos^2\theta+\sin^2\theta+\cot^2\theta+1$
$=1+\cot^2\theta+1$
$=\cot^2\theta+2$

57. $(\cos\theta-\sin\theta)^2+(\cos\theta+\sin\theta)^2$
$=\cos^2\theta-2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta$
$=\cos^2\theta+\sin^2\theta+\cos^2\theta+\sin^2\theta$
$=1+1=2$

58. $(3\cos\theta-4\sin\theta)^2+(4\cos\theta+3\sin\theta)^2$
$=9\cos^2\theta-24\cos\theta\sin\theta+16\sin^2\theta+$
$+16\cos^2\theta+24\cos\theta\sin\theta+9\sin^2\theta$
$=9\cos^2\theta+9\sin^2\theta+16\sin^2\theta+16\cos^2\theta$
$=9(\cos^2\theta+\sin^2\theta)+16(\sin^2\theta+\cos^2\theta)$
$=9(1)+16(1)$
$=25$

59. $\dfrac{\cos^2 x-\sin^2 x}{1-\tan^2 x}$
$=\dfrac{\cos^2 x-\sin^2 x}{1-\dfrac{\sin^2 x}{\cos^2 x}}=\dfrac{\cos^2 x-\sin^2 x}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}$
$=\dfrac{\cos^2 x-\sin^2 x}{1}\div\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}$
$=\dfrac{\cos^2 x-\sin^2 x}{1}\cdot\dfrac{\cos^2 x}{\cos^2 x-\sin^2 x}=\cos^2 x$

60. $\dfrac{\sin x+\cos x}{\sin x}-\dfrac{\cos x-\sin x}{\cos x}$
$=\dfrac{(\sin x+\cos x)\cos x}{\sin x\cos x}-\dfrac{(\cos x-\sin x)\sin x}{\sin x\cos x}$
$=\dfrac{\sin x\cos x+\cos^2 x-\cos x\sin x+\sin^2 x}{\sin x\cos x}$
$=\dfrac{\cos^2 x+\sin^2 x}{\sin x\cos x}$
$=\dfrac{1}{\sin x\cos x}$
$=\dfrac{1}{\sin x}\cdot\dfrac{1}{\cos x}$
$=\csc x\sec x$
$=\sec x\csc x$

61. Conjecture: left side is equal to $\cos x$

$$\begin{aligned}\frac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x}&=\frac{\sec^2 x-\tan^2 x}{\sec x}\\&=\frac{1}{\sec x}\\&=\cos x\end{aligned}$$

62. Conjecture: left side is equal to $\sin x$

$$\begin{aligned}\frac{\sec^2 x\csc x}{\sec^2 x+\csc^2 x}&=\frac{\frac{1}{\cos^2 x}\cdot\frac{1}{\sin x}}{\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}}\cdot\frac{\cos^2 x\sin^2 x}{\cos^2 x\sin^2 x}\\&=\frac{\sin x}{\sin^2 x+\cos^2 x}\\&=\frac{\sin x}{1}\\&=\sin x\end{aligned}$$

63. Conjecture: left side is equal to $2\sin x$

$$\begin{aligned}\frac{\cos x+\cot x\sin x}{\cot x}&=\frac{\cos x}{\cot x}+\frac{\cot x\sin x}{\cot x}\\&=\frac{\cos x}{\frac{\cos x}{\sin x}}+\frac{\cot x\sin x}{\cot x}\\&=\frac{\cos x\sin x}{\cos x}+\sin x\\&=\sin x+\sin x\\&=2\sin x\end{aligned}$$

64. Conjecture: left side is equal to $\cos x-1$

$$\begin{aligned}\frac{\cos x\tan x-\tan x+2\cos x-2}{\tan x+2}&=\frac{\cos x\tan x+2\cos x-\tan x-2}{\tan x+2}\\&=\frac{\cos x\tan x+2\cos x}{\tan x+2}+\frac{-\tan x-2}{\tan x+2}\\&=\frac{\cos x(\tan x+2)}{\tan x+2}-1\\&=\cos x-1\end{aligned}$$

65. Conjecture: left side is equal to $2\sec x$

$$\begin{aligned}\frac{1}{\sec x+\tan x}+\frac{1}{\sec x-\tan x}&=\frac{\sec^2 x-\tan^2 x}{\sec x+\tan x}+\frac{\sec^2 x-\tan^2 x}{\sec x-\tan x}\\&=\frac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x+\tan x}+\frac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x-\tan x}\\&=\sec x-\tan x+\sec x+\tan x\\&=2\sec x\end{aligned}$$

66. Conjecture: left side is equal to $2\csc x$

$$\frac{1+\cos x}{\sin x}+\frac{\sin x}{1+\cos x}=\frac{1+\cos x}{\sin x}\cdot\frac{1+\cos x}{1+\cos x}+\frac{\sin x}{1+\cos x}\cdot\frac{\sin x}{\sin x}$$
$$=\frac{1+2\cos x+\cos^2 x}{(\sin x)(1+\cos x)}+\frac{\sin^2 x}{(\sin x)(1+\cos x)}$$
$$=\frac{1+2\cos x+\cos^2 x+\sin^2 x}{(\sin x)(1+\cos x)}$$
$$=\frac{1+2\cos x+1}{(\sin x)(1+\cos x)}$$
$$=\frac{2+2\cos x}{(\sin x)(1+\cos x)}$$
$$=\frac{2(1+\cos x)}{(\sin x)(1+\cos x)}$$
$$=\frac{2}{\sin x}$$
$$=2\csc x$$

67. $$\frac{\tan x+\cot x}{\csc x}=\frac{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$
$$=\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)\frac{\sin x}{1}$$
$$=\frac{\sin^2 x}{\cos x}+\frac{\sin x\cos x}{\sin x}$$
$$=\frac{1-\cos^2 x}{\cos x}+\frac{\cos x}{1}$$
$$=\frac{1-\cos^2 x}{\cos x}+\frac{\cos^2 x}{\cos x}$$
$$=\frac{1}{\cos x}$$

68. $$\frac{\sec x+\csc x}{1+\tan x}=\left(\frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{1+\frac{\sin x}{\cos x}}\right)\frac{\sin x\cos x}{\sin x\cos x}$$
$$=\frac{\frac{\sin x\cos x}{\cos x}+\frac{\sin x\cos x}{\sin x}}{\sin x\cos x+\frac{\sin^2 x\cos x}{\cos x}}$$
$$=\frac{\sin x+\cos x}{\sin x\cos x+\sin^2 x}$$
$$=\frac{\sin x+\cos x}{\sin x(\cos x+\sin x)}$$
$$=\frac{1}{\sin x}$$

69. $\dfrac{\cos x}{1+\sin x}+\tan x=\dfrac{\cos x}{1+\sin x}\cdot\dfrac{\cos x}{\cos x}+\dfrac{\sin x}{\cos x}\cdot\dfrac{1+\sin x}{1+\sin x}$

$$=\frac{\cos^2 x}{(1+\sin x)(\cos x)}+\frac{\sin x+\sin^2 x}{(1+\sin x)(\cos x)}$$
$$=\frac{\cos^2 x+\sin x+\sin^2 x}{(1+\sin x)(\cos x)}$$
$$=\frac{\sin x+\cos^2 x+\sin^2 x}{(1+\sin x)(\cos x)}$$
$$=\frac{\sin x+1}{(1+\sin x)(\cos x)}$$
$$=\frac{1}{\cos x}$$

70. $\dfrac{1}{\sin x\cos x}-\cot x=\dfrac{1}{\sin x\cos x}-\dfrac{\sin x}{\cos x}$

$$=\frac{1}{\sin x\cos x}-\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}$$
$$=\frac{1-\cos^2 x}{\sin x\cos x}$$
$$=\frac{\sin^2 x}{\sin x\cos x}$$
$$=\frac{\sin x}{\cos x}$$
$$=\tan x$$
$$=\frac{1}{\cot x}$$

71. $\dfrac{1}{1-\cos x}-\dfrac{\cos x}{1+\cos x}=\dfrac{1}{1-\cos x}\cdot\dfrac{1+\cos x}{1+\cos x}-\dfrac{\cos x}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}$

$$=\frac{1+\cos x}{1-\cos^2 x}-\frac{\cos x-\cos^2 x}{1-\cos^2 x}$$
$$=\frac{1+\cos x-\cos x+\cos^2 x}{1-\cos^2 x}$$
$$=\frac{1+\cos^2 x}{\sin^2 x}$$
$$=\frac{1}{\sin^2 x}+\frac{\cos^2 x}{\sin^2 x}$$
$$=\csc^2 x+\cot^2 x$$
$$=\csc^2 x+\csc^2 x-1$$
$$=2\csc^2 x-1$$

72. $(\sec x+\csc x)(\sin x+\cos x)-2-\cot x=\sec x\sin x+\sec x\cos x+\csc x\sin x+\csc x\cos x-2-\cot x$

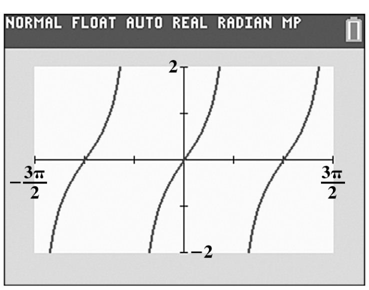
$$=\sec x\sin x+\sec x\cos x+\csc x\sin x+\csc x\cos x-2-\cot x$$
$$=\tan x+1+1+\cot x-2-\cot x$$
$$=\tan x$$

73. $$\frac{1}{\csc x-\sin x}=\frac{1}{\frac{1}{\sin x}-\sin x}$$
$$=\frac{1}{\frac{1}{\sin x}-\sin x}$$
$$=\frac{1}{\frac{1}{\sin x}-\frac{\sin^2 x}{\sin x}}$$
$$=\frac{1}{\frac{1-\sin^2 x}{\sin x}}$$
$$=\frac{1}{\frac{\cos^2 x}{\sin x}}$$
$$=\frac{\sin x}{\cos^2 x}$$
$$=\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}$$
$$=\sec x\tan x$$

74. $$\frac{1-\sin x}{1+\sin x}-\frac{1+\sin x}{1-\sin x}=\frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}-\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}$$
$$=\frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}-\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}$$
$$=\frac{1-2\sin x+\sin^2 x}{1-\sin^2 x}-\frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}$$
$$=\frac{-4\sin x}{1-\sin^2 x}$$
$$=\frac{-4\sin x}{\cos^2 x}$$
$$=\frac{-4}{\cos x}\cdot\frac{\sin x}{\cos x}$$
$$=-4\sec x\tan x$$

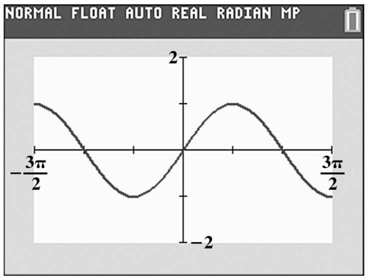
75. – 78. Answers may vary.

79.



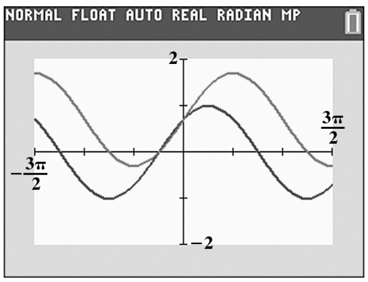
$$\sec x(\sin x-\cos x)+1=\frac{1}{\cos x}(\sin x-\cos x)+1$$
$$=\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}+1$$
$$=\tan x-1+1$$
$$=\tan x$$

80.



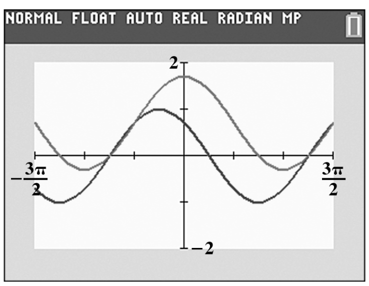
$$-\cos x\tan(-x) = -\cos x\cdot\frac{\sin(-x)}{\cos(-x)}$$
$$= -\cos x\cdot\frac{-\sin x}{\cos x} = \sin x$$

81.



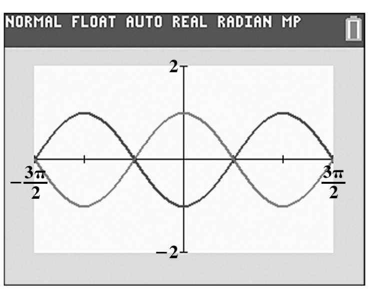
The graphs do not coincide.
Values for x may vary.

82.



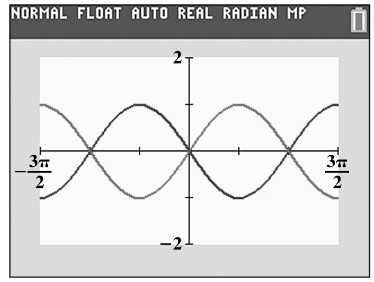
The graphs do not coincide.
Values for x may vary.

83.



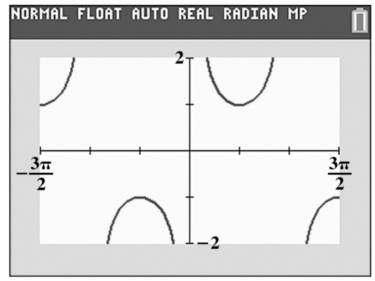
The graphs do not coincide.
Values for x may vary.

84.



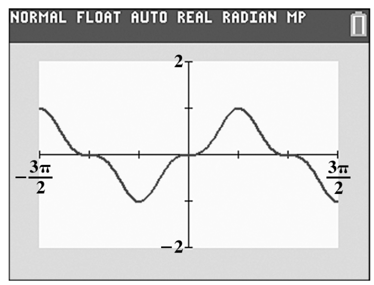
The graphs do not coincide.
Values for x may vary.

85.



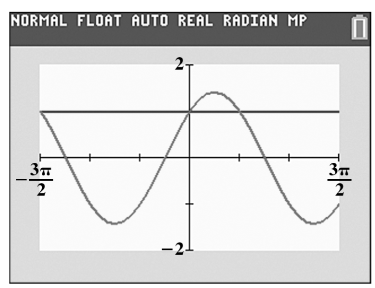
$$\frac{\sin x}{1-\cos^2 x} = \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x} = \csc x$$

86.



$$\sin x - \sin x\cos^2 x = \sin x(1-\cos^2 x)$$
$$= \sin x\left(\sin^2 x\right) = \sin^3 x$$

87.



The graphs do not coincide.
Values for x may vary.

88. makes sense

89. makes sense

90. makes sense

91. does not make sense; Explanations will vary. Sample explanation: The most efficient way to simplify the identity is to multiply out the numerator and then use a Pythagorean identity.

92.
$$\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}$$
$$=\frac{(\sin x-\cos x)(\sin^2 x+\sin x\cos x+\cos^2 x)}{\sin x-\cos x}$$
$$=\sin^2 x+\cos^2 x+\sin x\cos x$$
$$=1+\sin x\cos x$$

93.
$$\frac{\sin x-\cos x+1}{\sin x+\cos x-1}$$
$$=\frac{\sin x-\cos x+1}{\sin x+\cos x-1}\cdot\frac{\sin x-\cos x-1}{\sin x-\cos x-1}$$
$$=\frac{\sin^2 x-2\cos x\sin x+\cos^2 x-1}{\sin^2 x-2\sin x-\cos^2 x+1}$$
$$=\frac{\sin^2 x+\cos^2 x-2\cos x\sin x-1}{\sin x^2-2\sin x-(1-\sin^2 x)+1}$$
$$=\frac{1-2\cos x\sin x-1}{\sin^2 x-2\sin x+\sin x^2}$$
$$=\frac{-2\cos x\sin x}{2\sin^2 x-2\sin x}$$
$$=\frac{-2\sin x\cos x}{2\sin x(\sin x-1)}$$
$$=\frac{-\cos x}{\sin x-\cos x}$$
$$=\frac{-\cos x}{\sin x-1}\cdot\frac{\sin x+1}{\sin x+1}$$
$$=\frac{-\cos x(\sin x+1)}{\sin^2 x-1}$$
$$=\frac{-\cos x(\sin x+1)}{\cos^2 x-1-\cos^2 x}$$
$$=\frac{-\cos x(\sin x+1)}{\cos^2 x}$$
$$=\frac{\sin x+1}{\cos x}$$

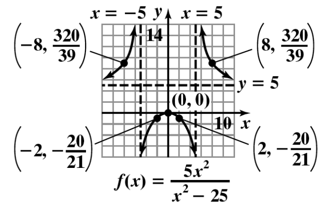
94.
$$-\ln|\cos x|=\ln|\cos x|^{-1}=\ln\frac{1}{|\cos x|}$$
$$=\ln\left|\frac{1}{\cos x}\right|=\ln|\sec x|$$

95.
$$\ln e^{\tan^2 x-\sec^2 x}$$
$$=\tan^2 x-\sec^2 x$$
$$=-(-\tan^2 x+\sec^2 x)$$
$$=-(\sec^2 x-\tan^2 x)$$
$$=-1$$

96. Answers may vary.

97. Answers may vary.

98. Graph:



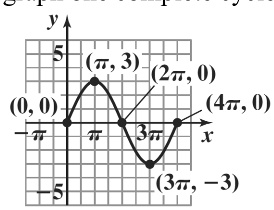
99. The equation $y=3\sin\frac{1}{2}x$ is of the form $y=A\sin Bx$ with $A = 3$ and $B=\frac{1}{2}$. The amplitude is $|A|=|3|=3$. The period is $\frac{2\pi}{B}=\frac{2\pi}{\frac{1}{2}}=2\pi\cdot 2=4\pi$. The quarter-period is $\frac{4\pi}{4}=\pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x=0$
$x=0+\pi=\pi$
$x=\pi+\pi=2\pi$
$x=2\pi+\pi=3\pi$
$x=3\pi+\pi=4\pi$

Evaluate the function at each value of x.

x	$y=3\sin\frac{1}{2}x$	coordinates
0	$y=3\sin\left(\frac{1}{2}\cdot 0\right)$ $=3\sin 0=3\cdot 0=0$	(0, 0)
π	$y=3\sin\left(\frac{1}{2}\cdot\pi\right)$ $=3\sin\frac{\pi}{2}=3\cdot 1=3$	$(\pi, 3)$
2π	$y=3\sin\left(\frac{1}{2}\cdot 2\pi\right)$ $=3\sin\pi=3\cdot 0=0$	$(2\pi, 0)$
3π	$y=3\sin\left(\frac{1}{2}\cdot 3\pi\right)$ $=3\sin\frac{3\pi}{2}$ $=3(-1)=-3$	$(3\pi, -3)$
4π	$y=3\sin\left(\frac{1}{2}\cdot 4\pi\right)$ $=3\sin 2\pi=3\cdot 0=0$	$(4\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin\frac{1}{2}x$

100. $f(x) = \frac{x-1}{x+1}, x \neq -1$

Replace $f(x)$ with y:

$y = \frac{x-1}{x+1}$

Interchange x and y:

$x = \frac{y-1}{y+1}$

Solve for y:

$$x = \frac{y-1}{y+1}$$
$$x(y+1) = y-1$$
$$xy + x = y - 1$$
$$xy - y = -x - 1$$
$$y(x-1) = -x - 1$$
$$y = \frac{-x-1}{x-1}$$
$$y = \frac{x+1}{1-x}$$

Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \frac{x+1}{1-x}$

101. $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin 30^\circ = \frac{1}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 90^\circ = 0$

$\sin 90^\circ = 1$

102. **a.** No, they are not equal.

$$\cos(30^\circ + 60^\circ) \neq \cos 30^\circ + \cos 60^\circ$$
$$\cos 90^\circ \neq \frac{\sqrt{3}}{2} + \frac{1}{2}$$
$$0 \neq \frac{1+\sqrt{3}}{2}$$

b. Yes, they are equal.

$$\cos(30^\circ + 60^\circ) = \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$$
$$\cos 90^\circ = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$0 = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$
$$0 = 0$$

103. **a.** No, they are not equal.

$$\sin(30^\circ + 60^\circ) \neq \sin 30^\circ + \sin 60^\circ$$
$$\sin 90^\circ \neq \frac{1}{2} + \frac{\sqrt{3}}{2}$$
$$1 \neq \frac{1+\sqrt{3}}{2}$$

b. Yes, they are equal.

$$\sin(30^\circ + 60^\circ) = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$
$$\sin 90^\circ = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$1 = \frac{1}{4} + \frac{3}{4}$$
$$1 = 1$$

Section 5.2

Check Point Exercises

1.
$$\begin{aligned}\cos 30^\circ &= \cos(90^\circ - 60^\circ) \\ &= \cos 90^\circ \cos 60^\circ + \sin 90^\circ \sin 60^\circ \\ &= 0 \cdot \frac{1}{2} + 1 \cdot \frac{\sqrt{3}}{2} \\ &= 0 + \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

2.
$$\begin{aligned}&\cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ \\ &= \cos(70 - 40^\circ) \\ &= \cos 30^\circ \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

3. $\dfrac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta}=\dfrac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$

$=\dfrac{\cos\alpha}{\cos\alpha}\cdot\dfrac{\cos\beta}{\cos\beta}+\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}$

$=1\cdot1+\tan\alpha\cdot\tan\beta$

$=1+\tan\alpha\tan\beta$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

4. $\sin\dfrac{5\pi}{12}=\sin\left(\dfrac{\pi}{6}+\dfrac{\pi}{4}\right)$

$=\sin\dfrac{\pi}{6}\cos\dfrac{\pi}{4}+\cos\dfrac{\pi}{6}\sin\dfrac{\pi}{4}$

$=\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}$

$=\dfrac{\sqrt{2}}{4}+\dfrac{\sqrt{6}}{4}$

$=\dfrac{\sqrt{2}+\sqrt{6}}{4}$

5. **a.** $\sin\alpha=\dfrac{4}{5}=\dfrac{y}{r}$

Find *x:*

$x^2+y^2=r^2$

$x^2+4^2=5^2$

$x^2+16=25$

$x^2=9$

Because α is in Quadrant II, *x* is negative.

$x=-\sqrt{9}=-3$

$\cos\alpha=\dfrac{x}{r}=\dfrac{-3}{5}=-\dfrac{3}{5}$

b. $\sin\beta=\dfrac{1}{2}=\dfrac{y}{r}$

Find *x:*

$x^2+y^2=r^2$

$x^2+1^2=2^2$

$x^2+1=4$

$x^2=3$

Because β is in Quadrant I, *x* is positive.

$x=\sqrt{3}$

$\cos\beta=\dfrac{x}{r}=\dfrac{\sqrt{3}}{2}$

c. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=-\dfrac{3}{5}\cdot\dfrac{\sqrt{3}}{2}-\dfrac{4}{5}\cdot\dfrac{1}{2}$

$=\dfrac{-3\sqrt{3}}{10}-\dfrac{4}{10}$

$=\dfrac{-3\sqrt{3}-4}{10}$

d. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\dfrac{4}{5}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{-3}{5}\cdot\dfrac{1}{2}$

$=\dfrac{4\sqrt{3}}{10}+\dfrac{-3}{10}$

$=\dfrac{4\sqrt{3}-3}{10}$

6. **a.** The graph appears to be the sine curve, $y=\sin x$.

It cycles through intercept, maximum, intercept, minimum and back to intercept. Thus, $y=\sin x$ also describes the graph.

b. $\cos\left(x+\dfrac{3\pi}{2}\right)=\cos x\cos\dfrac{3\pi}{2}-\sin x\sin\dfrac{3\pi}{2}$

$=\cos x\cdot0-\sin x\cdot(-1)$

$=\sin x$

This verifies our observation that $y=\cos\left(x+\dfrac{3\pi}{2}\right)$ and $y=\sin x$ describe the same graph.

7. $\tan(x+\pi)=\dfrac{\tan x+\tan\pi}{1-\tan x\tan\pi}$

$=\dfrac{\tan x+0}{1-\tan x\cdot0}$

$=\dfrac{\tan x}{1}$

$=\tan x$

Concept and Vocabulary Check 5.2

1. $\cos x\cos y-\sin x\sin y$

2. $\cos x\cos y+\sin x\sin y$

3. $\sin C\cos D+\cos C\sin D$

4. $\sin C\cos D-\cos C\sin D$

5. $\dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

6. $\dfrac{\tan\theta-\tan\phi}{1+\tan\theta\tan\phi}$

7. false

8. false

Exercise Set 5.2

1. $\cos(45°-30°)=\cos 45°\cos 30°+\sin 45°\sin 30°$
$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$
$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$
$=\frac{\sqrt{6}+\sqrt{2}}{4}$

2. $\cos(120^{\circ}-45^{\circ})$
$=\cos 120^{\circ}\cos 45^{\circ}+\sin 120^{\circ}\sin 45^{\circ}$
$=-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$
$=-\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$
$=\frac{-\sqrt{2}+\sqrt{6}}{4}$

3. $\cos\left(\frac{3\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{3\pi}{4}\cos\frac{\pi}{6}+\sin\frac{3\pi}{4}\sin\frac{\pi}{6}$
$=-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$
$=-\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$
$=\frac{\sqrt{2}-\sqrt{6}}{4}$

4. $\cos\left(\frac{2\pi}{3}-\frac{\pi}{6}\right)=\cos\frac{2\pi}{3}\cos\frac{\pi}{6}+\sin\frac{2\pi}{3}\sin\frac{\pi}{6}$
$=-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\frac{1}{2}$
$=\frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4}$
$=0$

5. **a.** $\cos 50°\cos 20°+\sin 50°\sin 20°$
$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
Thus, $\alpha=50°$ and $\beta=20°$.

b. $\cos 50°\cos 20°+\sin 50°\sin 20°$
$=\cos(50°-20°)$
$=\cos 30°$

c. $\cos 30°=\frac{\sqrt{3}}{2}$

6. **a.** $\cos 50^{\circ}\cos 5^{\circ}+\sin 50^{\circ}\sin 5^{\circ}$
$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
Thus, $\alpha=50^{\circ}$ and $\beta=5^{\circ}$

b. $\cos 50^{\circ}\cos 5^{\circ}+\sin 50^{\circ}\sin 5^{\circ}$
$=\cos(50^{\circ}-5^{\circ})$
$=\cos 45^{\circ}$

c. $\cos 45^{\circ}=\frac{\sqrt{2}}{2}$

7. **a.** $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$
$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
Thus, $\alpha=\frac{5\pi}{12}$ and $\beta=\frac{\pi}{12}$.

b. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$
$=\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$
$=\cos\frac{4\pi}{12}$
$=\cos\frac{\pi}{3}$

c. $\cos\frac{\pi}{3}=\frac{1}{2}$

8. **a.** $\cos\frac{5\pi}{18}\cos\frac{\pi}{9}+\sin\frac{5\pi}{18}\sin\frac{\pi}{9}$
$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$\alpha=\frac{5\pi}{18}$ and $\beta=\frac{\pi}{9}$

b. $\cos\frac{5\pi}{18}\cos\frac{\pi}{9}+\sin\frac{5\pi}{18}\sin\frac{\pi}{9}$
$=\cos\left(\frac{5\pi}{18}-\frac{\pi}{9}\right)$
$=\cos\frac{3\pi}{18}$
$=\cos\frac{\pi}{6}$

c. $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

9. $$\frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta}=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\sin\beta}$$
$$=\frac{\cos\alpha}{\cos\alpha}\cdot\frac{\cos\beta}{\sin\beta}-\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\sin\beta}$$
$$=1\cdot\cot\beta+\tan\alpha\cdot 1$$
$$=\tan\alpha+\cot\beta$$

10. $$\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta}=\frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$
$$=\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}+\frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$
$$=\frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\beta}{\sin\beta}+1$$
$$=\cot\alpha\cot\beta+1$$

11. $$\cos\left(x-\frac{\pi}{4}\right)=\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}$$
$$=\cos x\cdot\frac{\sqrt{2}}{2}+\sin x\cdot\frac{\sqrt{2}}{2}$$
$$=\frac{\sqrt{2}}{2}(\cos x+\sin x)$$

12. $$\cos\left(x-\frac{5\pi}{4}\right)=\cos x\cos\frac{5\pi}{4}+\sin x\sin\frac{5\pi}{4}$$
$$=\cos x\cdot-\frac{\sqrt{2}}{2}+\sin x\cdot-\frac{\sqrt{2}}{2}$$
$$=-\frac{\sqrt{2}}{2}(\cos x+\sin x)$$

13. $$\sin(45°-30°)=\sin 45°\cos 30°-\cos 45°\sin 30°$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$
$$=\frac{\sqrt{6}-\sqrt{2}}{4}$$

14. $$\sin(60^{\circ}-45^{\circ})$$
$$=\sin 60^{\circ}\cos 45^{\circ}-\cos 60^{\circ}\sin 45^{\circ}$$
$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$
$$=\frac{\sqrt{6}-\sqrt{2}}{4}$$

15.
$$\begin{aligned}\sin(105^\circ) &= \sin(60^\circ + 45^\circ)\\ &= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

16.
$$\begin{aligned}&\sin 75^\circ = \sin(30^\circ + 45^\circ)\\ &= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ\\ &= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\\ &= \frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

17.
$$\begin{aligned}\cos(135^\circ + 30^\circ) &= \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ\\ &= \cos(90^\circ + 45^\circ)\cos 30^\circ - \sin(90^\circ + 45^\circ)\sin 30^\circ\\ &= (\cos 90^\circ \cos 45^\circ - \sin 90^\circ \sin 45^\circ)\cos 30^\circ - (\sin 90^\circ \cos 45^\circ + \cos 90^\circ \sin 45^\circ)\sin 30^\circ\\ &= \left(0\cdot\frac{\sqrt{2}}{2} - 1\cdot\frac{\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} - \left(1\cdot\frac{\sqrt{2}}{2} + 0\cdot\frac{\sqrt{2}}{2}\right)\frac{1}{2}\\ &= \left(-\frac{\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} - \left(\frac{\sqrt{2}}{2}\right)\frac{1}{2}\\ &= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\ &= -\frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

18.
$$\begin{aligned}\cos(240^\circ + 45^\circ) &= \cos 240^\circ \cos 45^\circ - \sin 240^\circ \sin 45^\circ\\ &= \cos(180^\circ + 60^\circ)\cos 45^\circ - \sin(180^\circ + 60^\circ)\sin 45^\circ\\ &= (\cos 180^\circ \cos 60^\circ - \sin 180^\circ \sin 60^\circ)\cos 45^\circ - (\sin 180^\circ \cos 60^\circ + \cos 180^\circ \sin 60^\circ)\sin 45^\circ\\ &= \left(-1\cdot\frac{1}{2} - 0\cdot\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2} - \left(0\cdot\frac{1}{2} + (-1)\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}\\ &= \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} + \left(\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}\\ &= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

19. $\cos 75° = \cos(45° + 30°)$
$= \cos 45° \cos 30° - \sin 45° \sin 30°$
$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

20. $\cos 105° = \cos(45° + 60°)$
$= \cos 45° \cos 60° - \sin 45° \sin 60°$
$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$
$= \frac{\sqrt{2} - \sqrt{6}}{4}$

21. $\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$
$= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \cdot 1}$
$= \frac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}}$
$= \frac{\sqrt{3} + 3}{3 - \sqrt{3}}$
$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$
$= \frac{9 + 6\sqrt{3} + 3}{9 - 3}$
$= \frac{12 + 6\sqrt{3}}{6}$
$= 2 + \sqrt{3}$

22. $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$
$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1}$
$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$
$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$
$= \frac{1 + 2\sqrt{3} + 3}{1 - 3}$
$= \frac{4 + 2\sqrt{3}}{-2}$
$= -2 - \sqrt{3}$

23. $\tan\left(\frac{4\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{4\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{4\pi}{3}\tan\frac{\pi}{4}}$
$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$
$= \frac{-1 + \sqrt{3}}{1 + \sqrt{3}}$
$= \frac{-1 + \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$
$= \frac{-1 + 2\sqrt{3} - 3}{1 - 3}$
$= \frac{-4 + 2\sqrt{3}}{-2}$
$= 2 - \sqrt{3}$

24. $\tan\left(\frac{5\pi}{3}-\frac{\pi}{4}\right)=\dfrac{\tan\frac{5\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$=\dfrac{\tan\frac{5\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$=\dfrac{-\sqrt{3}-1}{1+\left(-\sqrt{3}\right)\cdot 1}$

$=\dfrac{-1-\sqrt{3}}{1-\sqrt{3}}$

$=\dfrac{-1-\sqrt{3}}{1-\sqrt{3}}\cdot\dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

$=\dfrac{-1-2\sqrt{3}-3}{1-3}$

$=\dfrac{-4-2\sqrt{3}}{-2}$

$=2+\sqrt{3}$

25. $\sin 25°\cos 5°+\cos 25°\sin 5°=\sin(25°+5°)$

$=\sin 30°$

$=\frac{1}{2}$

26. $\sin 40^{o}\cos 20^{o}+\cos 40^{o}\sin 20^{o}$

$=\sin(40^{o}+20^{o})$

$=\sin 60^{o}$

$=\frac{\sqrt{3}}{2}$

27. $\dfrac{\tan 10°+\tan 35°}{1-\tan 10°\tan 35°}=\tan(10°+35°)$

$=\tan 45°$

$=1$

28. $\dfrac{\tan 50^{o}-\tan 20^{o}}{1+\tan 50^{o}\tan 20^{o}}=\tan(50^{o}-20^{o})$

$=\tan 30^{o}$

$=\frac{\sqrt{3}}{3}$

29. $\sin\frac{5\pi}{12}\cos\frac{\pi}{4}-\cos\frac{5\pi}{12}\sin\frac{\pi}{4}=\sin\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)$

$=\sin\left(\frac{2\pi}{12}\right)$

$=\sin\left(\frac{\pi}{6}\right)$

$=\frac{1}{2}$

30. $\sin\frac{7\pi}{12}\cos\frac{\pi}{12}-\cos\frac{7\pi}{12}\sin\frac{\pi}{12}=\sin\left(\frac{7\pi}{12}-\frac{\pi}{12}\right)$

$=\sin\frac{6\pi}{12}$

$=\sin\frac{\pi}{2}$

$=1$

31. $\dfrac{\tan\frac{\pi}{5}-\tan\frac{\pi}{30}}{1+\tan\frac{\pi}{5}\tan\frac{\pi}{30}}=\tan\left(\frac{\pi}{5}-\frac{\pi}{30}\right)$

$=\tan\left(\frac{5\pi}{30}\right)=\tan\left(\frac{\pi}{6}\right)$

$=\frac{\sqrt{3}}{3}$

32. $\dfrac{\tan\frac{\pi}{5}+\tan\frac{4\pi}{5}}{1-\tan\frac{\pi}{5}\tan\frac{4\pi}{5}}=\tan\left(\frac{\pi}{5}+\frac{4\pi}{5}\right)$

$=\tan\frac{5\pi}{5}$

$=\tan\pi$

$=0$

33. $\sin\left(x+\frac{\pi}{2}\right)=\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}$

$=\sin x\cdot 0+\cos x\cdot 1$

$=\cos x$

34. $\sin\left(x+\frac{3\pi}{2}\right)=\sin x\cos\frac{3\pi}{2}+\cos x\sin\frac{3\pi}{2}$

$=\sin x\cdot 0+\cos x\cdot(-1)$

$=-\cos x$

35. $\cos\left(x-\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$

$=\cos x\cdot 0+\sin x\cdot 1$

$=\sin x$

36. $\cos(\pi-x)=\cos\pi\cos x+\sin\pi\sin x$

$=-1\cdot\cos x+0\cdot\sin x$

$=-\cos x$

37. $\tan(2\pi-x)=\dfrac{\tan 2\pi-\tan x}{1+\tan 2\pi\tan x}$

$=\dfrac{0-\tan x}{1+0\cdot\tan x}$

$=-\tan x$

38. $\tan(\pi-x)=\dfrac{\tan\pi-\tan x}{1+\tan\pi\tan x}$

$=\dfrac{0-\tan x}{1+0\cdot\tan x}$

$=-\tan x$

39. $\sin(\alpha+\beta)+\sin(\alpha-\beta)$
$= \sin\alpha\cos\beta+\cos\alpha\sin\beta$
$+\sin\alpha\cos\beta-\cos\alpha\sin\beta$
$= 2\sin\alpha\cos\beta$

40. $\cos(\alpha+\beta)+\cos(\alpha-\beta)$
$= \cos\alpha\cos\beta-\sin\alpha\sin\beta$
$+\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$= 2\cos\alpha\cos\beta$

41. $\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}=\dfrac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$=\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$=\tan\alpha\cdot 1-1\cdot\tan\beta$
$=\tan\alpha-\tan\beta$

42. $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$=\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$=\tan\alpha+\tan\beta$

43. $\tan\left(\theta+\dfrac{\pi}{4}\right)=\dfrac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$
$=\dfrac{\tan\theta+1}{1-\tan\theta}$
$=\dfrac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$
$=\dfrac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$
$=\dfrac{\sin\theta+\cos\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\cos\theta-\sin\theta}$
$=\dfrac{\sin\theta+\cos\theta}{\cos\theta-\sin\theta}$
$=\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

44. $\tan\left(\dfrac{\pi}{4}-\theta\right)=\dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$
$=\dfrac{1-\tan\theta}{1+1\cdot\tan\theta}$
$=\dfrac{1-\tan\theta}{1+\tan\theta}$
$=\dfrac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$
$=\dfrac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$
$=\dfrac{\cos\theta-\sin\theta}{\cos\theta}\div\dfrac{\cos\theta+\sin\theta}{\cos\theta}$
$=\dfrac{\cos\theta-\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\cos\theta+\sin\theta}$
$=\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

45. $\cos(\alpha+\beta)\cos(\alpha-\beta)$
$=(\cos\alpha\cos\beta-\sin\alpha\sin\beta)$
$\cdot(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$
$=\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta$
$=\left(1-\sin^2\alpha\right)\cos^2\beta-\sin^2\alpha\left(1-\cos^2\beta\right)$
$=\cos^2\beta-\sin^2\alpha\cos^2\beta$
$-\sin^2\alpha+\sin^2\alpha\cos^2\beta$
$=\cos^2\beta-\sin^2\alpha$

46. $\sin\left(a+\beta\right)\sin\left(\alpha-\beta\right)$
$=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)$
$\cdot(\sin\alpha\cos\beta-\cos\alpha\sin\beta)$
$=\sin^2\alpha\cos^2\beta-\cos^2\alpha\sin^2\beta$
$=(1-\cos^2\alpha)\cos^2\beta$
$-\cos^2\alpha(1-\cos^2\beta)$
$=\cos^2\beta-\cos^2\alpha\cos^2\beta$
$-\cos^2\alpha+\cos^2\alpha\cos^2\beta$
$=\cos^2\beta-\cos^2\alpha$

47. $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$
$=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}$
$=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}\cdot\dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$
$=\dfrac{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$=\dfrac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$=\dfrac{\tan\alpha\cdot 1+1\cdot\tan\beta}{\tan\alpha\cdot 1-1\cdot\tan\beta}$
$=\dfrac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

48. $\dfrac{\cos(\alpha+\beta)}{\cos\left(\alpha-\beta\right)}$
$=\dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$
$=\dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}\cdot\dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$
$=\dfrac{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$=\dfrac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$

49. $\dfrac{\cos(x+h)-\cos x}{h}$
$=\dfrac{\cos x\cos h-\sin x\sin h-\cos x}{h}$
$=\dfrac{\cos x\cos h-\cos x-\sin x\sin h}{h}$
$=\dfrac{\cos x(\cos h-1)-\sin x\sin h}{h}$
$=\cos x\cdot\dfrac{\cos h-1}{h}-\sin x\cdot\dfrac{\sin h}{h}$

50. $\dfrac{\sin(x+h)-\sin x}{h}$
$=\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$
$=\dfrac{\sin x\cos h-\sin x+\cos x\sin h}{h}$
$=\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$
$=\sin x\dfrac{\cos h-1}{h}+\cos x\dfrac{\sin h}{h}$

51. $\sin 2\alpha=\sin(\alpha+\alpha)$
$=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$
$=2\sin\alpha\cos\alpha$

52. $\cos 2\alpha=\cos(\alpha+\alpha)$
$=\cos\alpha\cos\alpha+\sin\alpha\sin\alpha$
$=\cos^2\alpha+\sin^2\alpha$

53. $\tan 2\alpha=\tan(\alpha+\alpha)$
$=\dfrac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$
$=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

54. $\tan\left(\dfrac{\pi}{4}+\alpha\right)-\tan\left(\dfrac{\pi}{4}-\alpha\right)$
$=\dfrac{\tan\frac{\pi}{4}+\tan\alpha}{1-\tan\frac{\pi}{4}\tan\alpha}-\dfrac{\tan\frac{\pi}{4}-\tan\alpha}{1+\tan\frac{\pi}{4}\tan\alpha}$
$=\dfrac{1+\tan\alpha}{1-\tan\alpha}-\dfrac{1-\tan\alpha}{1+\tan\alpha}$
$=\dfrac{(1+\tan\alpha)(1+\tan\alpha)}{(1-\tan\alpha)(1+\tan\alpha)}-\dfrac{(1-\tan\alpha)(1-\tan\alpha)}{(1+\tan\alpha)(1-\tan\alpha)}$
$=\dfrac{1+2\tan\alpha+\tan^2\alpha-(1-2\tan\alpha+\tan^2\alpha)}{1-\tan^2\alpha}$
$=\dfrac{4\tan\alpha}{1-\tan^2\alpha}$
$=\dfrac{2(2\tan\alpha)}{1-\tan^2\alpha}$
$=2\dfrac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$
$=2\tan(\alpha+\alpha)$
$=2\tan 2\alpha$

55. $\tan(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

$$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}\cdot\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$$

$$=\frac{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$=\frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

56. $\tan(\alpha-\beta)=\tan(\alpha+(-\beta))$

$$=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

57. $\sin\alpha=\dfrac{3}{5}=\dfrac{y}{r}$

$x^2+y^2=r^2$
$x^2+3^2=5^2$
$x^2+9=25$
$x^2=16$

Because α lies in quadrant I, x is positive.
$x=4$

Thus, $\cos\alpha=\dfrac{x}{r}=\dfrac{4}{5}$, and

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{3}{5}}{\frac{4}{5}}=\frac{3}{4}.$$

$\sin\beta=\dfrac{5}{13}=\dfrac{y}{r}$

$x^2+y^2=r^2$
$x^2+5^2=13^2$
$x^2+25=169$
$x^2=144$

Because β lies in quadrant II, x is negative.
$x=-12$

Thus, $\cos\beta=\dfrac{x}{r}=\dfrac{-12}{13}=-\dfrac{12}{13}$, and

$$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\frac{5}{13}}{-\frac{12}{13}}=-\frac{5}{12}.$$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\frac{4}{5}\cdot\left(-\frac{12}{13}\right)-\frac{3}{5}\cdot\frac{5}{13}=-\frac{63}{65}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\frac{3}{5}\cdot\left(-\frac{12}{13}\right)+\frac{4}{5}\cdot\frac{5}{13}=-\frac{16}{65}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{3}{4}+\left(-\frac{5}{12}\right)}{1-\frac{3}{4}\cdot\left(-\frac{5}{12}\right)}=\frac{\frac{4}{12}}{\frac{63}{48}}=\frac{16}{63}$$

58. $\sin\alpha=\dfrac{4}{5}=\dfrac{y}{r}$

$x^2+y^2=r^2$
$x^2+4^2=5^2$
$x^2+16=25$
$x^2=9$

Because α lies in quadrant I, x is positive.
$x=3$

$\cos\alpha=\dfrac{x}{r}=\dfrac{3}{5}$

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{3}$$

$\sin\beta=\dfrac{7}{25}=\dfrac{y}{r}$

$x^2+y^2=r^2$
$x^2+7^2=25^2$
$x^2+49=625$
$x^2=576$

Because β lies in quadrant II, x is negative.
$x=-24$

$\cos\beta=\dfrac{x}{r}=\dfrac{-24}{25}$

$$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\frac{7}{25}}{\frac{-24}{25}}=-\frac{7}{24}$$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\frac{3}{5}\cdot\left(-\frac{24}{25}\right)-\frac{4}{5}\cdot\frac{7}{25}=-\frac{4}{5}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\frac{4}{5}\cdot\left(-\frac{24}{25}\right)+\frac{3}{5}\cdot\frac{4}{25}=-\frac{3}{5}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{4}{3}-\frac{7}{24}}{1-\frac{4}{3}\cdot\left(-\frac{7}{24}\right)}=\frac{\frac{25}{24}}{\frac{25}{18}}=\frac{3}{4}$$

59. $\tan\alpha=-\dfrac{3}{4}=\dfrac{3}{-4}=\dfrac{y}{x}$

$$x^2+y^2=r^2$$
$$(-4)^2+3^2=r^2$$
$$16+9=r^2$$
$$25=r^2$$

Because r is a distance, it is positive.

$$r=5$$

Thus, $\cos\alpha=\dfrac{x}{r}=\dfrac{-4}{5}=-\dfrac{4}{5}$, and

$$\sin\alpha=\frac{y}{r}=\frac{3}{5}.$$

$$\cos\beta=\frac{1}{3}=\frac{x}{r}$$

$$x^2+y^2=r^2$$
$$1^2+y^2=3^2$$
$$1+y^2=9$$
$$y^2=8$$

Because β lies in quadrant I, y is positive.

$$y=\sqrt{8}=2\sqrt{2}$$

Thus, $\sin\beta=\dfrac{y}{r}=\dfrac{2\sqrt{2}}{3}$, and

$$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}}=2\sqrt{2}.$$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\left(-\frac{4}{5}\right)\cdot\frac{1}{3}-\frac{3}{5}\cdot\frac{2\sqrt{2}}{3}$$
$$=-\frac{4}{15}-\frac{6\sqrt{2}}{15}$$
$$=\frac{-4-6\sqrt{2}}{15}$$
$$=-\frac{4+6\sqrt{2}}{15}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\frac{3}{5}\cdot\frac{1}{3}+\left(-\frac{4}{5}\right)\cdot\frac{2\sqrt{2}}{3}$$
$$=\frac{3}{15}-\frac{8\sqrt{2}}{15}$$
$$=\frac{3-8\sqrt{2}}{15}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{-\frac{3}{4}+2\sqrt{2}}{1-\left(-\frac{3}{4}\right)\left(2\sqrt{2}\right)}$$
$$=\frac{\frac{-3+8\sqrt{2}}{4}}{\frac{4+6\sqrt{2}}{4}}$$
$$=\frac{-3+8\sqrt{2}}{4+6\sqrt{2}}\cdot\frac{\left(4-6\sqrt{2}\right)}{\left(4-6\sqrt{2}\right)}$$
$$=\frac{-108+50\sqrt{2}}{-56}$$
$$=\frac{54-25\sqrt{2}}{28}$$

60. $\tan\alpha=\dfrac{-4}{3}=\dfrac{4}{-3}=\dfrac{y}{x}$

$$x^2+y^2=r^2$$
$$(-3)^2+4^2=r^2$$
$$9+16=r^2$$
$$25=r^2$$

Because r is a distance, it is positive.

$$r=5$$

$$\cos\alpha=\frac{x}{r}=\frac{-3}{5}$$
$$\sin\alpha=\frac{y}{r}=\frac{4}{5}$$

$$\cos\beta=\frac{2}{3}=\frac{x}{r}$$

$$x^2+y^2=r^2$$
$$2^2+y^2=3^2$$
$$4+y^2=9$$
$$y^2=5$$

Because β lies in quadrant I, y is positive.

$$\sin\beta=\frac{y}{r}=\frac{\sqrt{5}}{3}$$

$$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}=\frac{\sqrt{5}}{2}$$

$$y=\sqrt{5}$$

a. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\frac{3}{5}\cdot\frac{2}{3} - \frac{4}{5}\cdot\frac{\sqrt{5}}{3} = \frac{-6-4\sqrt{5}}{15}$

b. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{4}{5}\cdot\frac{2}{3} + \left(-\frac{3}{5}\right)\cdot\frac{\sqrt{5}}{3} = \frac{8-3\sqrt{5}}{15}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \frac{-\frac{4}{3}+\frac{\sqrt{5}}{2}}{1-\left(-\frac{4}{3}\right)\cdot\frac{\sqrt{5}}{2}}$

$= \frac{\frac{-8+3\sqrt{5}}{6}}{\frac{6+4\sqrt{5}}{6}}$

$= \frac{-8+3\sqrt{5}}{6+4\sqrt{5}}$

$= \frac{-8+3\sqrt{5}}{6+4\sqrt{5}}\cdot\frac{6-4\sqrt{5}}{6-+4\sqrt{5}}$

$= \frac{-108+50\sqrt{5}}{-44}$

$= \frac{54-25\sqrt{5}}{22}$

61. $\cos\alpha = \frac{8}{17} = \frac{x}{r}$

$x^2 + y^2 = r^2$
$8^2 + y^2 = 17^2$
$64 + y^2 = 289$
$y^2 = 225$

Because α lies in quadrant IV, y is negative.
$y = -15$

Thus, $\sin\alpha = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$, and

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{15}{17}}{\frac{8}{17}} = -\frac{15}{8}.$

$\sin\beta = -\frac{1}{2} = \frac{-1}{2} = \frac{y}{r}$

$x^2 + y^2 = r^2$
$x^2 + (-1)^2 = 2^2$
$x^2 + 1 = 4$
$x^2 = 3$

Because β lies in quadrant III, x is negative.
$x = -\sqrt{3}$

Thus, $\cos\beta = \frac{x}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$, and

$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$

a. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{8}{17}\cdot\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{15}{17}\right)\cdot\left(-\frac{1}{2}\right)$

$= \frac{-8\sqrt{3}-15}{34}$

$= -\frac{8\sqrt{3}+15}{34}$

b. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(-\frac{15}{17}\right)\cdot\left(-\frac{\sqrt{3}}{2}\right) + \frac{8}{17}\cdot\left(-\frac{1}{2}\right)$

$= \frac{15\sqrt{3}-8}{34}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\frac{-\frac{15}{8}+\frac{\sqrt{3}}{3}}{1-\left(-\frac{15}{8}\right)\left(\frac{\sqrt{3}}{3}\right)}$

$= \frac{\frac{-45+8\sqrt{3}}{24}}{\frac{24+15\sqrt{3}}{24}}$

$= \frac{-45+8\sqrt{3}}{24+15\sqrt{3}}\cdot\frac{24-15\sqrt{3}}{24-15\sqrt{3}}$

$= \frac{-1440+867\sqrt{3}}{-99}$

$= \frac{489-289\sqrt{3}}{33}$

62. $\cos\alpha = \frac{1}{2} = \frac{x}{r}$

$x^2 + y^2 = r^2$
$1^2 + y^2 = 2^2$
$1 + y^2 = 4$
$y^2 = 3$

Because α lies in quadrant IV, y is negative.

$y = -\sqrt{3}$

$\sin\alpha = \frac{y}{r} = \frac{\sqrt{3}}{2}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$\sin\beta = -\frac{1}{3} = \frac{-1}{3} = \frac{y}{r}$

$x^2 + y^2 = r^2$
$x^2 + (-1)^2 = 3^2$
$x^2 + 1 = 9$
$x^2 = 8$

Because β lies in quadrant III, x is negative.

$x = -\sqrt{8} = -2\sqrt{2}$

$\cos\beta = \frac{x}{r} = \frac{-2\sqrt{2}}{3}$

$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

a. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{1}{2}\cdot\left(-\frac{2\sqrt{2}}{3}\right) - \left(\frac{-\sqrt{3}}{2}\right)\left(-\frac{1}{3}\right)$

$= \frac{-2\sqrt{6}-\sqrt{3}}{6}$

b. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{2\sqrt{2}}{3}\right) + \frac{1}{2}\left(-\frac{1}{3}\right)$

$= \frac{2\sqrt{6}-1}{6}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{-\sqrt{3} + \frac{\sqrt{2}}{4}}{1 - \left(-\sqrt{3}\right)\cdot\frac{\sqrt{2}}{4}}$

$= \frac{\frac{-4\sqrt{3}+\sqrt{2}}{4}}{\frac{4+\sqrt{6}}{4}}$

$= \frac{-4\sqrt{3}+\sqrt{2}}{4+\sqrt{6}}$

$= \frac{-4\sqrt{3}+\sqrt{2}}{4+\sqrt{6}}\cdot\frac{4-\sqrt{6}}{4+\sqrt{6}}$

$= \frac{-16\sqrt{3}+4\sqrt{18}+4\sqrt{2}-\sqrt{12}}{10}$

$= \frac{-16\sqrt{3}+12\sqrt{2}+4\sqrt{2}-2\sqrt{3}}{10}$

$= \frac{-18\sqrt{3}+16\sqrt{2}}{10}$

$= \frac{8\sqrt{2}-9\sqrt{3}}{5}$

63. $\tan\alpha = \frac{3}{4} = \frac{y}{x}$

Because α lies in quadrant III, x and y are negative.

$r^2 = x^2 + y^2$
$r^2 = (-4)^2 + (-3)^2$
$r^2 = 25$
$r = 5$

$\sin\alpha = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\cos\alpha = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$

$\cos\beta = \frac{1}{4} = \frac{x}{r}$

Because β lies in quadrant IV, y is negative.

$x^2 + y^2 = r^2$
$1^2 + y^2 = 4^2$
$y^2 = 15$
$y = -\sqrt{15}$

$\sin\beta = \frac{y}{r} = \frac{-\sqrt{15}}{4} = -\frac{\sqrt{15}}{4}$

$\tan\beta = \frac{y}{x} = \frac{-\sqrt{15}}{1} = -\sqrt{15}$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=-\frac{4}{5}\cdot\left(\frac{1}{4}\right)-\left(-\frac{3}{5}\right)\left(-\frac{\sqrt{15}}{4}\right)$$
$$=-\frac{4}{20}-\frac{3\sqrt{15}}{20}$$
$$=-\frac{4+3\sqrt{15}}{20}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\left(-\frac{3}{5}\right)\left(\frac{1}{4}\right)+\left(-\frac{4}{5}\right)\left(-\frac{\sqrt{15}}{4}\right)$$
$$=-\frac{3}{20}+\frac{4\sqrt{15}}{20}$$
$$=\frac{-3+4\sqrt{15}}{20}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{3}{4}+\left(-\sqrt{15}\right)}{1-\frac{3}{4}\left(-\sqrt{15}\right)}$$
$$=\frac{\frac{3}{4}-\frac{4\sqrt{15}}{4}}{\frac{4}{4}+\frac{3\sqrt{15}}{4}}$$
$$=\frac{3-4\sqrt{15}}{4+3\sqrt{15}}$$
$$=\frac{3-4\sqrt{15}}{4+3\sqrt{15}}\cdot\frac{4-3\sqrt{15}}{4-3\sqrt{15}}$$
$$=\frac{12-9\sqrt{15}-16\sqrt{15}+180}{16-135}$$
$$=\frac{192-25\sqrt{15}}{-119}$$
$$=\frac{-192+25\sqrt{15}}{119}$$

64. $\sin\alpha=\dfrac{5}{6}=\dfrac{y}{r}$

Because α lies in quadrant II, x is negative.

$$x^2+y^2=r^2$$
$$x^2+5^2=6^2$$
$$x^2=11$$
$$x=-\sqrt{11}$$

$$\cos\alpha=\frac{x}{r}=\frac{-\sqrt{11}}{6}=-\frac{\sqrt{11}}{6}$$
$$\tan\alpha=\frac{y}{x}=\frac{5}{-\sqrt{11}}=-\frac{5\sqrt{11}}{11}$$

$$\tan\beta=\frac{3}{7}=\frac{y}{x}$$

Because β lies in quadrant III, x and y are both negative.

$$r^2=x^2+y^2$$
$$r^2=7^2+3^2$$
$$r^2=58$$
$$r=\sqrt{58}$$

$$\sin\beta=\frac{y}{r}=\frac{-3}{\sqrt{58}}=-\frac{3\sqrt{58}}{58}$$
$$\cos\beta=\frac{x}{r}=\frac{-7}{\sqrt{58}}=-\frac{7\sqrt{58}}{58}$$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=-\frac{\sqrt{11}}{6}\cdot\left(-\frac{7\sqrt{58}}{58}\right)-\left(\frac{5}{6}\right)\left(-\frac{3\sqrt{58}}{58}\right)$$
$$=\frac{7\sqrt{638}}{348}+\frac{15\sqrt{58}}{348}$$
$$=\frac{7\sqrt{638}+15\sqrt{58}}{348}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$=\left(\frac{5}{6}\right)\left(-\frac{7\sqrt{58}}{58}\right)+\left(-\frac{\sqrt{11}}{6}\right)\left(-\frac{3\sqrt{58}}{58}\right)$$
$$=-\frac{35\sqrt{58}}{348}+\frac{3\sqrt{638}}{348}$$
$$=\frac{3\sqrt{638}-35\sqrt{58}}{348}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{-\frac{5\sqrt{11}}{11}+\frac{3}{7}}{1-\left(-\frac{5\sqrt{11}}{11}\right)\left(\frac{3}{7}\right)}$$
$$=\frac{-\frac{35\sqrt{11}}{77}+\frac{33}{77}}{\frac{77}{77}+\frac{15\sqrt{11}}{77}}$$
$$=\frac{-35\sqrt{11}+33}{77+15\sqrt{11}}$$
$$=\frac{33-35\sqrt{11}}{77+15\sqrt{11}}\cdot\frac{77-15\sqrt{11}}{77-15\sqrt{11}}$$
$$=\frac{2541-495\sqrt{11}-2695\sqrt{11}+5775}{5929-2475}$$
$$=\frac{8316-3190\sqrt{11}}{3454}$$
$$=\frac{22\left(378-145\sqrt{11}\right)}{22\cdot157}$$
$$=\frac{378-145\sqrt{11}}{157}$$

65. a. The graph appears to be the sine curve, $y = \sin x$. It cycles through intercept, maximum, minimum and back to intercept. Thus, $y = \sin x$ also describes the graph.

b. $\sin(\pi - x) = \sin\pi\cos x - \cos\pi\sin x$
$= 0\cdot\cos x - (-1)\cdot\sin x$
$= \sin x$

This verifies our observation that $y = \sin(\pi - x)$ and $y = \sin x$ describe the same graph.

66. a. The graph appears to be the cosine curve, $y = \cos x$. It cycles through maximum, intercept, minimum, intercept and back to maximum. Thus, $y = \cos x$ also describes the graph.

b. $\cos(x - 2\pi) = \cos x\cos 2\pi + \sin x\sin 2\pi$
$= \cos x\cdot 1 + \sin x\cdot 0$
$= \cos x$

This verifies our observation that $y = \cos(x - 2\pi)$ and $y = \cos x$ describe the same graph.

67. a. The graph appears to be 2 times the cosine curve, $y = 2\cos x$. It cycles through maximum, intercept, minimum, intercept and back to maximum. Thus $y = 2\cos x$ also describes the graph.

b.
$$\sin\left(x+\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}-x\right)$$
$$=\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}+\sin\frac{\pi}{2}\cos x$$
$$-\cos\frac{\pi}{2}\sin x$$
$$=\sin x\cdot 0+\cos x\cdot 1+1\cdot\cos x-0\cdot\sin x$$
$$=\cos x+\cos x$$
$$=2\cos x$$

This verifies our observation that $y=\sin\left(x+\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}-x\right)$ and $y = 2\cos x$ describe the same graph.

68. a. The graph appears to be 2 times the sine curve, $y = 2\sin x$. It cycles through intercept, maximum, intercept, minimum and back to intercept. Thus, $y = 2\sin x$ also describes the graph.

b.
$$\cos\left(x-\frac{\pi}{2}\right)-\cos\left(x+\frac{\pi}{2}\right)$$
$$=\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$$
$$-\left(\cos x\cos x\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}\right)$$
$$=2\sin x\sin\frac{\pi}{2}$$
$$=2\sin x\cdot 1$$
$$=2\sin x$$

This verifies our observation that $\cos\left(x-\frac{\pi}{2}\right)-\cos\left(x+\frac{\pi}{2}\right)$ and $y = 2\sin x$ describe the same graph.

69. $\cos(\alpha+\beta)\cos\beta+\sin(\alpha+\beta)\sin\beta$
$=\cos\left[(\alpha+\beta)-\beta\right]$
$=\cos\alpha$

70. $\sin(\alpha-\beta)\cos\beta+\cos(\alpha-\beta)\sin\beta$
$=\sin\left[(\alpha-\beta)+\beta\right]$
$=\sin\alpha$

71. $\dfrac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$$=\frac{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)-(\sin\alpha\cos\beta-\cos\alpha\sin\beta)}{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}$$
$$=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta-\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta}$$
$$=\frac{\cos\alpha\sin\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\cos\alpha\cos\beta}$$
$$=\frac{2\cos\alpha\sin\beta}{2\cos\alpha\cos\beta}$$
$$=\frac{\sin\beta}{\cos\beta}$$
$$=\tan\beta$$

72. $\dfrac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{-\sin(\alpha-\beta)+\sin(\alpha+\beta)}$

$$=\frac{(\cos\alpha\cos\beta+\sin\alpha\sin\beta)+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)}{-(\sin\alpha\cos\beta-\cos\alpha\sin\beta)+(\sin\alpha\cos\beta+\cos\alpha\sin\beta)}$$
$$=\frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\alpha\cos\beta-\sin\alpha\sin\beta}{-\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta+\cos\alpha\sin\beta}$$
$$=\frac{\cos\alpha\cos\beta+\cos\alpha\cos\beta}{\cos\alpha\sin\beta+\cos\alpha\sin\beta}$$
$$=\frac{2\cos\alpha\cos\beta}{2\cos\alpha\sin\beta}$$
$$=\frac{\cos\beta}{\sin\beta}$$
$$=\cot\beta$$

73. $\cos\left(\dfrac{\pi}{6}+\alpha\right)\cos\left(\dfrac{\pi}{6}-\alpha\right)-\sin\left(\dfrac{\pi}{6}+\alpha\right)\sin\left(\dfrac{\pi}{6}-\alpha\right)$

$$=\cos\left[\left(\frac{\pi}{6}+\alpha\right)+\left(\frac{\pi}{6}-\alpha\right)\right]$$
$$=\cos\left[\frac{\pi}{6}+\alpha+\frac{\pi}{6}-\alpha\right]$$
$$=\cos\frac{\pi}{3}$$
$$=\frac{1}{2}$$

74. $\sin\left(\frac{\pi}{3}-\alpha\right)\cos\left(\frac{\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right)$

$$= \sin\left[\left(\frac{\pi}{3}+\alpha\right)+\left(\frac{\pi}{3}-\alpha\right)\right]$$
$$= \sin\left[\frac{\pi}{3}+\alpha+\frac{\pi}{3}-\alpha\right]$$
$$= \sin\frac{2\pi}{3}$$
$$= \frac{\sqrt{3}}{2}$$

75. Conjecture: the left side is equal to $\cos 3x$.

$$\cos 2x\cos 5x+\sin 2x\sin 5x$$
$$= \cos(2x-5x)$$
$$= \cos(-3x)$$
$$= \cos 3x$$

76. Conjecture: the left side is equal to $\sin 3x$.

$$\sin 5x\cos 2x-\cos 5x\sin 2x$$
$$= \sin(5x-2x)$$
$$= \sin 3x$$

77. Conjecture: the left side is equal to $\sin\frac{x}{2}$.

$$\sin\frac{5x}{2}\cos 2x-\cos\frac{5x}{2}\sin 2x$$
$$= \sin\left(\frac{5x}{2}-2x\right)$$
$$= \sin\left(\frac{5x}{2}-\frac{4x}{2}\right)$$
$$= \sin\frac{x}{2}$$

78. Conjecture: the left side is equal to $\cos\frac{x}{2}$.

$$\cos\frac{5x}{2}\cos 2x+\sin\frac{5x}{2}\sin 2x$$
$$= \cos\left(\frac{5x}{2}-2x\right)$$
$$= \cos\left(\frac{5x}{2}-\frac{4x}{2}\right)$$
$$= \cos\frac{x}{2}$$

79. $\tan\theta = \dfrac{3}{2} = \dfrac{y}{x}$

$$x^2 + y^2 = r^2$$
$$2^2 + 3^3 = r^2$$
$$4 + 9 = r^2$$
$$13 = r^2$$

Because r is a distance, it is positive.

$$r = \sqrt{13}$$

Thus, $\sin\theta = \dfrac{y}{r} = \dfrac{3}{\sqrt{13}}$

and $\cos\theta = \dfrac{x}{r} = \dfrac{2}{\sqrt{13}}$

$$\begin{aligned}\sqrt{13}\cos(t-\theta) &= \sqrt{13}(\cos t\cos\theta + \sin t\sin\theta)\\ &= \sqrt{13}\left(\cos t\cdot\frac{2}{\sqrt{13}} + \sin t\cdot\frac{3}{\sqrt{13}}\right)\\ &= \cos t\cdot 2 + \sin t\cdot 3\\ &= 2\cos t + 3\sin t\end{aligned}$$

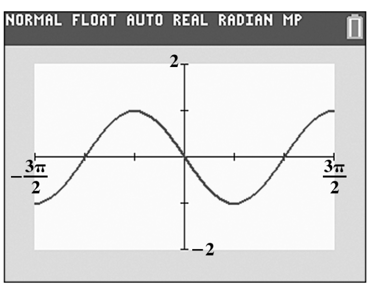
For the equation $y = \sqrt{13}\cos(t-\theta)$, the amplitude is $\left|\sqrt{13}\right| = \sqrt{13}$, and the period is $\dfrac{2\pi}{1} = 2\pi$.

80. **a.** $$\begin{aligned}\rho &= 3\sin 2t + 2\sin(2t+\pi)\\ &= 3\sin 2t + 2(\sin 2t\cos\pi + \cos 2t\cos\pi)\\ &= 3\sin 2t + 2(\sin 2t\cdot(-1) + \cos 2t\cdot 0)\\ &= 3\sin 2t - 2\sin 2t\\ &= \sin 2t\end{aligned}$$

b. No. The amplitude of p is 1.

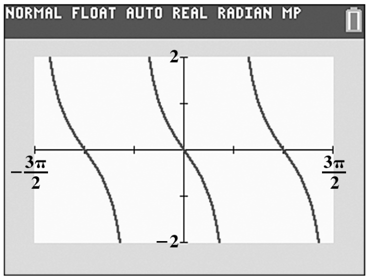
81. – 87. Answers may vary.

88.



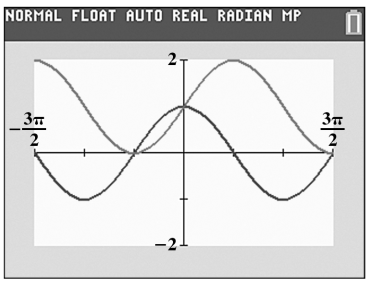
$$\begin{aligned}\cos\left(\frac{3\pi}{2} - x\right) &= \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x\\ &= 0\cdot\cos x + (-1)\sin x\\ &= -\sin x\end{aligned}$$

89.



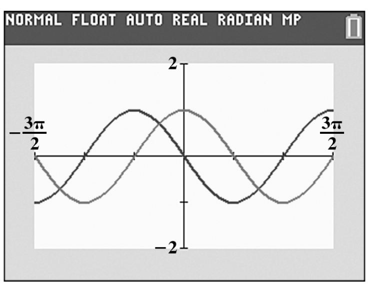
$$\tan(\pi - x) = \frac{\tan\pi - \tan x}{1 + \tan\pi\tan x}$$
$$= \frac{0 - \tan x}{1 + 0\cdot\tan x}$$
$$= \frac{-\tan x}{1}$$
$$= -\tan x$$

90.



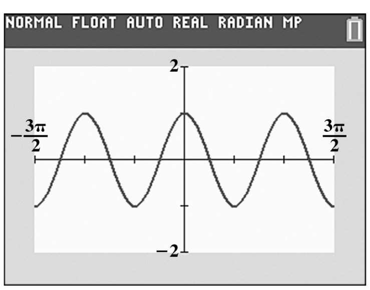
The graphs do not coincide.
Values for x may vary.

91.



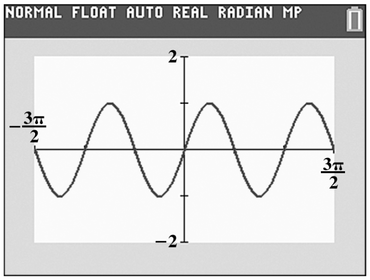
The graphs do not coincide.
Values for x may vary.

92.



$$\cos 1.2x\cos 0.8x - \sin 1.2x\sin 0.8x$$
$$= \cos(1.2 + 0.8) = \cos 2x$$

93.



$\sin 1.2x \cos 0.8x + \cos 1.2x \sin 0.8x$
$\sin(1.2x + 0.8x)$
$\sin 2x$

94. makes sense

95. makes sense

96. does not make sense; Explanations will vary. Sample explanation: The sum and difference formulas allow you to find exact values only for certain angles.

97. makes sense

98.

$$\frac{\sin(x-y)}{\cos x \cos y} + \frac{\sin(y-z)}{\cos y \cos z} + \frac{\sin(z-x)}{\cos z \cos x}$$
$$= \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} + \frac{\sin y \cos z - \cos y \sin z}{\cos y \cos z} + \frac{\sin z \cos x - \cos z \sin x}{\cos z \cos x}$$
$$= \frac{\sin x \cos y}{\cos x \cos y} - \frac{\cos x \sin y}{\cos y \cos y} + \frac{\sin y \cos z}{\cos y \cos z} - \frac{\cos y \sin z}{\cos y \cos z} + \frac{\sin z \cos x}{\cos z \cos x} - \frac{\cos z \sin x}{\cos z \cos x}$$
$$= \frac{\sin x}{\cos x} - \frac{\sin y}{\cos y} + \frac{\sin y}{\cos y} - \frac{\sin z}{\cos z} + \frac{\sin z}{\cos z} - \frac{\sin x}{\cos x}$$
$$= 0$$

99.

$\cos^{-1}\frac{1}{2}$
$x = 1$
$y = \sqrt{3}$
$r = 2$

$\sin^{-1}\frac{3}{5}$
$x = 4$
$y = 3$
$r = 5$

$$\sin\left(\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}\right)$$
$$= \sin\cos^{-1}\frac{1}{2}\cos\sin^{-1}\frac{3}{5} + \cos\cos^{-1}\frac{1}{2}\sin\sin^{-1}\frac{3}{5}$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{4}{5} + \frac{1}{2}\cdot\frac{3}{5}$$
$$= \frac{4\sqrt{3}+3}{10}$$

100. $\sin^{-1}\frac{3}{5}$

$y = 3, r = 5, x = 4$

$\cos^{-1}\left(-\frac{4}{5}\right)$

$x = -4, y = 3, r = 5$

$$\sin\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\left(-\frac{4}{5}\right)\right)$$
$$= \sin\sin^{-1}\frac{3}{5}\cos\cos^{-1}\left(-\frac{4}{5}\right) - \cos\sin^{-1}\frac{3}{5}\sin\cos^{-1}\left(-\frac{4}{5}\right)$$
$$= \frac{3}{5}\left(\frac{-4}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$
$$= \frac{-12}{25} - \frac{12}{25}$$
$$= -\frac{24}{25}$$

101. $\tan^{-1}\frac{4}{3}$

$x = 3$
$y = 4$
$r = 5$

$\cos^{-1}\frac{5}{13}$

$x = 5$
$y = 12$
$r = 13$

$$\cos\left(\tan^{-1}\frac{4}{3} + \cos^{-1}\frac{5}{13}\right)$$
$$= \cos\tan^{-1}\frac{4}{3}\cos\cos^{-1}\frac{5}{13}$$
$$\quad - \sin\tan^{-1}\frac{4}{3}\sin\cos^{-1}\frac{5}{13}$$
$$= \frac{3}{5}\cdot\frac{5}{13} - \frac{4}{5}\cdot\frac{12}{13}$$
$$= -\frac{33}{65}$$

102. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6} \qquad \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$
$$= \cos\left[\frac{5\pi}{6} - \left(-\frac{\pi}{6}\right)\right]$$
$$= \cos\pi$$
$$= -1$$

103. Let $\alpha = \sin^{-1} x$, where $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$. $\sin\alpha = x$

Because x is positive, sin α is positive. Thus α is in quadrant I. Using a right triangle in quadrant I with sin α = x, the third side can be found using the Pythagorean Theorem.

$$a^2 + x^2 = 1^2$$
$$a^2 = 1 - x^2$$
$$a = \sqrt{1-x^2}$$

Thus $\cos\alpha = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

Because y is positive, cos β is positive. Thus β is in quadrant I. Using a right triangle in quadrant I with cos β = y, the third side can be found using the Pythagorean Theorem.

$$b^2 + y^2 = 1^2$$
$$b^2 = 1 - x^2$$
$$a = \sqrt{1-y^2}$$

Thus $\cos\alpha = \frac{\sqrt{1-y^2}}{1} = \sqrt{1-y^2}$

$$\begin{aligned}\cos(\sin^{-1} x - \cos^{-1} y) &= \cos(\alpha - \beta)\\ &= \cos\alpha\cos\beta + \sin\alpha\sin\beta\\ &= \sqrt{1-x^2}\,y + x\sqrt{1-y^2}\\ &= y\sqrt{1-x^2} + x\sqrt{1-y^2}\end{aligned}$$

104. $\tan^{-1} x$
$y = x$
$x = 1$
$r = \sqrt{x^2+1}$

$\sin^{-1} x$
$y = y$
$r = 1$
$x = \sqrt{1-y^2}$

$$\begin{aligned}&\sin(\tan^{-1} x - \sin^{-1} y)\\ &= \sin\tan^{-1} x \cos\sin^{-1} y\\ &\quad - \cos\tan^{-1} x \sin\sin^{-1} y\\ &= \frac{x}{\sqrt{x^2+1}} \cdot \frac{\sqrt{1-y^2}}{1} - \frac{1}{\sqrt{x^2+1}} \cdot y\\ &= \frac{x\sqrt{1-y^2} - y}{\sqrt{x^2+1}}\end{aligned}$$

105. $\sin^{-1}$
$x = \sqrt{1-x^2}$
$y = x$
$r = 1$

$\cos^{-1} y$
$x = y$
$y = \sqrt{1-y^2}$
$r = 1$

$$\begin{aligned}&\tan(\sin^{-1} x + \cos^{-1} y)\\ &= \frac{\tan\sin^{-1} x + \tan\cos^{-1} y}{1 - \tan\sin^{-1} x \cdot \tan\cos^{-1} y}\\ &= \frac{\frac{x}{\sqrt{1-x^2}} + \frac{\sqrt{1-y^2}}{y}}{1 - \frac{x}{\sqrt{1-x^2}} \cdot \frac{\sqrt{1-y^2}}{y}}\\ &= \frac{xy + \sqrt{1-y^2}\sqrt{1-x^2}}{y\sqrt{1-x^2} - x\sqrt{1-y^2}}\end{aligned}$$

106. Answers may vary.

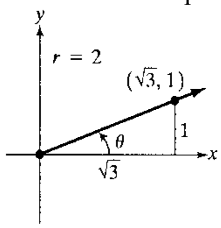
107. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, an unknown opposite side, a in the smaller triangle, b in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 37.1^\circ = \frac{a}{120}$$
$$a = 120\tan 37.1^\circ \approx 90.8$$
$$\tan 62.4^\circ = \frac{b}{120}$$
$$b = 120\tan 62.4^\circ \approx 229.5$$

The difference in the two heights is about 138.7 feet.

108. Let $\theta = \sin^{-1}\frac{1}{2}$, then $\sin\theta = \frac{1}{2}$.

Because $\sin\theta$ is positive, θ is in the first quadrant.



$$x^2 + y^2 = r^2$$
$$x^2 + 1^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

$$\sec\left(\sin^{-1}\frac{1}{2}\right) = \sec\theta = \frac{r}{x} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

109. **a.** domain: $(-4, \infty)$

b. range: $(-\infty, -1]$

c. y-intercept: -1

d. f is constant: $(-2, 3)$

e. f is increasing: $(-4, -2)$

f. f is decreasing: $(3, \infty)$

g. $f(-2) = -1$

110. $\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

111. **a.** No, they are not equal.

$$\sin(2\cdot 30^\circ) \neq 2\sin 30^\circ$$
$$\sin 60^\circ \neq 2\cdot\frac{1}{2}$$
$$\frac{\sqrt{3}}{2} \neq 1$$

b. Yes, they are equal.

$$\sin(2\cdot 30^\circ) = 2\sin 30^\circ\cos 30^\circ$$
$$\sin 60^\circ = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}$$
$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

112. **a.** No, they are not equal.

$$\cos(2\cdot 30^\circ) \neq 2\cos 30^\circ$$
$$\cos 60^\circ \neq 2\cdot\frac{\sqrt{3}}{2}$$
$$\frac{1}{2} \neq \sqrt{3}$$

b. Yes, they are equal.

$$\cos(2\cdot 30^\circ) = \cos^2 30^\circ - \sin^2 30^\circ$$
$$\cos 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$
$$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$$
$$\frac{1}{2} = \frac{1}{2}$$

Section 5.3

Check Point Exercises

1. $\sin\theta = \frac{4}{5} = \frac{y}{r}$

Because θ lies in quadrant II, x is negative.

$$x^2 + y^2 = r^2$$
$$x^2 + 4^2 = 5^2$$
$$x^2 = 5^2 - 4^2 = 9$$
$$x = -\sqrt{9} = -3$$

Now we use values for x, y, and r to find the required values.

a. $$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$$

b. $$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

c. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2} = \dfrac{-\frac{8}{3}}{1-\frac{16}{9}} = \dfrac{-\frac{8}{3}}{-\frac{7}{9}}$

$= \left(-\dfrac{8}{3}\right)\left(-\dfrac{9}{7}\right) = \dfrac{24}{7}$

2. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = 15°$.

$\cos^2 15° - \sin^2 15° = \cos(2\cdot 15°) = \cos 30° = \dfrac{\sqrt{3}}{2}$

3. $\sin 3\theta = \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (2\cos^2\theta - 1)\sin\theta$

$= 2\sin\theta\cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta$

$= 4\sin\theta\cos^2\theta - \sin\theta$

$= 4\sin\theta(1-\sin^2\theta) - \sin\theta$

$= 4\sin\theta - 4\sin^3\theta - \sin\theta$

$= 3\sin\theta - 4\sin^3\theta$

By working with the left side and expressing it in a form identical to the right side, we have verified the identity.

4. $\sin^4 x = \left(\sin^2 x\right)^2$

$= \left(\dfrac{1-\cos 2x}{2}\right)^2$

$= \dfrac{1-2\cos 2x + \cos^2 2x}{4}$

$= \dfrac{1}{4} - \dfrac{1}{2}\cos 2x + \dfrac{1}{4}\cos^2 2x$

$= \dfrac{1}{4} - \dfrac{1}{2}\cos 2x + \dfrac{1}{4}\left(\dfrac{1+\cos 2(2x)}{2}\right)$

$= \dfrac{1}{4} - \dfrac{1}{2}\cos 2x + \dfrac{1}{8} + \dfrac{1}{8}\cos 4x$

$= \dfrac{3}{8} - \dfrac{1}{2}\cos 2x + \dfrac{1}{8}\cos 4x$

5. Because 105° lies in quadrant II, $\cos 105° < 0$.

$\cos 105° = \cos\left(\dfrac{210°}{2}\right)$

$= -\sqrt{\dfrac{1+\cos 210°}{2}}$

$= -\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\dfrac{2-\sqrt{3}}{4}}$

$= -\dfrac{\sqrt{2-\sqrt{3}}}{2}$

6. $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{1+\left(1-2\sin^2\theta\right)}$

$= \dfrac{2\sin\theta\cos\theta}{2-2\sin^2\theta}$

$= \dfrac{2\sin\theta\cos\theta}{2\left(1-\sin^2\theta\right)}$

$= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

The right side simplifies to $\tan\theta$, the expression on the left side. Thus, the identity is verified.

7. $\dfrac{\sec\alpha}{\sec\alpha\csc\alpha + \csc\alpha} = \dfrac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha}\cdot\frac{1}{\sin\alpha} + \frac{1}{\sin\alpha}}$

$= \dfrac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha\sin\alpha} + \frac{\cos\alpha}{\cos\alpha\sin\alpha}}$

$= \dfrac{\frac{1}{\cos\alpha}}{\frac{1+\cos\alpha}{\cos\alpha\sin\alpha}}$

$= \dfrac{1}{\cos\alpha}\cdot\dfrac{\cos\alpha\sin\alpha}{1+\cos\alpha}$

$= \dfrac{\sin\alpha}{1+\cos\alpha}$

$= \tan\dfrac{\alpha}{2}$

We worked with the right side and arrived at the left side. Thus, the identity is verified.

Concept and Vocabulary Check 5.3

1. $2\sin x\cos x$

2. $\sin^2 A$; $2\cos^2 A$; $2\sin^2 A$

3. $\dfrac{2\tan B}{1-\tan^2 B}$

4. $1-\cos 2\alpha$

5. $1+\cos 2\alpha$

6. $1-\cos 2y$

7. $1-\cos x$

8. $1+\cos y$

9. $1-\cos\alpha$; $1-\cos\alpha$; $1+\cos\alpha$

10. false

11. false

12. false

13. +

14. −

15. +

Exercise Set 5.3

1. $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

3. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}}$

$= \frac{\frac{3}{2}}{\frac{7}{16}} = \left(\frac{3}{2}\right)\left(\frac{16}{7}\right) = \frac{24}{7}$

Use this information to solve problems 4, 5, and 6.

$\tan\alpha = \frac{7}{24} = \frac{y}{x}$

Because r is a distance it is positive.

$x^2 + y^2 = r^2$

$24^2 + 7^2 = r^2$

$576 + 49 = r^2$

$625 = r^2$

$r = 25$

$\sin\alpha = \frac{y}{r} = \frac{7}{25}$

$\cos\alpha = \frac{x}{r} = \frac{24}{25}$

4. $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{7}{25}\right)\left(\frac{24}{25}\right) = \frac{336}{625}$

5. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 = \frac{576}{625} - \frac{49}{625}$

$= \frac{527}{625}$

6. $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$= \frac{2\left(\frac{7}{24}\right)}{1-\left(\frac{7}{24}\right)^2} = \frac{\frac{14}{24}}{1-\frac{49}{576}} = \frac{\frac{14}{24}}{\frac{527}{576}}$

$= \left(\frac{14}{24}\right)\left(\frac{576}{527}\right) = \frac{336}{527}$

7. $\sin\theta = \frac{15}{17} = \frac{y}{r}$

Because θ lies in quadrant II, x is negative.

$x^2 + y^2 = r^2$

$x^2 + 15^2 = 17^2$

$x^2 = 17^2 - 15^2 = 64$

$x = -\sqrt{64} = -8$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{15}{17}\right)\left(-\frac{8}{17}\right) = -\frac{240}{289}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 = \frac{64}{289} - \frac{225}{289}$

$= -\frac{161}{289}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(-\frac{15}{8}\right)}{1-\left(-\frac{15}{8}\right)^2} = \frac{-\frac{15}{4}}{1-\frac{225}{64}} = \frac{-\frac{15}{4}}{-\frac{161}{64}}$

$= \left(-\frac{15}{4}\right)\left(-\frac{64}{161}\right) = \frac{240}{161}$

8. $\sin\theta = \frac{12}{13} = \frac{y}{r}$

Because θ lies in quadrant II, x is negative.

$$x^2 + y^2 = r^2$$
$$x^2 + 12^2 = 13^2$$
$$x^2 = 25$$
$$x = -\sqrt{25} = -5$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$$= 2\cdot\frac{12}{13}\cdot\left(-\frac{5}{13}\right)$$
$$= -\frac{120}{169}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$$= \left(-\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$
$$= \frac{25}{169} - \frac{144}{169}$$
$$= -\frac{119}{169}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$$= \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{\frac{-24}{5}}{1-\frac{144}{25}}$$
$$= \frac{-\frac{24}{5}}{-\frac{119}{5}} = \frac{120}{119}$$

9. $\cos\theta = \frac{24}{25} = \frac{x}{r}$

Because θ lies in quadrant IV, y is negative.

$$x^2 + y^2 = r^2$$
$$24^2 + y^2 = 25^2$$
$$y^2 = 25^2 - 24^2 = 49$$
$$y = -\sqrt{49} = -7$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$$= 2\left(-\frac{7}{25}\right)\left(\frac{24}{25}\right) = -\frac{336}{625}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$$= \left(\frac{24}{25}\right)^2 - \left(-\frac{7}{25}\right)^2$$
$$= \frac{576}{625} - \frac{49}{625} = \frac{527}{625}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$$= \frac{2\left(-\frac{7}{24}\right)}{1-\left(-\frac{7}{24}\right)^2} = \frac{-\frac{7}{12}}{1-\frac{49}{576}} = \frac{-\frac{7}{12}}{\frac{527}{576}}$$
$$= \left(-\frac{7}{12}\right)\left(\frac{576}{527}\right) = -\frac{336}{527}$$

10. $\cos\theta = \frac{40}{41} = \frac{x}{r}$

Because θ lies in quadrant IV, y is negative.

$$x^2 + y^2 = r^2$$
$$40^2 + y^2 = 41^2$$
$$y^2 = 81$$
$$y = -\sqrt{81} = -9$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$$= 2\left(-\frac{9}{41}\right)\left(\frac{40}{41}\right) = -\frac{720}{1681}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$$= \left(\frac{40}{41}\right) - \left(-\frac{9}{41}\right)^2 = \frac{1600}{1681} - \frac{81}{1681}$$
$$= \frac{1519}{1681}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$$= \frac{2\left(-\frac{9}{40}\right)}{1-\left(-\frac{9}{40}\right)^2} = \frac{-\frac{9}{20}}{1-\frac{81}{1600}}$$
$$= \frac{-\frac{9}{20}}{\frac{1519}{1600}} = \left(-\frac{9}{20}\right)\left(\frac{1600}{1519}\right) = -\frac{720}{1519}$$

11. $\cot\theta = 2 = \frac{-2}{-1} = \frac{x}{y}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$
$r^2 = (-2)^2 + (-1)^2$
$r^2 = 5$
$r = \sqrt{5}$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2$
$= \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$= \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}}$
$= (1)\left(\frac{4}{3}\right) = \frac{4}{3}$

12. $\cot\theta = 3 = \frac{-3}{-1} = \frac{x}{y}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$
$r^2 = (-3)^2 + (-1)^2$
$r^2 = 10$
$r = \sqrt{10}$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{1}{\sqrt{10}}\right)\left(-\frac{3}{\sqrt{10}}\right) = \frac{6}{10} = \frac{3}{5}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{3}{\sqrt{10}}\right)^2 - \left(-\frac{1}{\sqrt{10}}\right)^2$
$= \frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$= \frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}}$
$= \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2}{3}\cdot\frac{9}{8} = \frac{3}{4}$

13. $\sin\theta = -\frac{9}{41} = \frac{-9}{41} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$x^2 + y^2 = r^2$
$x^2 + (-9)^2 = 41^2$
$x^2 = 1600$
$x = -\sqrt{1600}$
$x = -40$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{9}{41}\right)\left(-\frac{40}{41}\right) = \frac{720}{1681}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{40}{41}\right)^2 - \left(-\frac{9}{41}\right)^2$
$= \frac{1600}{1681} - \frac{81}{1681}$
$= \frac{1519}{1681}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$= \frac{2\left(\frac{9}{40}\right)}{1-\left(\frac{9}{40}\right)^2} = \frac{\frac{9}{20}}{1-\frac{81}{1600}} = \frac{\frac{9}{20}}{\frac{1519}{1600}}$
$= \left(\frac{9}{20}\right)\left(\frac{1600}{1519}\right) = \frac{720}{1519}$

14. $\sin\theta = -\frac{2}{3} = \frac{-2}{3} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$x^2 + y^2 = r^2$
$x^2 + (-2)^2 = 3^2$
$x^2 = 5$
$x = -\sqrt{5}$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = \frac{4\sqrt{5}}{9}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(-\frac{\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2$
$= \frac{5}{9} - \frac{4}{6} = \frac{1}{9}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
$= \frac{2\cdot\frac{2}{\sqrt{5}}}{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \frac{\frac{4}{\sqrt{5}}}{1-\frac{4}{5}}$
$= \frac{\frac{4}{\sqrt{5}}}{\frac{1}{5}} = \frac{4\sqrt{5}}{4}\cdot\frac{5}{1} = 4\sqrt{5}$

15. The given expression is the right side of the formula for $\sin 2\theta$ with $\theta = 15°$.
$2\sin 15°\cos 15° = \sin(2\cdot 15°)$
$= \sin 30° = \frac{1}{2}$

16. The given expression is the right side of the formula for sin 2θ with θ = 22.5°.
$2\sin 22.5^{\circ}\cos 22.5^{\circ} = \sin(2\cdot 22.5^{\circ})$
$= \sin 45^{\circ} = \frac{\sqrt{2}}{2}$

17. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = 75°$.
$\cos^2 75° - \sin^2 75° = \cos(2\cdot 75°)$
$= \cos 150° = -\frac{\sqrt{3}}{2}$

18. The given expression is the right side of the formula for cos 2θ with θ = 105°
$\cos^2 105^{\circ} - \sin^2 105^{\circ} = \cos(2\cdot 105^{\circ})$
$= \cos 210^{\circ} = -\frac{\sqrt{3}}{2}$

19. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = \frac{\pi}{8}$.
$2\cos^2\frac{\pi}{8} - 1 = \cos\left(2\cdot\frac{\pi}{8}\right)$
$= \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

20. The given expression is the right side of the formula for cos 2θ with θ = $\frac{\pi}{12}$.
$1 - 2\sin^2\frac{\pi}{12} = \cos\left(2\cdot\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

21. The given expression is the right side of the formula for $\tan 2\theta$ with $\theta = \frac{\pi}{12}$.
$\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}} = \tan\left(2\cdot\frac{\pi}{12}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

22. The given expression is the right side of the formula for tan 2θ with θ = $\frac{\pi}{8}$.
$\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} = \tan\left(2\cdot\frac{\pi}{8}\right) = \tan\frac{\pi}{4} = 1$

23. $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\cdot\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}}$
$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$
$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$
$= \frac{2\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{1}$
$= 2\sin\theta\cos\theta$
$= \sin 2\theta$

24. $$\frac{2\cot\theta}{1+\cot^2\theta} = \frac{2\cdot\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}} = \frac{\frac{2\cos\theta}{\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}} = \frac{\frac{2\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}} = \frac{2\cos\theta}{\sin\theta}\cdot\frac{\sin^2\theta}{1} = 2\cos\theta\sin\theta = \sin 2\theta$$

25. $$(\sin\theta+\cos\theta)^2 = \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta = \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = 1+2\sin\theta\cos\theta = 1+\sin 2\theta$$

26. $$(\sin\theta-\cos\theta)^2 = \sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta = \sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta = 1-2\sin\theta\cos\theta = 1-\sin 2\theta$$

27. $$\sin^2 x+\cos 2x = \sin^2 x+\cos^2 x-\sin^2 x = \cos^2 x$$

28. $$\frac{\cos 2x}{\cos^2 x} = \frac{1-2\sin^2 x}{\cos^2 x} = \frac{1-\sin^2 x-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x} = 1-\tan^2 x$$

29. $$\frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x\cos x}{1-\left(\cos^2 x-\sin^2 x\right)} = \frac{2\sin x\cos x}{1-\cos^2 x+\sin^2 x} = \frac{2\sin x\cos x}{\sin^2+\sin^2 x} = \frac{2\sin x\cos x}{2\sin^2 x} = \frac{\cos x}{\sin x} = \cot x$$

30. $$\frac{1+\cos 2x}{\sin 2x} = \frac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x} = \frac{1-\sin^2 x+\cos^2 x}{2\sin x\cos x} = \frac{\cos^2 x+\cos^2 x}{2\sin x\cos x} = \frac{2\cos^2 x}{2\sin x\cos x} = \frac{\cos x}{\sin x} = \cot x$$

31. $$\tan t\cos 2t = \frac{\sin t}{\cos t}\cdot\left(2\cos^2 t-1\right) = \frac{2\sin t\cos^2 t}{\cos t}-\frac{\sin t}{\cos t} = 2\sin t\cos t-\tan t = \sin 2t-\tan t$$

32. $$-\cos t\cos 2t = \frac{-\cos t}{\sin t}\left(1-2\sin^2 t\right) = -\frac{\cos t}{\sin t}+\frac{2\cos t\sin^2 t}{\sin t} = -\cot t+2\cos t\sin t = 2\cos t\sin t-\cot t = \sin 2t-\cot t$$

33. $$\sin 4t = \sin(2t+2t) = \sin 2t\cos 2t+\cos 2t\sin 2t = \cos 2t(\sin 2t+\sin 2t) = \cos 2t\cdot 2\sin 2t = \left(\cos^2 t-\sin^2 t\right)\cdot 2\cdot 2\sin t\cos t = 4\sin t\cos^3 t-4\sin^3 t\cos t$$

34. $$\cos 4t = \cos 2(2t) = 2\cos^2 2t-1 = 2(2\cos^2 t-1)^2-1 = 2(4\cos^4 t-4\cos^2 t+1)-1 = 8\cos^4 t-8\cos^2 t+2-1 = 8\cos^4 t-8\cos^2 t+1$$

35. $6\sin^4 x$

$$= 6\left(\frac{1-\cos 2x}{2}\right)^2$$
$$= 6\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right)$$
$$= \frac{6-12\cos 2x+6\cos^2 2x}{4}$$
$$= \frac{3}{4}-3\cos 2x+\frac{3}{2}\cos^2 2x$$
$$= \frac{3}{4}-3\cos 2x+\frac{3}{2}\left(\frac{1+\cos 4x}{2}\right)$$
$$= \frac{3}{4}-3\cos 2x+\frac{3}{2}\left(\frac{1}{2}+\frac{\cos 4x}{2}\right)$$
$$= \frac{3}{4}-3\cos 2x+\frac{3}{4}+\frac{3}{4}\cos 4x$$
$$= \frac{9}{4}-3\cos 2x+\frac{3}{4}\cos 4x$$

36. $10\cos^2 x\cos^2 x = 10\left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

$$= \frac{10(1+2\cos 2x+\cos^2 2x)}{4}$$
$$= \frac{10+20\cos 2x+10\cos^2 2x}{4}$$
$$= \frac{10}{4}+\frac{20\cos 2x}{4}+\frac{10\cos^2 2x}{4}$$
$$= \frac{5}{2}+5\cos 2x+\frac{5}{4}(1+\cos 4x)$$
$$= \frac{5}{2}+5\cos 2x+\frac{5}{4}+\frac{5}{4}\cos 4x$$
$$= \frac{15}{4}+5\cos 2x+\frac{5}{4}\cos 4x$$

37. $\sin^2 x\cos^2 x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

$$= \frac{1-\cos^2 2x}{4}$$
$$= \frac{1}{4}-\frac{1}{4}\cos^2 2x$$
$$= \frac{1}{4}-\frac{1}{4}\left(\frac{1+\cos(2\cdot 2x)}{2}\right)$$
$$= \frac{1}{4}-\frac{1}{8}(1+\cos 4x)$$
$$= \frac{1}{4}-\frac{1}{8}-\frac{1}{8}\cos 4x$$
$$= \frac{1}{8}-\frac{1}{8}\cos 4x$$

38. $8\sin^2 x\cos^2 x = 8\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

$$= \frac{8(1-\cos^2 2x)}{4}$$
$$= \frac{8}{4}-\frac{8(\cos^2 2x)}{4}$$
$$= 2-2\left(\frac{1+\cos 2\cdot 2x}{2}\right)$$
$$= 2-1-\cos 4x$$
$$= 1-\cos 4x$$

39. Because 15° lies in quadrant I, $\sin 15° > 0$.

$$\sin 15° = \sin\frac{30°}{2}$$
$$= \sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$
$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

40. Because 22.5° lies in quadrant I, $\cos 22.5° > 0$.

$$\cos 22.5° = \cos\frac{45°}{2} = \sqrt{\frac{1+\cos 45°}{2}}$$
$$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$
$$= \sqrt{\frac{2+\sqrt{2}}{4}}$$
$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

41. Because 157.5° lies in quadrant II, $\cos 157.5° < 0$.

$$\cos 157.5° = \cos\frac{315°}{2} = -\sqrt{\frac{1+\cos 315°}{2}}$$
$$= -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}}$$
$$= -\frac{\sqrt{2+\sqrt{2}}}{2}$$

42. Because 105° lies in quadrant II, $\sin 105° > 0$.

$$\sin 105° = \sin\frac{210°}{2} = \sqrt{\frac{1-\cos 210°}{2}}$$
$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$
$$= \sqrt{\frac{2+\sqrt{3}}{4}}$$
$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

43. Because 75° lies in quadrant I, $\tan 75° > 0$.

$$\tan 75° = \tan\frac{150°}{2} = \frac{1-\cos 150°}{\sin 150°}$$
$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = 2+\sqrt{3}$$

44. Because 112.5° lies in quadrant II, $\tan 112.5^0 < 0$.

$$\tan 125^{\text{o}} = \tan\frac{225^{\text{o}}}{2}$$
$$= \frac{1-\cos 225^{\text{o}}}{\sin 225^{\text{o}}}$$
$$= \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{-\frac{\sqrt{2}}{2}}$$
$$= \frac{2+\sqrt{2}}{-\sqrt{2}}$$
$$= -\frac{2}{\sqrt{2}} - 1$$
$$= -\sqrt{2} - 1$$

45. Because $\frac{7\pi}{8}$ lies in quadrant II, $\tan\frac{7\pi}{8} < 0$.

$$\tan\frac{7\pi}{8} = \tan\left(\frac{\frac{7\pi}{4}}{2}\right) = \frac{1-\cos\frac{7\pi}{4}}{\sin\frac{7\pi}{4}}$$
$$= \frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} + 1$$
$$= -\sqrt{2} + 1$$

46. Because $\frac{3\pi}{8}$ lies in quadrant I, $\tan\frac{3\pi}{8} > 0$.

$$\tan\frac{3\pi}{8} = \tan\frac{\frac{3\pi}{4}}{2}$$
$$= \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}}$$
$$= \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}}$$
$$= \frac{2}{\sqrt{2}} + 1$$
$$= \sqrt{2} + 1$$

47. $$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$
$$= \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{1}{10}}$$
$$= \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

48. $$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$
$$= \sqrt{\frac{1+\frac{4}{5}}{2}}$$
$$= \sqrt{\frac{\frac{9}{5}}{2}}$$
$$= \sqrt{\frac{9}{10}}$$
$$= \frac{3}{\sqrt{10}}$$
$$= \frac{3\sqrt{10}}{10}$$

49. $$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$
$$= \frac{1-\frac{4}{5}}{\frac{3}{5}}$$
$$= \frac{1}{3}$$

Use this information to solve problems 50, 51, 52 and 54.

$$\tan\alpha = \frac{7}{24} = \frac{y}{x}$$

Because r is a distance, it is positive.

$$r^2 = x^2 + y^2$$
$$r^2 = 24^2 + 7^2$$
$$r^2 = 625$$
$$r = 25$$

$$\sin\alpha = \frac{y}{r} = \frac{7}{25}$$
$$\cos\alpha = \frac{x}{r} = \frac{24}{25}$$

50. $\sin\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{2}}$
$=\sqrt{\frac{1-\frac{24}{25}}{2}}$
$=\sqrt{\frac{\frac{1}{25}}{2}}$
$=\sqrt{\frac{1}{50}}$
$=\frac{1}{5\sqrt{2}}$
$=\frac{\sqrt{2}}{10}$

51. $\cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}=\sqrt{\frac{1+\frac{24}{25}}{2}}=\sqrt{\frac{49}{50}}$
$=\frac{7}{5\sqrt{2}}=\frac{7\sqrt{2}}{10}$

52. $\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}$
$=\frac{1-\frac{24}{25}}{\frac{7}{25}}$
$=\frac{\frac{1}{25}}{\frac{7}{25}}$
$=\frac{1}{7}$

53. $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}=2\cdot\sqrt{\frac{1-\cos\theta}{2}}\cdot\sqrt{\frac{1+\cos\theta}{2}}$
$=2\sqrt{\frac{1-\frac{4}{5}}{2}}\cdot\sqrt{\frac{1+\frac{4}{5}}{2}}$
$=2\cdot\sqrt{\frac{1}{10}}\cdot\sqrt{\frac{9}{10}}$
$=2\cdot\frac{1}{\sqrt{10}}\cdot\frac{3}{\sqrt{10}}$
$=\frac{6}{10}=\frac{3}{5}$

54. $2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}=2\sqrt{\frac{1-\cos\alpha}{2}}\cdot\sqrt{\frac{1+\cos\alpha}{2}}$
$=2\sqrt{\frac{1-\frac{24}{25}}{2}}\cdot\sqrt{\frac{1+\frac{24}{25}}{2}}$
$=2\sqrt{\frac{1}{50}}\cdot\sqrt{\frac{49}{50}}$
$=2\cdot\frac{1}{\sqrt{50}}\cdot\frac{7}{\sqrt{50}}$
$=\frac{7}{25}$

55. $\tan\alpha=\frac{4}{3}=\frac{-4}{-3}=\frac{y}{x}$

Because r is a distance, it is positive.

$r^2=x^2+y^2$
$r^2=(-4)^2+(-3)^2$
$r^2=25$
$r=5$

Since $180^\circ<\alpha<270^\circ$, then $90^\circ<\frac{\alpha}{2}<135^\circ$.

Therefore $\frac{\alpha}{2}$ lies in quadrant II.

Thus, $\sin\frac{\alpha}{2}>0$, $\cos\frac{\alpha}{2}<0$, and $\tan\frac{\alpha}{2}<0$.

a. $\sin\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{2}}=\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}$
$=\sqrt{\frac{\frac{8}{5}}{2}}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

b. $\cos\frac{\alpha}{2}=-\sqrt{\frac{1+\cos\alpha}{2}}=-\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}$
$=-\sqrt{\frac{\frac{2}{5}}{2}}=-\sqrt{\frac{1}{5}}=-\frac{1}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$

c. $\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}=\frac{1-\left(-\frac{3}{5}\right)}{-\frac{4}{5}}$
$=\frac{\frac{8}{5}}{-\frac{4}{5}}=\frac{8}{-4}=-2$

56. $\tan\alpha = \frac{8}{15} = \frac{-8}{-15} = \frac{y}{x}$

Because r is a distance, it is positive.

$$r^2 = x^2 + y^2$$
$$r^2 = (-15)^2 + (-8)^2$$
$$r^2 = 289$$
$$r = \sqrt{289}$$
$$r = 17$$

Since $180^\circ < \alpha < 270^\circ$, then $90^\circ < \frac{\alpha}{2} < 135^\circ$.

Therefore $\frac{\alpha}{2}$ lies in quadrant II.

Thus, $\sin\frac{\alpha}{2} > 0$, $\cos\frac{\alpha}{2} < 0$ and $\tan\frac{\alpha}{2} < 0$.

a. $$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\frac{-15}{17}}{2}} = \sqrt{\frac{\frac{32}{17}}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$

b. $$\cos\frac{\alpha}{2} = -\sqrt{\frac{1+\left(-\frac{15}{17}\right)}{2}} = -\sqrt{\frac{\frac{2}{17}}{2}} = -\sqrt{\frac{1}{17}} = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$$

c. $$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\left(-\frac{15}{17}\right)}{-\frac{8}{17}} = \frac{\frac{32}{17}}{-\frac{8}{17}} = \frac{32}{-8} = -4$$

57. $\sec\alpha = -\frac{13}{5} = \frac{13}{-5} = \frac{r}{x}$

Because α lies in quadrant II, y is positive.

$$x^2 + y^2 = r^2$$
$$(-5)^2 + y^2 = (13)^2$$
$$y^2 = 144$$
$$y = 12$$

Since $\frac{\pi}{2} < \alpha < \pi$, then $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$. Therefore $\frac{\alpha}{2}$ lies in quadrant I.

Thus, $\sin\frac{\alpha}{2} > 0$, $\cos\frac{\alpha}{2} > 0$, and $\tan\frac{\alpha}{2} > 0$.

a. $$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}} = \sqrt{\frac{18}{26}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

b. $$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\left(-\frac{5}{13}\right)}{2}} = \sqrt{\frac{8}{26}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

c. $$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\left(-\frac{5}{13}\right)}{\frac{12}{13}} = \frac{13+5}{12} = \frac{18}{12} = \frac{3}{2}$$

58. $\sec\alpha = -3 = \dfrac{3}{-1} = \dfrac{r}{x}$

Because α lies in quadrant II, y is positive.

$$x^2 + y^2 = r^2$$
$$(-1)^2 + y^2 = 3^2$$
$$y^2 = 8$$
$$y = \sqrt{8}$$
$$y = 2\sqrt{2}$$

Since $\dfrac{\pi}{2} < \alpha < \pi,$ then $\dfrac{\pi}{4} < \dfrac{\alpha}{2} < \dfrac{\pi}{2}.$ Therefore $\dfrac{\alpha}{2}$ lies in quadrant I.

Thus, $\sin\dfrac{\alpha}{2} > 0,$ $\cos\dfrac{\alpha}{2} > 0,$ and $\tan\dfrac{\alpha}{2} > 0.$

a. $$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{1}{3}\right)}{\frac{1}{3}}} = \sqrt{\frac{\frac{4}{3}}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

b. $$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\left(-\frac{1}{3}\right)}{\frac{1}{3}}} = \sqrt{\frac{\frac{2}{3}}{2}} = \sqrt{\frac{2}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

c. $$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\left(-\frac{1}{3}\right)}{\frac{2\sqrt{2}}{3}} = \frac{3+1}{2\sqrt{2}} = \frac{4}{2\sqrt{2}} = \sqrt{2}$$

59. $$\sin^2\frac{\theta}{2} = \frac{1-\cos 2\left(\frac{\theta}{2}\right)}{2} = \frac{1-\cos\theta}{2}\cdot\frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}} = \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{2}{\cos\theta}} = \frac{\frac{1}{\cos\theta}-\frac{\cos\theta}{\cos\theta}}{2\cdot\frac{1}{\cos\theta}} = \frac{\sec\theta-1}{2\sec\theta}$$

60. $$\sin^2\frac{\alpha}{2} = \frac{1-\cos 2\left(\frac{\alpha}{2}\right)}{2} = \frac{1-\cos\alpha}{2}\cdot\frac{\frac{1}{\sin\alpha}}{\frac{1}{\sin\alpha}} = \frac{\frac{1-\cos\alpha}{\sin\alpha}}{\frac{2}{\sin\alpha}} = \frac{\frac{1}{\sin\alpha}-\frac{\cos\alpha}{\sin\alpha}}{2\cdot\frac{1}{\sin\alpha}} = \frac{\csc\alpha-\cot\alpha}{2\csc\alpha}$$

61. $$\cos^2\frac{\theta}{2}=\frac{1+\cos 2\left(\frac{\theta}{2}\right)}{2}=\frac{1+\cos\theta}{2}=\frac{1+\cos\theta}{2}\cdot\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}=\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{2\cdot\frac{\sin\theta}{\cos\theta}}=\frac{\tan\theta+\sin\theta}{2\tan\theta}=\frac{\sin\theta+\tan\theta}{2\tan\theta}$$

62. $$\cos^2\frac{\theta}{2}=\frac{1+\cos 2\left(\frac{\theta}{2}\right)}{2}=\frac{1+\cos\theta}{2}\cdot\frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}=\frac{\frac{1}{\cos\theta}+1}{2\cdot\frac{1}{\cos\theta}}=\frac{\sec\theta+1}{2\sec\theta}$$

63. $$\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{\sin\alpha}{1+\cos\alpha}\cdot\frac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha}}=\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1+\cos\alpha}{\cos\alpha}}=\frac{\tan\alpha}{\frac{1}{\cos\alpha}+\frac{\cos\alpha}{\cos\alpha}}=\frac{\tan\alpha}{\sec\alpha+1}$$

64. $$\frac{\sin^2\alpha+1-\cos^2\alpha}{\sin\alpha(1+\cos\alpha)}=\frac{\sin^2\alpha+\sin^2\alpha}{\sin\alpha(1+\cos\alpha)}=\frac{2\sin^2\alpha}{\sin\alpha(1+\cos\alpha)}=\frac{2\sin\alpha}{1+\cos\alpha}=2\left(\frac{\sin\alpha}{1+\cos a}\right)=2\tan\frac{\alpha}{2}$$

65. $$\frac{\sin x}{1-\cos x}=\frac{\sin x}{1-\cos x}\cdot\frac{\frac{1}{\sin x}}{\frac{1}{\sin x}}=\frac{\frac{\sin x}{\sin x}}{\frac{1-\cos x}{\sin x}}=\frac{1}{\tan\frac{x}{2}}=\cot\frac{x}{2}$$

66. $$\frac{1+\cos x}{\sin x}=\frac{1+\cos x}{\sin x}\cdot\frac{\frac{1}{1+\cos x}}{\frac{1}{1+\cos x}}=\frac{1}{\frac{\sin x}{1+\cos x}}=\frac{1}{\tan\frac{x}{2}}=\cot\frac{x}{2}$$

67. $$\tan\frac{x}{2}+\cot\frac{x}{2}=\frac{1-\cos x}{\sin x}+\frac{1}{\tan\frac{x}{2}}=\frac{1-\cos x}{\sin x}+\frac{1}{\frac{\sin x}{1+\cos x}}=\frac{1-\cos x}{\sin x}+\frac{1+\cos x}{\sin x}=\frac{1-\cos x+1+\cos x}{\sin x}=\frac{2}{\sin x}=2\csc x$$

68. $$\tan\frac{x}{2}-\cot\frac{x}{2}=\frac{1-\cos x}{\sin x}-\frac{1}{\tan\frac{x}{2}}=\frac{1-\cos x}{\sin x}-\frac{1}{\frac{\sin x}{1+\cos x}}=\frac{1-\cos x}{\sin x}-\frac{1+\cos x}{\sin x}=\frac{-2\cos x}{\sin x}=-2\cdot\frac{\cos x}{\sin x}=-2\cot x$$

69. Conjecture: The left side is equal to $\cos 2x$.

$$\frac{\cot x-\tan x}{\cot x+\tan x}=\frac{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}}$$
$$=\frac{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}$$
$$=\frac{\frac{\cos^2 x}{\sin x\cos x}-\frac{\sin^2 x}{\sin x\cos x}}{\frac{\cos^2 x}{\sin x\cos x}+\frac{\sin^2 x}{\sin x\cos x}}$$
$$=\frac{\cos^2 x-\sin^2 x}{\cos^2 x+\sin^2 x}$$
$$=\frac{\cos 2x}{1}$$
$$=\cos 2x$$

70. Conjecture: The left side is equal to $\sin 2x$.

$$\frac{2(\tan x-\cot x)}{\tan^2 x-\cot^2 x}=\frac{2(\tan x-\cot x)}{(\tan x+\cot x)(\tan x-\cot x)}$$
$$=\frac{2}{\tan x+\cot x}$$
$$=\frac{2}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$
$$=\frac{2}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\cdot\frac{\sin x\cos x}{\sin x\cos x}$$
$$=\frac{2\sin x\cos x}{\sin^2 x+\cos^2 x}$$
$$=\frac{\sin 2x}{1}$$
$$=\sin 2x$$

71. Conjecture: The left side is equal to $\sin x+1$.

$$\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2=\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}$$
$$=2\sin\frac{x}{2}\cos\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}$$
$$=\left[2\sin\frac{x}{2}\cos\frac{x}{2}\right]+\left[\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right]$$
$$=\sin\left(2\cdot\frac{x}{2}\right)+1$$
$$=\sin x+1$$

72. Conjecture: The left side is equal to $-\cos x$.

$$\sin^2\frac{x}{2}-\cos^2\frac{x}{2}=-\left(-\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)$$
$$=-\cos\left(2\cdot\frac{x}{2}\right)$$
$$=-\cos x$$

73. Conjecture: The left side is equal to $\sec x$.

$$\frac{\sin 2x}{\sin x}-\frac{\cos 2x}{\cos x}=\frac{2\sin x\cos x}{\sin x}-\frac{2\cos^2 x-1}{\cos x}$$
$$=2\cos x-\frac{2\cos^2 x}{\cos x}+\frac{1}{\cos x}$$
$$=2\cos x-2\cos x+\sec x$$
$$=\sec x$$

74. Conjecture: The left side is equal to $2\sin x$.

$$\sin 2x\sec x=2\sin x\cos x\cdot\frac{1}{\cos x}$$
$$=2\sin x$$

75. Conjecture: The left side is equal to $2\csc 2x$.

$$\frac{\csc^2 x}{\cot x}=\frac{\frac{1}{\sin^2 x}}{\frac{\cos x}{\sin x}}$$
$$=\frac{1}{\sin^2 x}\frac{\sin x}{\cos x}$$
$$=\frac{1}{\sin x\cos x}$$
$$=\frac{2}{2\sin x\cos x}$$
$$=\frac{2}{\sin 2x}$$
$$=2\csc 2x$$

76. Conjecture: The left side is equal to $2\csc 2x$.

$$\tan x+\cot x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}$$
$$=\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}$$
$$=\frac{\sin^2 x}{\sin x\cos x}+\frac{\cos^2 x}{\sin x\cos x}$$
$$=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}$$
$$=\frac{1}{\sin x\cos x}$$
$$=\frac{2}{2\sin x\cos x}$$
$$=\frac{2}{\sin 2x}$$
$$=2\csc 2x$$

77. Conjecture: The left side is equal to $\sin 3x$.

$$\sin x\left(4\cos^2 x-1\right)=\sin x\left(2\cos^2 x+2\cos^2 x-1\right)$$
$$=\sin x\left(2\cos^2 x+\cos 2x\right)$$
$$=2\sin x\cos^2 x+\sin x\cos 2x$$
$$=2\sin x\cos x\cos x+\sin x\cos 2x$$
$$=\sin 2x\cos x+\sin x\cos 2x$$
$$=\sin(2x+x)$$
$$=\sin 3x$$

78. Conjecture: The left side is equal to $\cos 4x$.

$$\begin{aligned}1-8\sin^2 x\cos^2 x &= 1-(2\cdot 2\sin x\cos x\cdot 2\sin x\cos x)\\ &= 1-2\sin 2x\cdot\sin 2x\\ &= 1-2\sin^2 2x\\ &= \cos^2 2x-\sin^2 2x\\ &= \cos 2x\cos 2x-\sin 2x\sin 2x\\ &= \cos(2x+2x)\\ &= \cos 4x\end{aligned}$$

79. **a.**

$$\begin{aligned}d &= \frac{v_o^2}{16}\sin\theta\cos\theta\\ &= \frac{v_o^2}{32}\cdot 2\sin\theta\cos\theta\\ &= \frac{v_o^2}{32}\cdot\sin 2\theta\end{aligned}$$

b. $\sin\alpha$ is at a maximum in the interval $[0, 2\pi]$ when $\alpha=\frac{\pi}{2}$, so $\sin 2\theta$ is at a maximum when $2\theta=\frac{\pi}{2}$ or $\theta=\frac{\pi}{4}$.

80. $\theta=\frac{\pi}{6}$

$$\begin{aligned}\sin\frac{\theta}{2} &= \sqrt{\frac{1-\cos\theta}{2}}\\ &= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}\\ &= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}\\ &= \sqrt{\frac{2-\sqrt{3}}{4}}\\ &= \frac{\sqrt{2-\sqrt{3}}}{2}\end{aligned}$$

$$\begin{aligned}\sin\frac{\theta}{2} &= \frac{1}{M}\\ \frac{\sqrt{2-\sqrt{3}}}{2} &= \frac{1}{M}\\ M &= \frac{2}{\sqrt{2-\sqrt{3}}}\\ &= \frac{2\sqrt{2-\sqrt{3}}}{2-\sqrt{3}}\\ &= \frac{2\sqrt{2-\sqrt{3}}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}\\ &= \frac{4\sqrt{2-\sqrt{3}}+2\sqrt{3}\sqrt{2-\sqrt{3}}}{4-3}\\ &= 4\sqrt{2-\sqrt{3}}+2\sqrt{3}\sqrt{2-\sqrt{3}}\approx 3.9\end{aligned}$$

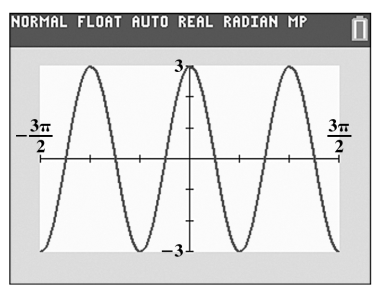
81. $\theta=\frac{\pi}{4}$

$$\begin{aligned}\sin\frac{\theta}{2} &= \sqrt{\frac{1-\cos\theta}{2}}\\ &= \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}\\ &= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}\\ &= \sqrt{\frac{2-\sqrt{2}}{4}}\\ &= \frac{\sqrt{2-\sqrt{2}}}{2}\end{aligned}$$

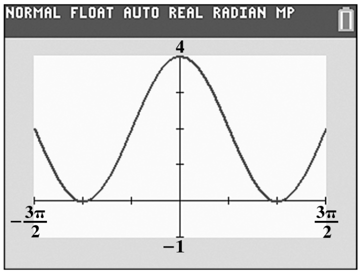
$$\begin{aligned}\sin\frac{\theta}{2} &= \frac{1}{M}\\ \frac{\sqrt{2-\sqrt{2}}}{2} &= \frac{1}{M}\\ M &= \frac{2}{\sqrt{2-\sqrt{2}}}\\ &= \frac{2\sqrt{2-\sqrt{2}}}{2-\sqrt{2}}\\ &= \frac{2\sqrt{2-\sqrt{2}}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}\\ &= \frac{4\sqrt{2-\sqrt{2}}+2\sqrt{2}\sqrt{2-\sqrt{2}}}{4-2}\\ &= \frac{2\left(2\sqrt{2-\sqrt{2}}+\sqrt{2}\sqrt{2-\sqrt{2}}\right)}{2}\\ &= 2\sqrt{2-\sqrt{2}}+\sqrt{2}\sqrt{2-\sqrt{2}}\\ &= \sqrt{2-\sqrt{2}}\cdot\left(2+\sqrt{2}\right)\approx 2.6\end{aligned}$$

82. – 94. Answers may vary.

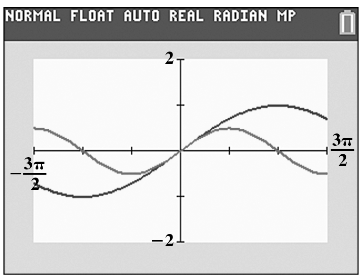
95.



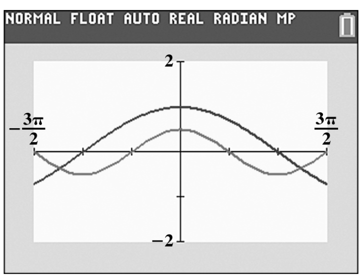
$$\begin{aligned}3-6\sin^2 x &= 3-6\left(\frac{1-\cos 2x}{2}\right)\\ &= 3-3(1-\cos 2x)\\ &= 3-3+3\cos 2x\\ &= 3\cos 2x\end{aligned}$$

96. 

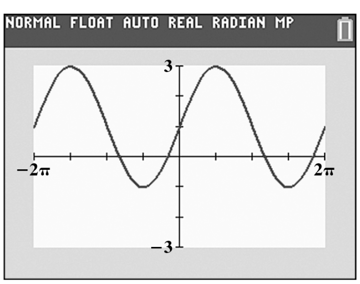
$$4\cos^2\frac{x}{2} = 4\left(\frac{1+\cos 2\left(\frac{x}{2}\right)}{2}\right)$$
$$= 2(1+\cos x)$$
$$= 2+2\cos x$$

97. 

The graphs do not coincide.
Values for x may vary.

98. 

The graphs do not coincide.
Values for x may vary.

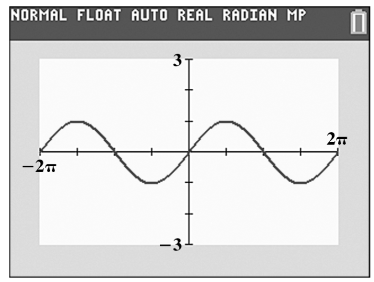
99. 

a. The graph appears to be the sum of 1 and 2 times the sine curve, $y = 1+2\sin x$. If you subtract 1 from the graph, it cycles through intercept, maximum, intercept, minimum, and back to intercept. Thus, $y = 1+2\sin x$ also describes the graph.

b. $\dfrac{1-2\cos 2x}{2\sin x-1}$

$$= \frac{1-2\left(1-2\sin^2 x\right)}{2\sin x-1} = \frac{1-2+4\sin^2 x}{2\sin x-1}$$
$$= \frac{4\sin^2 x-1}{2\sin x-1} = \frac{(2\sin x-1)(2\sin x+1)}{2\sin x-1}$$
$$= 2\sin x+1 = 1+2\sin x$$

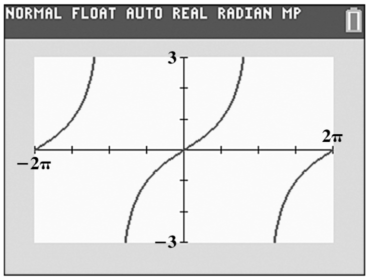
This verifies our observation that $y = \dfrac{1-2\cos 2x}{2\sin x-1}$ and $y = 1+2\sin x$ describe the same graph.

100. 

a. The graph appears to be the sine curve, y = sin x. It cycles through intercept, maximum, intercept, minimum and back to intercept. Thus, $y = \sin x$ also describes the graph.

b. $$\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2\left(\frac{\sin x}{1+\cos x}\right)}{1+\left(\frac{1-\cos 2\left(\frac{x}{2}\right)}{1+\cos 2\left(\frac{x}{2}\right)}\right)}$$
$$= \frac{\frac{2\sin x}{1+\cos x}}{\frac{1+\cos x}{1+\cos x}+\frac{1-\cos x}{1+\cos x}}$$
$$= \frac{\frac{2\sin x}{1+\cos x}}{\frac{2}{1+\cos x}}$$
$$= \frac{2\sin x}{1+\cos x}\cdot\frac{1+\cos x}{2}$$
$$= \sin x$$

This verifies our observation that $y = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $y = \sin x$ describe the same graph.

101. 

a. The graph appears to be the tangent of half the angle. It cycles from negative infinity through intercept to positive infinity. Thus, $y = \tan\frac{x}{2}$ also describes the graph.

b. $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x$

This verifies our observation that $y = \csc x - \cot x$ and $y = \tan\frac{x}{2}$ describe the same graph.

102. makes sense

103. does not make sense; Explanations will vary. Sample explanation: That procedure is not algebraically sound.

104. does not make sense; Explanations will vary. Sample explanation: An angle and its half-angle do not necessarily lie in the same quadrant.

105. does not make sense; Explanations will vary. Sample explanation: That method will not work well because 200° is not an angle with known trigonometric values.

106. $(\sin x + \cos x)\left(1 - \frac{\sin 2x}{2}\right)$

$$\begin{aligned}
&= (\sin x + \cos x)\left(1 - \frac{2\sin x\cos x}{2}\right) \\
&= (\sin x + \cos x)(1 - \sin x\cos x) \\
&= \sin x + \cos x - \sin^2 x\cos x - \sin x\cos^2 x \\
&= \sin x + \cos x - (1-\cos^2 x)\cos x - \sin x\left(1 - \sin^2 x\right) \\
&= \sin x + \cos x - \cos x + \cos^3 x - \sin x + \sin^3 x \\
&= \sin^3 x + \cos^3 x
\end{aligned}$$

107. $$\begin{aligned}
\sin\left(2\sin^{-1}\frac{\sqrt{3}}{2}\right) &= \sin\left(2\cdot\frac{\pi}{3}\right) \\
&= \sin\frac{2\pi}{3} \\
&= \frac{\sqrt{3}}{2}
\end{aligned}$$

108. $\cos\left[2\tan^{-1}\left(-\frac{4}{3}\right)\right]$

Let $\theta = \tan^{-1}\left(-\frac{4}{3}\right)$

Since θ is in quadrant IV, x is positive and y is negative.

$$\tan\theta = \frac{y}{x} = \frac{-4}{3}$$

$$\begin{aligned}
r^2 &= x^2 + y^2 \\
r^2 &= 3^2 + (-4)^2 \\
r^2 &= 25 \\
r &= 5
\end{aligned}$$

$$\begin{aligned}
\cos\left[2\tan^{-1}\left(-\frac{4}{3}\right)\right] &= \cos 2\theta \\
&= 1 - 2\sin^2\theta \\
&= 1 - 2\left(\frac{y}{r}\right)^2 \\
&= 1 - 2\left(\frac{-4}{5}\right)^2 \\
&= 1 - 2\cdot\frac{16}{25} \\
&= -\frac{7}{25}
\end{aligned}$$

109. $\cos^2\left[\frac{1}{2}\sin^{-1}\frac{3}{5}\right]$

Let $\theta = \sin^{-1}\frac{3}{5}$, then $\sin\theta = \frac{y}{r} = \frac{3}{5}$

Since θ is in quadrant I, x is positive.

$$\begin{aligned}
x^2 + y^2 &= r^2 \\
x^2 + 3^2 &= 5^2 \\
x^2 &= 16 \\
x &= 4
\end{aligned}$$

$$\begin{aligned}
\cos^2\ \frac{1}{2}\sin^{-1}\frac{3}{5} &= \cos^2\ \frac{1}{2}\theta \\
&= \frac{1 + \cos\ 2\cdot\frac{1}{2}\theta}{2} \\
&= \frac{1+\cos\theta}{2} \\
&= \frac{1 + \frac{4}{5}}{2} \\
&= \frac{1 + \frac{x}{r}}{2} \\
&= \frac{9}{10}
\end{aligned}$$

110. $\sin^2\left[\frac{1}{2}\cos^{-1}\frac{3}{5}\right]$

Let $\theta = \cos^{-1}\frac{3}{5}$, then $\cos\theta = \frac{3}{5}$

$$\sin^2\ \frac{1}{2}\cos^{-1}\frac{3}{5}\ = \sin^2\ \frac{1}{2}\theta\ = \frac{1-\cos\ \ 2\cdot\frac{1}{2}\theta}{2} = \frac{1-\cos\theta}{2} = \frac{1-\frac{3}{5}}{2} = \frac{1}{5}$$

111. Let $\alpha = \sin^{-1}x$ where $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.

$\sin\alpha = x$

Because x is positive, sin α is positive. Thus, α is in quadrant I. Using a right triangle in quadrant I with $\sin\alpha = x = \frac{x}{1}$ the third side a can be found using the Pythagorean Theorem.

$\alpha^2 + x^2 = 1^2$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\cos\alpha = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

$\sin(2\sin^{-1}x) = \sin 2\alpha = 2x\sqrt{1-x^2}$

112. $\sin^6 x = \left(\sin^2 x\right)^3 = \left(\frac{1-\cos 2x}{2}\right)^3$

$$= \frac{1-3\cos 2x + 3\cos^2 2x - \cos^3 2x}{8}$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos 2x\cos^2 2x$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{8}\left[\frac{1+\cos(2\cdot 2x)}{2}\right] - \frac{1}{8}\cos 2x\left[\frac{1+\cos(2\cdot 2x)}{2}\right]$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{16}(1+\cos 4x) - \frac{1}{16}\cos 2x(1+\cos 4x)$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{16} + \frac{3}{16}\cos 4x - \frac{1}{16}\cos 2x - \frac{1}{16}\cos 2x\cos 4x$$

$$= \frac{5}{16} - \frac{7}{16}\cos 2x + \frac{3}{16}\cos 4x - \frac{1}{16}\cos 2x\cos 4x$$

113. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find *b*.

$$a^2 + b^2 = c^2$$
$$1^2 + b^2 = 2^2$$
$$1 + b^2 = 4$$
$$b^2 = 3$$
$$b = \sqrt{3} = \sqrt{3}$$

Note that side *a* is opposite θ and side *b* is adjacent to θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{2}{1} = 2$$
$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$
$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

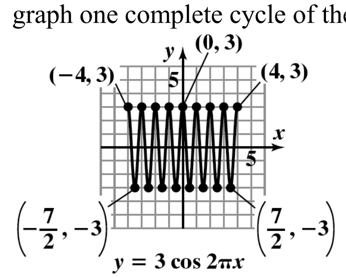
114. The equation $y = 3\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 3$ and $B = 2\pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate *x*-values for the key points.

$$x = 0$$
$$x = 0 + \frac{1}{4} = \frac{1}{4}$$
$$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$
$$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$
$$x = \frac{3}{4} + \frac{1}{4} = 1$$

Evaluate the function at each value of *x*.

x	$y = 3\cos 2\pi x$	coordinates
0	$y = 3\cos(2\pi\cdot 0) = 3\cos 0 = 3\cdot 1 = 3$	(0, 3)
$\frac{1}{4}$	$y = 3\cos\left(2\pi\cdot\frac{1}{4}\right) = 3\cos\frac{\pi}{2} = 3\cdot 0 = 0$	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	$y = 3\cos\left(2\pi\cdot\frac{1}{2}\right) = 3\cos\pi = 3\cdot(-1) = -3$	$\left(\frac{1}{2}, -3\right)$
$\frac{3}{4}$	$y = 3\cos\left(2\pi\cdot\frac{3}{4}\right) = 3\cos\frac{3\pi}{2} = 3\cdot 0 = 0$	$\left(\frac{3}{4}, 0\right)$
1	$y = 3\cos(2\pi\cdot 1) = 3\cos 2\pi = 3\cdot 1 = 3$	(1, 3)

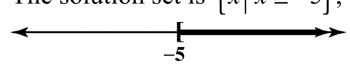
Connect the five points with a smooth curve and graph one complete cycle of the given function.



115. $\frac{2x-3}{8} \le \frac{3x}{8} + \frac{1}{4}$

$$2x - 3 \le 3x + 2$$
$$-x \le 5$$
$$x \ge -5$$

The solution set is $\{x \mid x \ge -5\}$, or $[-5, \infty)$.



116. $\sin 60^\circ \sin 30^\circ = \frac{1}{2}\left[\cos(60^\circ - 30^\circ) - \cos(60^\circ + 30^\circ)\right]$

$$\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{1}{2}\left[\cos 30^\circ - \cos 90^\circ\right]$$
$$\frac{\sqrt{3}}{4} = \frac{1}{2}\left[\frac{\sqrt{3}}{2} - 0\right]$$
$$\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$$

117. $\cos\frac{\pi}{2}\cos\frac{\pi}{3} = \frac{1}{2}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right]$

$$0\cdot\frac{1}{2} = \frac{1}{2}\left[\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{5\pi}{6}\right)\right]$$
$$0 = \frac{1}{2}\left[\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right]$$
$$0 = \frac{1}{2}[0]$$
$$0 = 0$$

118. $\sin\pi\cos\frac{\pi}{2} = \frac{1}{2}\left[\sin\left(\pi+\frac{\pi}{2}\right)+\sin\left(\pi-\frac{\pi}{2}\right)\right]$

$$0\cdot 0 = \frac{1}{2}\left[\sin\left(\frac{3\pi}{2}\right)+\sin\left(\frac{\pi}{2}\right)\right]$$
$$0\cdot 0 = \frac{1}{2}[-1+1]$$
$$0 = \frac{1}{2}[0]$$
$$0 = 0$$

Mid-Chapter 5 Check Point

1. $\cos x(\tan x + \cot x)$

$$= \cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$
$$= \cos x\left(\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}\right)$$
$$= \cos x\left(\frac{\sin^2 x}{\sin x\cos x}+\frac{\cos^2 x}{\sin x\cos x}\right)$$
$$= \cos x\left(\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)$$
$$= \cos x\left(\frac{1}{\sin x\cos x}\right)$$
$$= \frac{\cos x}{\sin x\cos x}$$
$$= \frac{1}{\sin x}$$
$$= \csc x$$

2. $\frac{\sin(x+\pi)}{\cos\left(x+\frac{3\pi}{2}\right)} = \frac{-\sin x}{\cos\left((x+\pi)+\frac{\pi}{2}\right)}$

$$= \frac{-\sin x}{-\sin(x+\pi)}$$
$$= \frac{-\sin x}{\sin x}$$
$$= -1$$
$$= -\left(\sec^2 x-\tan^2 x\right)$$
$$= \tan^2 x-\sec^2 x$$

3. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$

$$= \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$$
$$+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$$
$$= \sin^2\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta$$
$$= 1+1$$
$$= 2$$

4. $\frac{\sin t-1}{\cos t} = \frac{\sin t-1}{\cos t}\cdot\frac{\cot t}{\cot t}$

$$= \frac{\sin t\cot t-\cot t}{\cos t\cot t}$$
$$= \frac{\sin t\cdot\frac{\cos t}{\sin t}-\cot t}{\cos t\cot t}$$
$$= \frac{\cos t-\cot t}{\cos t\cot t}$$

5. $\frac{1-\cos 2x}{\sin 2x} = \frac{1-2\cos^2 x-1}{2\sin x\cos x}$

$$= \frac{2\cos^2 x}{2\sin x\cos x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \tan x$$

6. $\sin\theta\cos\theta+\cos^2\theta$

$$= \cos\theta(\sin\theta+\cos\theta)$$
$$= \cos\theta(\sin\theta+\cos\theta)\cdot\frac{\csc\theta}{\csc\theta}$$
$$= \frac{\cos\theta(\sin\theta\csc\theta+\cos\theta\csc\theta)}{\csc\theta}$$
$$= \frac{\cos\theta\left(\sin\theta\cdot\frac{1}{\sin\theta}+\cos\theta\cdot\frac{1}{\sin\theta}\right)}{\csc\theta}$$
$$= \frac{\cos\theta(1+\tan\theta)}{\csc\theta}$$

7. $\frac{\sin x}{\tan x}+\frac{\cos x}{\cot x} = \frac{\sin x}{\frac{\sin x}{\cos x}}+\frac{\cos x}{\frac{\cos x}{\sin x}}$

$$= \sin x\cdot\frac{\cos x}{\sin x}+\cos x\cdot\frac{\sin x}{\cos x}$$
$$= \cos x+\sin x$$
$$= \sin x+\cos x$$

8. $\sin^2\frac{t}{2}=\left(\sin\frac{t}{2}\right)^2$

$=\left(\pm\sqrt{\frac{1-\cos t}{2}}\right)^2$

$=\frac{1-\cos t}{2}$

$=\frac{1-\cos t}{2}\cdot\frac{\tan t}{\tan t}$

$=\frac{\tan t-\cos t\tan t}{2\tan t}$

$=\frac{\tan t-\cos t\cdot\frac{\sin t}{\cos t}}{2\tan t}$

$=\frac{\tan t-\sin t}{2\tan t}$

9. $\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$

$=\frac{1}{2}[\sin\alpha\cos\beta+\cos\alpha\sin\beta+$
$\sin\alpha\cos\beta-\cos\alpha\sin\beta]$

$=\frac{1}{2}[2\sin\alpha\cos\beta]$

$=\sin\alpha\cos\beta$

10. $\frac{1+\csc x}{\sec x}-\cot x=\frac{1+\frac{1}{\sin x}}{\frac{1}{\cos x}}-\frac{\cos x}{\sin x}$

$=\cos x\left(1+\frac{1}{\sin x}\right)-\frac{\cos x}{\sin x}$

$=\cos x+\frac{\cos x}{\sin x}-\frac{\cos x}{\sin x}$

$=\cos x$

11. $\frac{\cot x-1}{\cot x+1}=\frac{\frac{\cot x}{\cot x}-\frac{1}{\cot x}}{\frac{\cot x}{\cot x}+\frac{1}{\cot x}}$

$=\frac{1-\tan x}{1+\tan x}$

12. $2\sin^3\theta\cos\theta+2\sin\theta\cos^3\theta$

$=2\sin\theta\cos\theta\left(\sin^2\theta+\cos^2\theta\right)$

$=2\sin\theta\cos\theta$

$=\sin 2\theta$

13. $\frac{\sin t+\cos t}{\sec t+\csc t}=\frac{\sin t+\cos t}{\frac{1}{\cos t}+\frac{1}{\sin t}}$

$=\frac{\sin t+\cos t}{\frac{1}{\cos t}\cdot\frac{\sin t}{\sin t}+\frac{1}{\sin t}\cdot\frac{\cos t}{\cos t}}$

$=\frac{\sin t+\cos t}{\frac{\sin t+\cos t}{\sin t\cos t}}$

$=(\sin t+\cos t)\frac{\sin t\cos t}{\sin t+\cos t}$

$=\sin t\cos t$

$=\sin t\frac{1}{\sec t}$

$=\frac{\sin t}{\sec t}$

14. $\frac{\sec^2 x}{2-\sec^2 x}=\frac{\frac{\sec^2 x}{\sec^2 x}}{\frac{2}{\sec^2 x}-\frac{\sec^2 x}{\sec^2 x}}$

$=\frac{1}{2\cos^2 x-1}$

$=\frac{1}{\cos 2x}$

$=\sec 2x$

15. $\tan(\alpha+\beta)\tan(\alpha-\beta)$

$=\tan(\alpha+\beta)\tan(\alpha-\beta)$

$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\cdot\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$=\frac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$

16. $\frac{\sin\theta}{1-\cos\theta}=\frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$

$=\frac{\sin\theta+\sin\theta\cos\theta}{1-\cos^2\theta}$

$=\frac{\sin\theta+\sin\theta\cos\theta}{\sin^2\theta}$

$=\frac{\sin\theta}{\sin^2\theta}+\frac{\sin\theta\cos\theta}{\sin^2\theta}$

$=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$

$=\csc\theta+\cot\theta$

17. $\frac{1}{\csc 2x} = \sin 2x$
$= 2\sin x\cos x$
$= 2\sin x\cos x \cdot \frac{\cos x}{\cos x}$
$= \frac{2\sin x\cos^2 x}{\cos x}$
$= \frac{2\sin x}{\cos x}\cdot\cos^2 x$
$= 2\tan x \cdot \frac{1}{\sec^2 x}$
$= \frac{2\tan x}{\sec^2 x}$
$= \frac{2\tan x}{1+\tan^2 x}$

18. $\frac{\sec t - 1}{t\sec t} = \frac{\sec t - 1}{t\sec t}\cdot\frac{\cos t}{\cos t}$
$= \frac{\sec t\cos t - \cos t}{t\sec t\cos t}$
$= \frac{\frac{1}{\cos t}\cos t - \cos t}{t\frac{1}{\cos t}\cos t}$
$= \frac{1-\cos t}{t}$

19. Use $\sin\alpha = \frac{3}{5} = \frac{y}{r}$ to find $\cos\alpha$ and $\tan\alpha$.

Because α is in Quadrant II, x is negative.

$x^2 + y^2 = r^2$
$x^2 + 3^2 = 5^2$
$x^2 = 16$
$x = -\sqrt{16}$
$x = -4$

Thus, $\cos\alpha = \frac{-4}{5} = -\frac{4}{5}$ and $\tan\alpha = \frac{-3}{4} = -\frac{3}{4}$.

Use $\cos\beta = \frac{-12}{13} = \frac{x}{r}$ to find $\sin\beta$ and $\tan\beta$.

Because β is in Quadrant III, x and y are negative.

$x^2 + y^2 = r^2$
$(-12)^2 + y^2 = 13^2$
$y^2 = 25$
$y = -\sqrt{25}$
$y = -5$

Thus, $\sin\beta = \frac{-5}{13} = -\frac{5}{13}$ and $\tan\beta = \frac{-5}{-12} = \frac{5}{12}$.

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(-\frac{5}{13}\right)$
$= \frac{33}{65}$

20. In exercise 19 it was shown that

$\tan\alpha = -\frac{3}{4}$ and $\tan\beta = \frac{5}{12}$. Thus,

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$
$= \frac{-\frac{3}{4}+\frac{5}{12}}{1-\left(-\frac{3}{4}\right)\frac{5}{12}}$
$= -\frac{16}{63}$

21. In exercise 19 it was shown that

$\sin\alpha = \frac{3}{5}$ and $\cos\alpha = -\frac{4}{5}$.

Thus, $\sin 2\alpha = 2\sin\alpha\cos\alpha$
$= 2\cdot\frac{3}{5}\left(-\frac{4}{5}\right)$
$= -\frac{24}{25}$

22. $\cos\beta = -\frac{12}{13}$.

Since β is in quadrant III, $\frac{\beta}{2}$ is in quadrant II.

The cosine is negative in quadrant II.

$\cos\frac{\beta}{2} = -\sqrt{\frac{1+\cos\beta}{2}}$
$= -\sqrt{\frac{1+\left(-\frac{12}{13}\right)}{2}}$
$= -\sqrt{\frac{1}{26}}$
$= -\frac{1}{\sqrt{26}}$
$= -\frac{\sqrt{26}}{26}$

23. $\sin\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right)$
$= \sin\frac{3\pi}{4}\cos\frac{5\pi}{6}+\cos\frac{3\pi}{4}\sin\frac{5\pi}{6}$
$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
$= -\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$
$= -\frac{\sqrt{6}+\sqrt{2}}{4}$

24. $\cos^2 15° - \sin^2 15° = \cos(2\cdot 15°)$
$= \cos 30°$
$= \frac{\sqrt{3}}{2}$

25. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$
$= \cos\frac{4\pi}{12}$
$= \cos\frac{\pi}{3}$
$= \frac{1}{2}$

26. $\tan 22.5° = \tan\frac{45°}{2}$
$= \frac{\sin 45°}{1+\cos 45°}$
$= \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$
$= \frac{\sqrt{2}}{2+\sqrt{2}}$
$= \frac{\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}$
$= \frac{2\sqrt{2}-2}{4-2}$
$= \frac{2\sqrt{2}-2}{2}$
$= \sqrt{2}-1$

Section 5.4

Check Point Exercises

1. **a.** $\sin 5x \sin 2x$
$= \frac{1}{2}[\cos(5x-2x)-\cos(5x+2x)]$
$= \frac{1}{2}[\cos 3x - \cos 7x]$

b. $\cos 7x \cos x$
$= \frac{1}{2}[\cos(7x-x)+\cos(7x+x)]$
$= \frac{1}{2}[\cos 6x + \cos 8x]$

2. **a.** $\sin 7x + \sin 3x$
$= 2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)$
$= 2\sin\left(\frac{10x}{2}\right)\cos\left(\frac{4x}{2}\right)$
$= 2\sin 5x \cos 2x$

b. $\cos 3x + \cos 2x$
$= 2\cos\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right)$
$= 2\cos\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)$

3. $\frac{\cos 3x - \cos x}{\sin 3x + \sin x} = \frac{-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-2x}{2}\right)}{2\sin\frac{3x+x}{2}\cos\left(\frac{3x-x}{2}\right)}$
$= \frac{-2\sin 2x \sin x}{2\sin 2x \cos x}$
$= \frac{-\sin x}{\cos x}$
$= -\tan x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

Concept and Vocabulary Check 5.4

1. product; difference

2. product; sum

3. product; sum

4. product; difference

5. sum; product

6. difference; product

7. sum; product

8. difference; product

Exercise Set 5.4

1. $\sin 6x \sin 2x = \frac{1}{2}[\cos(6x-2x) - \cos(6x+2x)]$
$= \frac{1}{2}[\cos 4x - \cos 8x]$

2. $\sin 8x \sin 4x = \frac{1}{2}[\cos(8x-4x) - \cos(8x+4x)]$
$= \frac{1}{2}(\cos 4x - \cos 12x)$

3. $\cos 7x \cos 3x = \frac{1}{2}[\cos(7x-3x) + \cos(7x+3x)]$
$= \frac{1}{2}[\cos 4x + \cos 10x]$

4. $\cos 9x \cos 2x = \frac{1}{2}[\cos(9x-2x) + \cos(9x+2x)]$
$= \frac{1}{2}[\cos 7x + \cos 11x]$

5. $\sin x \cos 2x = \frac{1}{2}[\sin(x+2x) + \sin(x-2x)]$
$= \frac{1}{2}[\sin 3x + \sin(-x)]$
$= \frac{1}{2}[\sin 3x - \sin x]$

6. $\sin 2x \cos 3x = \frac{1}{2}[\cos(2x+3x) + \sin(2x-3x)]$
$= \frac{1}{2}[\cos 5x + \sin(-x)]$
$= \frac{1}{2}(\cos 5x - \sin x)$

7. $\cos\frac{3x}{2}\sin\frac{x}{2} = \frac{1}{2}\left[\sin\left(\frac{3x}{2}+\frac{x}{2}\right) - \sin\left(\frac{3x}{2}-\frac{x}{2}\right)\right]$
$= \frac{1}{2}\left[\sin\left(\frac{4x}{2}\right) - \sin\left(\frac{2x}{2}\right)\right]$
$= \frac{1}{2}[\sin 2x - \sin x]$

8. $\cos\frac{5x}{2}\sin\frac{x}{2} = \frac{1}{2}\left[\sin\left(\frac{5x}{2}+\frac{x}{2}\right) - \sin\left(\frac{5x}{2}-\frac{x}{2}\right)\right]$
$= \frac{1}{2}\left[\sin\left(\frac{6x}{2}\right) - \sin\left(\frac{4x}{2}\right)\right]$
$= \frac{1}{2}[\sin 3x - \sin 2x]$

9. $\sin 6x + \sin 2x = 2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right)$
$= 2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{4x}{2}\right)$
$= 2\sin 4x \cos 2x$

10. $\sin 8x + \sin 2x = 2\sin\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right)$
$= 2\sin\left(\frac{10x}{2}\right)\cos\left(\frac{6x}{2}\right)$
$= 2\sin 5x \cos 3x$

11. $\sin 7x - \sin 3x = 2\sin\left(\frac{7x-3x}{2}\right)\cos\left(\frac{7x+3x}{2}\right)$
$= 2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{10x}{2}\right)$
$= 2\sin 2x \cos 5x$

12. $\sin 11x - \sin 5x = 2\sin\left(\frac{11x-5x}{2}\right)\cos\left(\frac{11x+5x}{2}\right)$
$= 2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{16x}{2}\right)$
$= 2\sin 3x \cos 8x$

13. $\cos 4x + \cos 2x = 2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)$
$= 2\cos\left(\frac{6x}{2}\right)\cos\left(\frac{2x}{2}\right)$
$= 2\cos 3x \cos x$

14. $\cos 9x - \cos 7x = -2\sin\left(\frac{9x+7x}{2}\right)\sin\left(\frac{9x-7x}{2}\right)$
$= -2\sin\left(\frac{16x}{2}\right)\sin\left(\frac{2x}{2}\right)$
$= -2\sin 8x \sin x$

15. $\sin x + \sin 2x = 2\sin\left(\frac{x+2x}{2}\right)\cos\left(\frac{x-2x}{2}\right)$
$= 2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{-x}{2}\right)$
$= 2\sin\frac{3x}{2}\cos\frac{x}{2}$

16. $\sin x - \sin 2x = 2\sin\left(\frac{x-2x}{2}\right)\cos\left(\frac{x+2x}{2}\right)$
$= 2\sin\left(\frac{-x}{2}\right)\cos\left(\frac{3x}{2}\right)$
$= -2\sin\frac{x}{2}\cos\frac{3x}{2}$

17. $\cos\frac{3x}{2}+\cos\frac{x}{2}=2\cos\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right)$
$=2\cos\left(\frac{4x}{4}\right)\cos\left(\frac{2x}{4}\right)$
$=2\cos x\cos\frac{x}{2}$

18. $\sin\frac{3x}{2}+\sin\frac{x}{2}=2\sin\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right)$
$=2\sin\left(\frac{4x}{4}\right)\cos\left(\frac{2x}{4}\right)$
$=2\sin x\cos\frac{x}{2}$

19. $\sin 75^\circ+\sin 15^\circ$
$=2\sin\left(\frac{75^\circ+15^\circ}{2}\right)\cos\left(\frac{75^\circ-15^\circ}{2}\right)$
$=2\sin(45^\circ)\cos(30^\circ)$
$=2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
$=\frac{\sqrt{6}}{2}$

20. $\cos 75^\circ-\cos 15^\circ$
$=-2\sin\left(\frac{75^\circ+15^\circ}{2}\right)\sin\left(\frac{75^\circ-15^\circ}{2}\right)$
$=-2\sin(45^\circ)\sin(30^\circ)$
$=-2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)=-\frac{\sqrt{2}}{2}$

21. $\sin\frac{\pi}{12}-\sin\frac{5\pi}{12}$
$=2\sin\left(\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}\right)\cos\left(\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\right)$
$=2\sin\left(-\frac{4\pi}{24}\right)\cos\left(\frac{6\pi}{24}\right)$
$=-2\sin\frac{\pi}{6}\cos\frac{\pi}{4}$
$=-2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$=-\frac{\sqrt{2}}{2}$

22. $\cos\frac{\pi}{12}-\cos\frac{5\pi}{12}$
$=-2\sin\left(\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\right)\sin\left(\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}\right)$
$=-2\sin\left(\frac{6\pi}{24}\right)\sin\left(-\frac{4\pi}{24}\right)$
$=2\sin\frac{\pi}{4}\sin\frac{\pi}{6}$
$=2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{2}}{2}$

23. $\frac{\sin 3x-\sin x}{\cos 3x-\cos x}$
$=\frac{2\sin\left(\frac{3x-x}{2}\right)\cos\left(\frac{3x+x}{2}\right)}{-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}$
$=\frac{2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{4x}{2}\right)}{-2\sin\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}$
$=\frac{2\sin x\cos 2x}{-2\sin 2x\sin x}$
$=-\frac{\cos 2x}{\sin 2x}=-\cot 2x$

24. $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$
$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$
$=\frac{2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right)}$
$=\frac{2\sin 2x\cos(-x)}{2\cos 2x\cos(-x)}$
$=\frac{\sin 2x}{\cos 2x}=\tan 2x$

25. $\dfrac{\sin 2x+\sin 4x}{\cos 2x+\cos 4x}$

$=\dfrac{2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)}{2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)}$

$=\dfrac{2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{2\cos\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right)}$

$=\dfrac{2\sin 3x\cos(-x)}{2\cos 3x\cos(-x)}$

$=\dfrac{\sin 3x}{\cos 3x}$

$=\tan 3x$

26. $\dfrac{\cos 4x-\cos 2x}{\sin 2x-\sin 4x}$

$=\dfrac{-2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)}{2\sin\left(\frac{2x-4x}{2}\right)\cos\left(\frac{2x+4x}{2}\right)}$

$=\dfrac{-2\sin\left(\frac{6x}{2}\right)\sin\left(\frac{2x}{2}\right)}{2\sin\left(\frac{-2x}{2}\right)\cos\left(\frac{6x}{2}\right)}$

$=\dfrac{-2\sin 3x\sin x}{2\sin(-x)\cos 3x}$

$=\dfrac{-\sin x\sin 3x}{-\sin x\cos 3x}$

$=\dfrac{\sin 3x}{\cos 3x}$

$=\tan 3x$

27. $\dfrac{\sin x-\sin y}{\sin x+\sin y}=\dfrac{2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$

$=\dfrac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}\cdot\dfrac{\cos\left(\frac{x+y}{2}\right)}{\sin\left(\frac{x+y}{2}\right)}$

$=\tan\dfrac{x-y}{2}\cot\dfrac{x+y}{2}$

28. $\dfrac{\sin x+\sin y}{\sin x-\sin y}=\dfrac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)}$

$=\dfrac{\sin\left(\frac{x+y}{2}\right)}{\cos\left(\frac{x+y}{2}\right)}\cdot\dfrac{\cos\left(\frac{x-y}{2}\right)}{\sin\left(\frac{x-y}{2}\right)}$

$=\tan\dfrac{x+y}{2}\cot\dfrac{x-y}{2}$

29. $\dfrac{\sin x+\sin y}{\cos x+\cos y}=\dfrac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$

$=\dfrac{\sin\left(\frac{x+y}{2}\right)}{\cos\left(\frac{x+y}{2}\right)}$

$=\tan\dfrac{x+y}{2}$

30. $\dfrac{\sin x-\sin y}{\cos x-\cos y}=\dfrac{2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)}{-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}$

$=-\dfrac{\sin\left(\frac{x-y}{2}\right)}{\sin\left(\frac{x-y}{2}\right)}\cdot\dfrac{\cos\left(\frac{x+y}{2}\right)}{\sin\left(\frac{x+y}{2}\right)}$

$=-\cot\dfrac{x+y}{2}$

31. a. $y = \cos x$ also describes the graph.

b. $$\frac{\sin x + \sin 3x}{2\sin 2x} = \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\sin 2x} = \frac{2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{2\sin 2x}$$
$$= \frac{2\sin 2x\cos(-x)}{2\sin 2x} = \cos(-x) = \cos x$$

32. a. $y = \tan x$ also describes the graph.

b. $$\frac{\cos x - \cos 3x}{\sin x + \sin 3x} = \frac{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$
$$= \frac{-2\sin 3x\sin(-x)}{2\sin 3x\cos(-x)}$$
$$= \frac{-\sin(-x)}{\cos x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

33. a. $y = \tan 2x$ also describes the graph.

b. $$\frac{\cos x - \cos 5x}{\sin x + \sin 5x} = \frac{-2\sin\left(\frac{x+5x}{2}\right)\sin\left(\frac{x-5x}{2}\right)}{2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)} = \frac{-2\sin\left(\frac{6x}{2}\right)\sin\left(\frac{-4x}{2}\right)}{2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{-4x}{2}\right)}$$
$$= \frac{-2\sin 3x\sin(-2x)}{2\sin 3x\cos(-2x)} = \frac{-\sin(-2x)}{\cos 2x} = \frac{\sin 2x}{\cos 2x} = \tan 2x$$

34. a. $y = -\tan x$ also describes the graph.

b. $$\frac{\cos 5x - \cos 3x}{\sin 5x + \sin 3x} = \frac{-2\sin\frac{5x+3x}{2}\sin\frac{5x-3x}{2}}{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}} = \frac{-2\sin 4x\sin x}{2\sin 4x\cos x} = \frac{-\sin x}{\cos x} = -\tan x$$

35. a. $y = -\cot 2x$ also describes the graph.

b. $$\frac{\sin x - \sin 3x}{\cos x - \cos 3x} = \frac{2\sin\frac{x-3x}{2}\cos\frac{x+3x}{2}}{-2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}} = \frac{2\sin(-x)\cos 2x}{-2\sin 2x\sin(-x)} = \frac{\cos 2x}{-\sin 2x} = -\cot 2x$$

36. a. $y = -\cot 2x$ also describes the graph.

b. $$\frac{\sin 2x + \sin 6x}{\cos 6x - \cos 2x} = \frac{2\sin\frac{2x+6x}{2}\cos\frac{2x-6x}{2}}{-2\sin\frac{6x+2x}{2}\sin\frac{6x-2x}{2}} = \frac{2\sin 4x\cos(-2x)}{-2\sin 4x\sin 2x} = \frac{\cos 2x}{-\sin 2x} = -\cot 2x$$

37. a. The low frequency is l = 852 cycles per second and the high frequency is h = 1209 cycles per second. The sound produced by touching 7 is described by $y = \sin 2\pi(852)t + \sin 2\pi(1209)t$, or $y = \sin 1704\pi t + \sin 2418\pi t$.

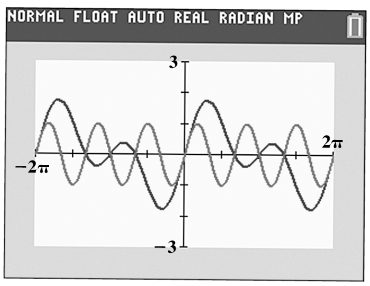
b. $$\begin{aligned} y &= \sin 1704\pi t + \sin 2418\pi t \\ &= 2\sin\left(\frac{1704\pi t + 2418\pi t}{2}\right)\cdot\cos\left(\frac{1704\pi t - 2418\pi t}{2}\right) \\ &= 2\sin 2061\pi t \cdot \cos(-357\pi t) \\ &= 2\sin 2061\pi t \cdot \cos 357\pi t \end{aligned}$$

38. a. The low frequency is l = 697 cycles per second and the high frequency is h = 1477 cycles per second. The sound produced by touching 3 is described by $y = \sin 2\pi(697)t + \sin 2\pi(1477)t$, or $y = \sin 1394\pi t + \sin 2954\pi t$.

b. $$\begin{aligned} y &= \sin 2954\pi t + \sin 1394\pi t \\ &= 2\sin\left(\frac{2954\pi t + 1394\pi t}{2}\right)\cdot\cos\left(\frac{2954\pi t - 1394\pi t}{2}\right) \\ &= 2\sin 2174\pi t \cdot \cos 780\pi t \end{aligned}$$

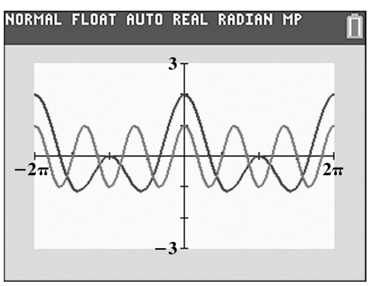
39. – 44. Answers may vary.

45.



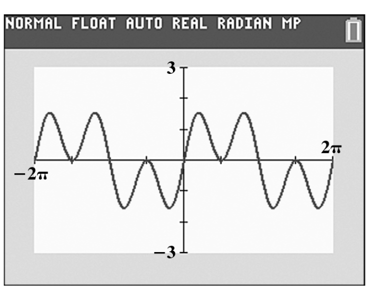
The graphs do not coincide.
Values for x may vary.

46.



The graphs do not coincide. Values for x may vary.

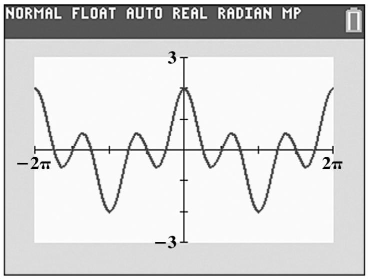
47.



$$\begin{aligned} \sin x + \sin 3x &= 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right) \\ &= 2\sin 2x\cos(-x) \\ &= 2\sin 2x\cos x \end{aligned}$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

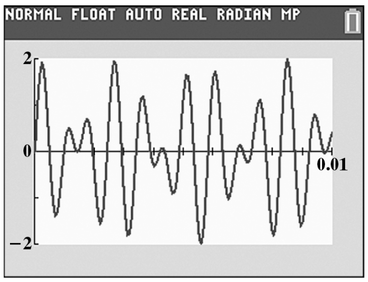
48.



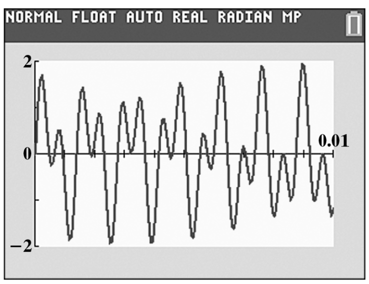
$$\cos x+\cos 3x = 2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)$$
$$= 2\cos 2x\cos(-x)$$
$$= 2\cos 2x\cos x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

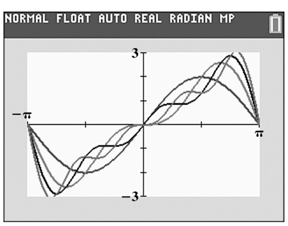
49.



50.

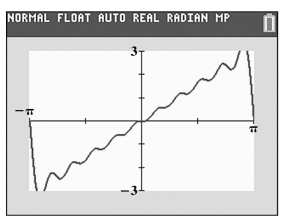


51. **a.**



Answers may vary.

b.



Answers may vary.

c. When $x=\frac{\pi}{2}$,

$$\frac{\pi}{2} = 2\left(\frac{\sin\frac{\pi}{2}}{1}-\frac{\sin\left(2\cdot\frac{\pi}{2}\right)}{2}+\frac{\sin\left(3\cdot\frac{\pi}{2}\right)}{3}-\frac{\sin\left(4\cdot\frac{\pi}{2}\right)}{4}+\frac{\sin\left(5\cdot\frac{\pi}{2}\right)}{5}-\frac{\sin\left(6\cdot\frac{\pi}{2}\right)}{6}+\frac{\sin\left(7\cdot\frac{\pi}{2}\right)}{7}-\frac{\sin\left(8\cdot\frac{\pi}{2}\right)}{8}+\cdots\right)$$
$$= 2\left(1-0+\left(-\frac{1}{3}\right)-0+\frac{1}{5}-0+\left(-\frac{1}{7}\right)+\cdots\right)$$
$$= 2-\frac{2}{3}+\frac{2}{5}-\frac{2}{7}+\cdots$$

Multiplying both sides by 2 gives: $\pi = 4-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\cdots$

52. makes sense

53. makes sense

54. makes sense

55. makes sense

56.
$$\begin{array}{r} \sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \\ +\ [\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta] \\ \hline \sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta \end{array}$$

Solve for $\sin\alpha\cos\beta$ by multiplying both sides by $\frac{1}{2}$: $\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]=\sin\alpha\cos\beta$

57.
$$\begin{array}{r} \sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \\ -[\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta] \\ \hline \sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta \end{array}$$

Solve for $\cos\alpha\sin\beta$ by multiplying both sides by $\frac{1}{2}$: $\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]=\cos\alpha\sin\beta$

58.
$$\begin{aligned} 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} &= 2\cdot\frac{1}{2}\left[\sin\left(\frac{\alpha-\beta}{2}+\frac{\alpha+\beta}{2}\right)+\sin\left(\frac{\alpha-\beta}{2}-\frac{\alpha+\beta}{2}\right)\right] \\ &= \sin\left(\frac{2\alpha}{2}\right)+\sin\left(\frac{-2\beta}{2}\right) \\ &= \sin\alpha+\sin(-\beta) \\ &= \sin\alpha-\sin\beta \end{aligned}$$

59.
$$\begin{aligned} 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} &= 2\cdot\frac{1}{2}\left[\cos\left(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right)\right] \\ &= \cos\left(\frac{2\beta}{2}\right)+\cos\left(\frac{2\alpha}{2}\right) \\ &= \cos\beta+\cos\alpha \\ &= \cos\alpha+\cos\beta \end{aligned}$$

60.
$$\begin{aligned} \frac{\sin 2x+(\sin 3x+\sin x)}{\cos 2x+(\cos 3x+\cos x)} &= \frac{\sin 2x+2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{\cos 2x+2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)} \\ &= \frac{\sin 2x+2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)}{\cos 2x+2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)} \\ &= \frac{\sin 2x+2\sin 2x\cos x}{\cos 2x+2\cos 2x\cos x} \\ &= \frac{\sin 2x(1+2\cos x)}{\cos 2x(1+2\cos x)} \\ &= \frac{\sin 2x}{\cos 2x} \\ &= \tan 2x \end{aligned}$$

61.
$$\begin{aligned}
\sin 2x+\sin 4x+\sin 6x &= \sin 4x+(\sin 2x+\sin 6x)\\
&= \sin 4x+2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\\
&= \sin 4x+2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{-4x}{2}\right)\\
&= \sin 4x+2\sin 4x\cos(-2x)\\
&= \sin 4x+2\sin 4x\cos 2x\\
&= \sin(2\cdot 2x)+2\sin 4x\cos 2x\\
&= 2\sin 2x\cos 2x+2\sin 4x\cos 2x\\
&= 2\cos 2x(\sin 2x+\sin 4x)\\
&= 2\cos 2x\left(2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)\right)\\
&= 2\cos 2x\cdot 2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right)\\
&= 2\cos 2x\cdot 2\sin 3x\cos(-x)\\
&= 4\cos 2x\sin 3x\cos x\\
&= 4\cos x\cos 2x\sin 3x
\end{aligned}$$

62. Answers may vary.

63. The formula $s=r\theta$ can only be used when θ is expressed in radians. Thus, we begin by converting 150° to radians.

Multiply by $\frac{\pi \text{ radians}}{180°}$.

$$\begin{aligned}
150° &= 150°\cdot\frac{\pi \text{ radians}}{180°}=\frac{150}{180}\pi \text{ radians}\\
&= \frac{5}{6}\pi \text{ radians}
\end{aligned}$$

Now we can use the formula $s=r\theta$ to find the length of the arc. The circle's radius is 8 inches :

r = 8 inches. The measure of the central angle in radians is $\frac{5}{6}\pi : \theta=\frac{5}{6}\pi$. The length of the arc intercepted by this central angle is

$$\begin{aligned}
s &= r\theta\\
&= (8 \text{ inches})\left(\frac{5}{6}\pi\right)\\
&= \frac{20\pi}{3} \text{ inches}\\
&\approx 20.94 \text{ inches.}
\end{aligned}$$

64. Using $y=A\cos Bx$ the amplitude is 2 and $A=2$, The period is 8 and thus

$$B=\frac{2\pi}{\text{period}}=\frac{2\pi}{8}=\frac{\pi}{4}$$
$$y=A\cos Bx$$
$$y=2\cos\left(\frac{\pi}{4}x\right)$$

65. $\pm 1, \pm 2, \pm 3, \pm 6$ are possible rational zeros

1	1	–2	–5	6
		1	–1	–6
	1	–1	–6	0

1 is a zero.

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \quad \text{or} \quad x = -2$$

The zeros are $3, -2,$ and 1.

66.
$$2(1-u^2) + 3u = 0$$
$$2 - 2u^2 + 3u = 0$$
$$2u^2 - 3u - 2 = 0$$
$$(2u+1)(u-2) = 0$$
$$2u+1 = 0 \quad \text{and} \quad u-2=0$$
$$2u = -1 \qquad u = 2$$
$$u = -\frac{1}{2}$$

The solution set is $\left\{-\frac{1}{2}, 2\right\}$.

67.
$$u^3 - 3u = 0$$
$$u(u^2-3) = 0$$
$$u = 0 \quad \text{or} \quad u^2 - 3 = 0$$
$$u^2 = 3$$
$$u = \pm\sqrt{3}$$

The solution set is $\left\{-\sqrt{3}, 0, \sqrt{3}\right\}$.

68.
$$u^2 - u - 1 = 0$$
$$a = 1, \quad b = -1, \quad c = -1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{5}}{2}$$

The solution set is $\left\{\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right\}$.

Section 5.5

Check Point Exercises

1.

$$5\sin x = 3\sin x + \sqrt{3}$$
$$5\sin x - 3\sin x = 3\sin x - 3\sin x + \sqrt{3}$$
$$2\sin x = \sqrt{3}$$
$$\sin x = \frac{\sqrt{3}}{2}$$

Because $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, the solutions for $\sin x = \frac{\sqrt{3}}{2}$ in $[0, 2\pi)$ are

$$x = \frac{\pi}{3}$$
$$x = \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}.$$

Because the period of the sine function is 2π, the solutions are given by

$$x = \frac{\pi}{3} + 2n\pi \quad \text{or}$$
$$x = \frac{2\pi}{3} + 2n\pi$$

where n is any integer.

2. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\sqrt{3}$ is $\frac{\pi}{3}$. All the solutions to $\tan 2x = \sqrt{3}$ are given by

$$2x = \frac{\pi}{3} + n\pi$$
$$x = \frac{\pi}{6} + \frac{n\pi}{2}$$

where n is any integer. In the interval $[0, 2\pi)$, we obtain solutions as follows:

Let $n = 0$. $x = \frac{\pi}{6} + \frac{0\pi}{2} = \frac{\pi}{6}$

Let $n = 1$. $x = \frac{\pi}{6} + \frac{1\pi}{2} = \frac{\pi}{6} + \frac{3\pi}{6} = \frac{2\pi}{3}$

Let $n = 2$. $x = \frac{\pi}{6} + \frac{2\pi}{2} = \frac{\pi}{6} + \frac{6\pi}{6} = \frac{7\pi}{6}$

Let $n = 3$. $x = \frac{\pi}{6} + \frac{3\pi}{2} = \frac{\pi}{6} + \frac{9\pi}{6} = \frac{5\pi}{3}$

In the interval $[0, 2\pi)$, the solutions are $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6},$ and $\frac{5\pi}{3}$.

3. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine is positive in quadrant II. Thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$. All the solutions to $\sin\frac{x}{3} = \frac{1}{2}$ are given by

$$\frac{x}{3} = \frac{\pi}{6} + 2n\pi$$
$$x = \frac{\pi}{2} + 6n\pi$$

or

$$\frac{x}{3} = \frac{5\pi}{6} + 2n\pi$$
$$x = \frac{5\pi}{2} + 6n\pi$$

where n is any integer. In the interval $[0, 2\pi)$, we obtain solutions as follows:

Let $n = 0$. $x = \frac{\pi}{2}$ or $x = \frac{5\pi}{2}$

The value $\frac{5\pi}{2}$ exceeds 2π.

If we let $n = 1$, we are adding 6π to each of these expressions. These values of x exceed 2π. Thus in the interval $[0, 2\pi)$, the solution set is $\left\{\frac{\pi}{2}\right\}$

4. The given equation is in quadratic form $2t^2 - 3t + 1 = 0$ with $t = \sin x$.

$$2\sin^2 x - 3\sin x + 1 = 0$$
$$(2\sin x - 1)(\sin x - 1) = 0$$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$2\sin x = 1$ $\quad$ $\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad$ $x = \frac{\pi}{2}$

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{\pi}{2},$ and $\frac{5\pi}{6}.$

5. $4\cos^2 x - 3 = 0$

$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\sqrt{\frac{3}{4}}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$

$\cos x = \frac{\sqrt{3}}{2}$ or $\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ $\quad$ $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}.$

6. $\sin x \tan x = \sin x$

$$\sin x \tan x - \sin x = 0$$
$$\sin x(\tan x - 1) = 0$$

$\sin x = 0$ or $\tan x - 1 = 0$

$x = 0 \quad x = \pi$ $\quad$ $\tan x = 1$

$x = \frac{\pi}{4}$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{4}, \pi,$ and $\frac{5\pi}{4}.$

7. $2\sin^2 x - 3\cos x = 0$

$$2\left(1-\cos^2 x\right) - 3\cos x = 0$$
$$2 - 2\cos^2 x - 3\cos x = 0$$
$$-2\cos^2 x - 3\cos x + 2 = 0$$
$$2\cos^2 x + 3\cos x - 2 = 0$$
$$(2\cos x - 1)(\cos x + 2) = 0$$

$2\cos x - 1 = 0$ or $\cos x + 2 = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -2$

~~$\cos x = -2$~~

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ This equation has no solution.

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}.$

8. $\cos 2x + \sin x = 0$

$$1 - 2\sin^2 x + \sin x = 0$$
$$-2\sin^2 x + \sin x + 1 = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$

$2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$2\sin x = -1$ $\quad$ $\sin x = 1$

$\sin x = -\frac{1}{2}$ $\quad$ $x = \frac{\pi}{2}$

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ or

$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}.$

9. $\sin x \cos x = -\frac{1}{2}$

$$2\sin x \cos x = -1$$
$$\sin 2x = -1$$

The period of the sine function is $2\pi.$ In the interval $[0, 2\pi),$ the sine function is –1 at $\frac{3\pi}{2}.$ All the solutions to $\sin 2x$ are given by

$$2x = \frac{3\pi}{2} + 2n\pi$$
$$x = \frac{3\pi}{4} + n\pi,$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$. The solutions are $\frac{3\pi}{4}$ and $\frac{7\pi}{4}.$

10.
$$\cos x - \sin x = -1$$
$$(\cos x - \sin x)^2 = (-1)^2$$
$$\cos^2 x - 2\cos x \sin x + \sin^2 x = 1$$
$$\cos^2 x + \sin^2 x - 2\cos x \sin x = 1$$
$$1 - 2\cos x \sin x = 1$$
$$-2\cos x \sin x = 0$$
$$\cos x \sin x = 0$$

$\cos x = 0$ or $\sin x = 0$

$x = \frac{\pi}{2}$ $\quad x = 0$

$x = \frac{3\pi}{2}$ $\quad x = \pi$

We check these proposed solutions to see if any are extraneous.

Check 0: $\cos 0 - \sin 0 = -1$
$1 - 0 = -1$
$1 = -1$, false

Check $\frac{\pi}{2}$: $\cos\frac{\pi}{2} - \sin\frac{\pi}{2} = -1$
$0 - 1 = -1$
$-1 = -1$, true

Check π: $\cos\frac{\pi}{2} - \sin\frac{\pi}{2} = -1$
$-1 - 0 = -1$
$-1 = -1$, true

Check $\frac{3\pi}{2}$: $\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2} = -1$
$0 - (-1) = -1$
$1 = -1$, false

The actual solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}$ and π.

11. **a.** $\tan x = 3.1044$

Be sure calculator is in radian mode and find the inverse tangent of 3.1044. This gives the first quadrant reference angle.

$\theta = \tan^{-1} 3.1044 \approx 1.2592$

The tangent is positive in quadrants I and III thus,

$x \approx 1.2592$ or $x \approx \pi + 1.2592$
$x \approx 4.4008$

b. $\sin x = -0.2315$

Be sure calculator is in radian mode and find the inverse sine of +0.2315. This gives the first quadrant reference angle.

$\theta = \sin^{-1}(0.2315) \approx 0.2336$

The sine is negative in quadrants III and IV thus,

$x \approx \pi + 0.2336$ or $x \approx 2\pi - 1.2592$
$x \approx 3.3752$ $\quad x \approx 6.0496$

12. $\cos^2 x + 5\cos x + 3 = 0$

Use the quadratic formula to solve for cos*x*.

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\cos x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(3)}}{2(1)}$$
$$\cos x = \frac{-5 \pm \sqrt{13}}{2}$$

$\cos x \approx -0.6972$ or $\cos x \approx -4.3028$

~~$\cos x \approx -4.3028$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of +0.6972. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.6972 \approx 0.7993$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.7993$ or $x \approx \pi + 0.7993$
$x \approx 2.3423$ $\quad x \approx 3.9409$

Concept and Vocabulary Check 5.5

1. $\frac{3\pi}{4}$; $\frac{\pi}{4} + 2n\pi$; $\frac{3\pi}{4} + 2n\pi$

2. $\frac{2\pi}{3}$; $x = \frac{2\pi}{3} + n\pi$

3. false

4. true

5. false

6. $2\cos x + 1$; $\cos x - 5$; $\cos x - 5 = 0$

7. $\cos x$; $2\sin x + \sqrt{2}$

8. $\cos^2 x$; $1 - \sin^2 x$

9. π; 2π

Exercise Set 5.5

1. $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$ is true.

Thus, $\frac{\pi}{4}$ is a solution.

2. $\tan\frac{\pi}{3}=\sqrt{3}$

$\sqrt{3}=\sqrt{3}$ is true.

Thus, $\frac{\pi}{3}$ is a solution.

3. $\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$\frac{1}{2}=\frac{\sqrt{3}}{2}$ is false.

Thus, $\frac{\pi}{6}$ is not a solution.

4. $\sin\frac{\pi}{3}=\frac{\sqrt{2}}{2}$

$\frac{\sqrt{3}}{2}=\frac{\sqrt{2}}{2}$ is false.

Thus, $\frac{\pi}{3}$ is not a solution.

5. $\cos\frac{2\pi}{3}=-\frac{1}{2}$

$-\frac{1}{2}=-\frac{1}{2}$ is true.

Thus, $\frac{2\pi}{3}$ is a solution.

6. $\cos\frac{4\pi}{3}=-\frac{1}{2}$

$-\frac{1}{2}=-\frac{1}{2}$ is true.

Thus, $\frac{4\pi}{3}$ is a solution.

7. $\tan\left(2\cdot\frac{5\pi}{12}\right)=-\frac{\sqrt{3}}{3}$

$\tan\frac{5\pi}{6}=-\frac{\sqrt{3}}{3}$

$-\frac{\sqrt{3}}{3}=-\frac{\sqrt{3}}{3}$ is true.

Thus, $\frac{5\pi}{12}$ is a solution.

8. $\cos\frac{2\pi}{3}=-\frac{1}{2}$

$-\frac{1}{2}=-\frac{1}{2}$ is true.

Thus, $\frac{2\pi}{3}$ is a solution.

9. $\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\frac{1}{2}=\frac{\sqrt{3}}{2}$ is false.

Thus, $\frac{\pi}{3}$ is not a solution.

10. $\cos\frac{\pi}{6}+2=\sqrt{3}\cdot\sin\frac{\pi}{6}$

$\frac{\sqrt{3}}{2}+2=\sqrt{3}\cdot\frac{1}{2}$

$\frac{4+\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$ is false.

Thus, $\frac{\pi}{6}$ is not a solution.

11. $\sin x=\frac{\sqrt{3}}{2}$

Because $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, the solutions for $\sin x=\frac{\sqrt{3}}{2}$ in $[0,2\pi)$ are

$x=\frac{\pi}{3}$

$x=\pi-\frac{\pi}{3}=\frac{3\pi}{3}-\frac{\pi}{3}=\frac{2\pi}{3}$.

Because the period of the sine function is 2π, the solutions are given by

$x=\frac{\pi}{3}+2n\pi$ or $x=\frac{2\pi}{3}+2n\pi$

where n is any integer.

12. $\cos x = \frac{\sqrt{3}}{2}$

Because $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, the solutions for $\cos x = \frac{\sqrt{3}}{2}$ in $[0, 2\pi)$ are

$x = \frac{\pi}{6}$

$x = 2\pi - \frac{\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}.$

Because the period of the cosine function is 2π, the solutions are given by

$x = \frac{\pi}{6} + 2n\pi$ or $x = \frac{11\pi}{6} + 2n\pi$

where n is any integer.

13. $\tan x = 1$

Because $\tan\frac{\pi}{4} = 1$, the solution for $\tan x = 1$ in $[0, \pi)$ is

$x = \frac{\pi}{4}.$

Because the period of the tangent function is π, the solutions are given by

$x = \frac{\pi}{4} + n\pi$

where n is any integer.

14. $\tan x = \sqrt{3}$

Because $\tan\frac{\pi}{3} = \sqrt{3}$, the solution for $\tan x = \sqrt{3}$ in $[0, \pi)$ is

$x = \frac{\pi}{3}.$

Because the period of the tangent function is π, the solutions are given by

$x = \frac{\pi}{3} + n\pi$ where n is any integer.

15. $\cos x = -\frac{1}{2}$

Because $\cos\frac{\pi}{3} = \frac{1}{2}$, the solutions for $\cos x = -\frac{1}{2}$ in $[0, 2\pi)$ are

$x = \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

$x = \pi + \frac{\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}.$

Because the period of the cosine function is 2π, the solutions are given by

$x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$

where n is any integer.

16. $\sin x = -\frac{\sqrt{2}}{2}$

Because $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, the solutions for $\sin x = -\frac{\sqrt{2}}{2}$ in $[0, 2\pi)$ are

$x = \pi + \frac{\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = 2\pi - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}.$

Because the period of the sine function is 2π, the solutions are given by

$x = \frac{5\pi}{4} + 2n\pi$ or $x = \frac{7\pi}{4} + 2n\pi$

where n is any integer.

17. $\tan x = 0$

Because $\tan 0 = 0$, the solution for $\tan x = 0$ in $[0, \pi)$ is

$x = 0.$

Because the period of the tangent function is π, the solutions are given by

$x = 0 + n\pi = n\pi$

where n is any integer.

18. $\sin x = 0$

Because $\sin 0 = 0$, the solutions for $\sin x = 0$ in $[0, 2\pi)$ are

$x = 0$

$x = \pi + 0 = \pi.$

Because the period of the sine function is 2π, the solutions are given by

$x = 0 + n\pi = n\pi$ or $x = \pi + 2n\pi$

where n is any integer.

19. $2\cos x+\sqrt{3}=0$
$2\cos x=-\sqrt{3}$
$\cos x=-\frac{\sqrt{3}}{2}$

Because $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$, the solutions for $\cos x=-\frac{\sqrt{3}}{2}$ in $[0,2\pi)$ are

$x=\pi-\frac{\pi}{6}=\frac{6\pi}{6}-\frac{\pi}{6}=\frac{5\pi}{6}$

$x=\pi+\frac{\pi}{6}=\frac{6\pi}{6}+\frac{\pi}{6}=\frac{7\pi}{6}.$

Because the period of the cosine function is 2π, the solutions are given by

$x=\frac{5\pi}{6}+2n\pi$ or $x=\frac{7\pi}{6}+2n\pi$

where n is any integer.

20. $2\sin x+\sqrt{3}=0$
$2\sin x=-\sqrt{3}$
$\sin x=-\frac{\sqrt{3}}{2}$

Because $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, the solutions for $\sin x=-\frac{\sqrt{3}}{2}$ in $[0,2\pi)$ are

$x=\pi+\frac{\pi}{3}=\frac{3\pi}{3}+\frac{\pi}{3}=\frac{4\pi}{3}$

$x=2\pi-\frac{\pi}{3}=\frac{6\pi}{3}-\frac{\pi}{3}=\frac{5\pi}{3}.$

Because the period of the sine function is 2π, the solutions are given by

$x=\frac{4\pi}{3}+2n\pi$ or $x=\frac{5\pi}{3}+2n\pi$

where n is any integer.

21. $4\sin\theta-1=2\sin\theta$
$4\sin\theta-2\sin\theta=1$
$2\sin\theta=1$
$\sin\theta=\frac{1}{2}$

Because $\sin\frac{\pi}{6}=\frac{1}{2}$, the solutions for $\sin\theta=\frac{1}{2}$ in $[0,2\pi)$ are

$\theta=\frac{\pi}{6}$

$\theta=\pi-\frac{\pi}{6}=\frac{6\pi}{6}-\frac{\pi}{6}=\frac{5\pi}{6}.$

Because the period of the sine function is 2π, the solutions are given by

$\theta=\frac{\pi}{6}+2n\pi$ or $\theta=\frac{5\pi}{6}+2n\pi$

where n is any integer.

22. $5\sin\theta+1=3\sin\theta$
$5\sin\theta-3\sin\theta=-1$
$2\sin\theta=-1$
$\sin\theta=-\frac{1}{2}$

Because $\sin\frac{\pi}{6}=\frac{1}{2}$, the solutions for $\sin\theta=-\frac{1}{2}$ in $[0,2\pi)$ are

$\theta=\pi+\frac{\pi}{6}=\frac{6\pi}{6}+\frac{\pi}{6}=\frac{7\pi}{6}$

$\theta=2\pi-\frac{\pi}{6}=\frac{12\pi}{6}-\frac{\pi}{6}=\frac{11\pi}{6}.$

Because the period of the sine function is 2π, the solutions are given by

$\theta=\frac{7\pi}{6}+2n\pi$ or $\theta=\frac{11\pi}{6}+2n\pi$

where n is any integer.

23. $3\sin\theta+5=-2\sin\theta$
$3\sin\theta+2\sin\theta=-5$
$5\sin\theta=-5$
$\sin\theta=-1$

Because $\sin\frac{\pi}{2}=1$, the solutions for $\sin\theta=-1$ in $[0,2\pi)$ are

$\theta=\pi+\frac{\pi}{2}=\frac{2\pi}{2}+\frac{\pi}{2}=\frac{3\pi}{2}$

$\theta=2\pi-\frac{\pi}{2}=\frac{4\pi}{2}-\frac{\pi}{2}=\frac{3\pi}{2}.$

Because the period of the sine function is 2π, the solutions are given by

$\theta=\frac{3\pi}{2}+2n\pi$

where n is any integer.

24. $7\cos\theta+9=-2\cos\theta$
$7\cos\theta+2\cos\theta=-9$
$9\cos\theta=-9$
$\cos\theta=-1$

Because $\cos\pi=-1$, the solution for $\cos\theta=-1$ in $[0,2\pi)$ is
$x=\pi.$

Because the period of the cosine function is 2π, the solutions are given by
$\theta=\pi+2n\pi$
where n is any integer.

25. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{3}}{2}$. One is $\frac{\pi}{3}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$. All the solutions to $\sin 2x = \frac{\sqrt{3}}{2}$ are given by

$$2x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{2\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{6} + n\pi \qquad x = \frac{\pi}{3} + n\pi$$

Where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.
The solutions are $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}$, and $\frac{4\pi}{3}$.

26. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the cosine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The cosine is positive in quadrant IV; thus, the other value is $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$. All the solutions to $\cos 2x = \frac{\sqrt{2}}{2}$ are given by

$$2x = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad 2x = \frac{7\pi}{4} + 2n\pi$$

$$x = \frac{\pi}{8} + n\pi \qquad x = \frac{7\pi}{8} + n\pi$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.
The solutions are $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}$, and $\frac{15\pi}{8}$.

27. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the cosine function is $-\frac{\sqrt{3}}{2}$. One is $\frac{5\pi}{6}$. The cosine is negative in quadrant III; thus, the other value is $2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$. All the solutions to $\cos 4x = -\frac{\sqrt{3}}{2}$ are given by

$$4x = \frac{5\pi}{6} + 2n\pi \quad \text{or} \quad 4x = \frac{7\pi}{6} + 2n\pi$$

$$x = \frac{5\pi}{24} + \frac{n\pi}{2} \qquad x = \frac{7\pi}{24} + \frac{n\pi}{2}$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, and $n = 3$.
The solutions are $\frac{5\pi}{24}, \frac{7\pi}{24}, \frac{17\pi}{24}, \frac{19\pi}{24}, \frac{29\pi}{24}, \frac{31\pi}{24}, \frac{41\pi}{24}$ and $\frac{43\pi}{24}$.

28. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{\sqrt{2}}{2}$. One is $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$. The sine is negative in quadrant IV; thus, the other value is $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$. All solutions to $\sin 4x = -\frac{\sqrt{2}}{2}$ are given by

$$4x = \frac{5\pi}{4} + 2n\pi \quad \text{or} \quad 4x = \frac{7\pi}{4} + 2n\pi$$

$$x = \frac{5\pi}{16} + \frac{n\pi}{2} \qquad x = \frac{7\pi}{16} + \frac{n\pi}{2}$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$,
$n = 2$, and $n = 3$. The solutions are $\frac{5\pi}{16}, \frac{7\pi}{16}, \frac{13\pi}{16}, \frac{15\pi}{16}, \frac{21\pi}{16}, \frac{23\pi}{16}, \frac{29\pi}{16}$ and $\frac{31\pi}{16}$.

29. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

All the solutions to $\tan 3x = \frac{\sqrt{3}}{3}$ are given by

$$3x = \frac{\pi}{6} + n\pi$$

$$x = \frac{\pi}{18} + \frac{n\pi}{3}$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, $n = 3$, $n = 4$, and $n = 5$.
The solutions are $\frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}$, and $\frac{31\pi}{18}$.

30. The period of the tangent function is π. In the interval $[0,\pi)$, the only value for which the tangent function is $\sqrt{3}$ is $\frac{\pi}{3}$.

All the solutions to $\tan 3x=\sqrt{3}$ are given by

$$3x=\frac{\pi}{3}+n\pi$$
$$x=\frac{\pi}{9}+\frac{n\pi}{3}$$

where n is any integer.

The solutions in the interval $[0,2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, $n = 3$, $n = 4$, and $n = 5$.

The solutions are $\frac{\pi}{9},\frac{4\pi}{9},\frac{7\pi}{9},\frac{10\pi}{9},\frac{13\pi}{9}$, and $\frac{16\pi}{9}$.

31. The period of the tangent function is π. In the interval $[0,\pi)$, the only value for which the tangent function is $\sqrt{3}$ is $\frac{\pi}{3}$.

All the solutions to $\tan\frac{x}{2}=\sqrt{3}$ are given by

$$\frac{x}{2}=\frac{\pi}{3}+n\pi$$
$$x=\frac{2\pi}{3}+2n\pi$$ where n is any integer. The solution in the interval $[0,2\pi)$ is obtained by letting $n = 0$.

The only solution is $\frac{2\pi}{3}$.

32. The period of the tangent function is π. In the interval $[0,\pi)$, the only value for which the tangent function is $\frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

All the solutions to $\tan\frac{x}{2}=\frac{\sqrt{3}}{3}$ are given by

$$\frac{x}{2}=\frac{\pi}{6}+n\pi$$
$$x=\frac{\pi}{3}+2n\pi$$

where n is any integer.

The solution in the interval $[0,2\pi)$ is obtained by letting $n = 0$.

The only solution is $\frac{\pi}{3}$.

33. The period of the sine function is 2π. In the interval $[0,2\pi)$, the only value for which the sine function is -1 is $\frac{3\pi}{2}$.

All the solutions to $\sin\frac{2\theta}{3}=-1$ are given by

$$\frac{2\theta}{3}=\frac{3\pi}{2}+2n\pi$$
$$\theta=\frac{9\pi}{4}+3n\pi$$ where n is any integer. All values of θ exceed 2π or are less than zero.

Thus, in the interval $[0,2\pi)$ there is no solution.

34. The period of the cosine function is 2π. In the interval $[0,2\pi)$, the only value for which the cosine function is –1 is π. All the solutions to $\cos\frac{2\theta}{3}=-1$ are given by

$$\frac{2\theta}{3}=\pi+2n\pi$$
$$\theta=\frac{3\pi}{2}+3n\pi$$

where n is any integer.

The solution in the interval $\left[0,\,2\pi\right)$ is obtained by letting $n = 0$.

The only solution is $\frac{3\pi}{2}$.

35. The period of the secant function is 2π. In the interval $[0,2\pi)$, there are two values at which the secant function is –2. One is $\frac{2\pi}{3}$. The secant is negative in quadrant III; thus, the other value is $2\pi-\frac{2\pi}{3}=\frac{4\pi}{3}$. All the solutions to $\sec\frac{3\theta}{2}=-2$ are given by

$$\frac{3\theta}{2}=\frac{2\pi}{3}+2n\pi \quad \text{or} \quad \frac{3\theta}{2}=\frac{4\pi}{3}+2n\pi$$
$$\theta=\frac{4\pi}{9}+\frac{4n\pi}{3} \qquad \theta=\frac{8\pi}{9}+\frac{4n\pi}{3}$$

where n is any integer. The solutions in the interval $[0,2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

Since $\frac{20\pi}{9}$ is not in $[0,2\pi)$, the solutions are $\frac{4\pi}{9},\frac{8\pi}{9}$, and $\frac{16\pi}{9}$.

36. The period of the cotangent function is π. In the interval $[0, \pi)$, the only value for which the cotangent function is $-\sqrt{3}$ is $\frac{5\pi}{6}$

All the solutions to $\cot\frac{3\theta}{2} = -\sqrt{3}$ are given by

$$\frac{3\theta}{2} = \frac{5\pi}{6} + n\pi$$
$$\theta = \frac{5\pi}{9} + \frac{2n\pi}{3}$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, and $n = 2$.
The solutions are $\frac{5\pi}{9}, \frac{11\pi}{9}$, and $\frac{17\pi}{9}$.

37. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine is positive in quadrant II; Thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

All the solutions to $\sin\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$ are given by

$$2x + \frac{\pi}{6} = \frac{\pi}{6} + 2n\pi$$
$$2x = 2n\pi$$
$$x = n\pi \quad \text{or}$$
$$2x + \frac{\pi}{6} = \frac{5\pi}{6} + 2n\pi$$
$$2x = \frac{4\pi}{6} + 2n\pi$$
$$x = \frac{2\pi}{6} + n\pi$$
$$x = \frac{\pi}{3} + n\pi$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $0, \frac{\pi}{3}, \pi$, and $\frac{4\pi}{3}$.

38. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The sine is positive in quadrant II; Thus, the other value is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$. All the solutions to $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ are given by

$$2x - \frac{\pi}{4} = \frac{\pi}{4} + 2n\pi$$
$$2x = \frac{2\pi}{4} + 2n\pi \quad \text{or}$$
$$x = \frac{\pi}{4} + n\pi$$
$$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2n\pi \text{ where } n \text{ is any integer.}$$
$$2x = \frac{4\pi}{4} + 2n\pi$$
$$x = \frac{\pi}{2} + n\pi$$

The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.
The solutions are $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, and $\frac{3\pi}{2}$.

39.
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$2\sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$
$$2\sin x = -1 \qquad \sin x = 1$$
$$\sin x = -\frac{1}{2}$$
$$x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6} \qquad x = \frac{\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

40.
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$
$$2\sin x = 1 \qquad \sin x = -1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

41. $2\cos^2 x+3\cos x+1=0$

$(2\cos x+1)(\cos x+1)=0$

$2\cos x+1=0$ or $\cos x+1=0$

$2\cos x=-1$ $\quad$ $\cos x=-1$

$\cos x=-\frac{1}{2}$

$x=\frac{2\pi}{3}$ $\quad x=\frac{4\pi}{3}$ $\quad x=\pi$

The solutions in the interval $[0,2\pi)$ are $\frac{2\pi}{3}$, π, and $\frac{4\pi}{3}$.

42. $\cos^2 x+2\cos x-3=0$

$(\cos x-1)(\cos x+3)=0$

$\cos x-1=0$ or $\cos x+3=0$

$\cos x=1$ $\quad$ $\cos x=-3$

$x=0$

$\cos x$ cannot be less than –1.

The solution in the interval $[0,2\pi)$ is 0.

43. $2\sin^2 x=\sin x+3$

$2\sin^2 x-\sin x-3=0$

$(2\sin x-3)(\sin x+1)=0$

$2\sin x-3=0$ or $\sin x+1=0$

$2\sin x=3$ $\quad$ $\sin x=-1$

$\sin x=\frac{3}{2}$ $\quad$ $x=\frac{3\pi}{2}$

$\sin x$ cannot be greater than 1.

The solution in the interval $[0,2\pi)$ is $\frac{3\pi}{2}$.

44. $2\sin^2 x=4\sin x+6$

$2\sin^2 x-4\sin x-6=0$

$(2\sin x+2)(\sin x-3)=0$

$2\sin x+2=0$ or $\sin x-3=0$

$2\sin x=-2$ $\quad$ $\sin x=3$

$\sin x=-1$

$x=\frac{3\pi}{2}$

$\sin x$ cannot be greater than 1.

The solution in the interval $[0,2\pi)$ is $\frac{3\pi}{2}$.

45. $\sin^2\theta-1=0$

$(\sin\theta-1)(\sin\theta+1)=0$

$\sin\theta-1=0$ or $\sin\theta+1=0$

$\sin\theta=1$ $\quad$ $\sin\theta=-1$

$\theta=\frac{\pi}{2}$ $\quad$ $\theta=\frac{3\pi}{2}$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

46. $\cos^2\theta-1=0$

$(\cos\theta-1)(\cos\theta+1)=1$

$\cos\theta-1=0$ or $\cos\theta+1=0$

$\cos\theta=1$ $\quad$ $\cos\theta=-1$

$\theta=0$ $\quad$ $\theta=\pi$

The solutions in the interval $[0,2\pi)$ are 0 and π.

47. $4\cos^2 x-1=0$

$(2\cos x+1)(2\cos x-1)=1$

$2\cos x+1=0$ or $2\cos x-1=0$

$\cos x=-\frac{1}{2}$ $\quad$ $\cos x=\frac{1}{2}$

$x=\frac{2\pi}{3},\frac{4\pi}{3}$ $\quad$ $x=\frac{\pi}{3},\frac{5\pi}{3}$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

48. $4\sin^2 x-3=0$

$\sin^2 x=\frac{3}{4}$

$\sin x=\pm\sqrt{\frac{3}{4}}$

$\sin x=\pm\frac{\sqrt{3}}{2}$

$\sin x=\frac{\sqrt{3}}{2}$ or $\sin x=-\frac{\sqrt{3}}{2}$

$x=\frac{\pi}{3},\frac{2\pi}{3}$ $\quad$ $x=\frac{4\pi}{3},\frac{5\pi}{3}$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

49. $9\tan^2 x - 3 = 0$

$\tan^2 x = \frac{3}{9}$

$\tan x = \pm\sqrt{\frac{3}{9}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$\tan x = \frac{\sqrt{3}}{3}$ or $\tan x = -\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$ $\quad x = \frac{5\pi}{6}, \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

50. $3\tan^2 x - 9 = 0$

$\tan^2 x = \frac{9}{3}$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \sqrt{3}$ or $\tan x = -\sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ $\quad x = \frac{2\pi}{3}, \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

51. $\sec^2 x - 2 = 0$

$\sec^2 x = 2$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\cos x = \frac{\sqrt{2}}{2}$ or $\cos x = -\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$ $\quad x = \frac{3\pi}{4}, \frac{5\pi}{4}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$.

52. $4\sec^2 x - 2 = 0$

$\sec^2 x = \frac{2}{4}$

$\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$

No solution.

53. $(\tan x - 1)(\cos x + 1) = 0$

$\tan x - 1 = 0$ or $\cos x + 1 = 0$

$\tan x = 1$ $\quad \cos x = -1$

$x = \frac{\pi}{4}$ $\quad x = \frac{5\pi}{4}$ $\quad x = \pi$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{4}, \pi$, and $\frac{5\pi}{4}$

54. $(\tan x + 1)(\sin x - 1) = 0$

$\tan x + 1 = 0$ or $\sin x - 1 = 0$

$\tan x = -1$ $\quad \sin x = 1$

$x = \frac{3\pi}{4}$ $\quad x = \frac{7\pi}{4}$ $\quad x = \frac{\pi}{2}$

The solutions in the interval $[0, 2\pi)$ are $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ since tan is undefined at $\frac{\pi}{2}$.

55. $\left(2\cos x + \sqrt{3}\right)(2\sin x + 1) = 0$

$2\cos x + \sqrt{3} = 0$ or $2\sin x + 1 = 0$

$2\cos x = -\sqrt{3}$ $\quad 2\sin x = -1$

$\cos x = -\frac{\sqrt{3}}{2}$ $\quad \sin x = -\frac{1}{2}$

$x = \frac{5\pi}{6}$ $\quad x = \frac{7\pi}{6}$ $\quad x = \frac{7\pi}{6}$ $\quad x = \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

56. $\left(2\cos x - \sqrt{3}\right)(2\sin x - 1) = 0$

$2\cos x - \sqrt{3} = 0$ or $2\sin x - 1 = 0$

$2\cos x = \sqrt{3}$ $\quad 2\sin x = 1$

$\cos x = \frac{\sqrt{3}}{2}$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad x = \frac{11\pi}{6}$ $\quad x = \frac{\pi}{6}$ $\quad x = \frac{5\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{11\pi}{6}$.

57. $\cot x(\tan x - 1) = 0$
$\cot x = 0$ or $\tan x - 1 = 0$
$\tan x = 1$
$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \quad x = \frac{\pi}{4} \quad x = \frac{5\pi}{4}$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ since tan is undefined for $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

58. $\cot x(\tan x + 1) = 0$
$\cot x = 0$ or $\tan x + 1 = 0$
$\tan x = -1$
$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \quad x = \frac{3\pi}{4} \quad x = \frac{7\pi}{4}$
The solutions in the interval $[0, 2\pi)$ are $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ since tan is undefined at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

59. $\sin x + 2\sin x\cos x = 0$
$\sin x(1 + 2\cos x) = 0$
$\sin x = 0$ or $1 + 2\cos x = 0$
$2\cos x = -1$
$\cos x = -\frac{1}{2}$
$x = 0 \quad x = \pi \quad x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3}$
The solutions in the interval $[0, 2\pi)$ are $0, \frac{2\pi}{3}, \pi$, and $\frac{4\pi}{3}$.

60. $\cos x - 2\sin x\cos x = 0$
$\cos x(1 - 2\sin x) = 0$
$\cos x = 0$ or $1 - 2\sin x = 0$
$-2\sin x = -1$
$\sin x = \frac{1}{2}$
$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \quad x = \frac{\pi}{6} \quad x = \frac{5\pi}{6}$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

61. $\tan^2 x\cos x = \tan^2 x$
$\tan^2 x\cos x - \tan^2 x = 0$
$\tan^2 x(\cos x - 1) = 0$
$\tan^2 x = 0$ or $\cos x - 1 = 0$
$\tan x = 0 \qquad \cos x = 1$
$x = 0 \quad x = \pi \qquad x = 0$
The solutions in the interval $[0, 2\pi)$ are 0 and π.

62. $\cot^2 x\sin x = \cot^2 x$
$\cot^2 x\sin x - \cot^2 x = 0$
$\cot 2(\sin x - 1) = 0$
$\cot^2 x = 0$ or $\sin x - 1 = 0$
$\cot x = 0 \qquad \sin x = 1$
$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \qquad x = \frac{\pi}{2}$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

63. $2\cos^2 x + \sin x - 1 = 0$
$2\left(1 - \sin^2 x\right) + \sin x - 1 = 0$
$2 - 2\sin^2 x + \sin x - 1 = 0$
$-2\sin^2 x + \sin x + 1 = 0$
$2\sin^2 x - \sin x - 1 = 0$
$(2\sin x + 1)(\sin x - 1) = 0$
$2\sin x + 1 = 0$ or $\sin x - 1 = 0$
$2\sin x = -1 \qquad \sin x = 1$
$\sin x = -\frac{1}{2}$
$x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6} \qquad x = \frac{\pi}{2}$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

64.

$$\begin{aligned}
2\cos^2 x - \sin x - 1 &= 0\\
2\left(1-\sin^2 x\right) - \sin x - 1 &= 0\\
2 - 2\sin^2 x - \sin x - 1 &= 0\\
-2\sin^2 x - \sin x + 1 &= 0\\
2\sin^2 x + \sin x - 1 &= 0\\
(2\sin x - 1)(\sin x + 1) &= 0
\end{aligned}$$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1$ $\quad \sin x = -1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad x = \frac{5\pi}{6}$ $\quad x = \frac{3\pi}{2}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

65.

$$\begin{aligned}
\sin^2 x - 2\cos x - 2 &= 0\\
1 - \cos^2 x - 2\cos x - 2 &= 0\\
-\cos^2 x - 2\cos x - 1 &= 0\\
\cos^2 x + 2\cos x + 1 &= 0\\
(\cos x + 1)(\cos x + 1) &= 0\\
\cos x + 1 &= 0\\
\cos x &= -1\\
x &= \pi
\end{aligned}$$

The solution in the interval $[0, 2\pi)$ is π.

66.

$$\begin{aligned}
4\sin^2 x + 4\cos x - 5 &= 0\\
4\left(1-\cos^2 x\right) + 4\cos x - 5 &= 0\\
4 - 4\cos^2 x + 4\cos x - 5 &= 0\\
-4\cos^2 x + 4\cos x - 1 &= 0\\
4\cos^2 x - 4\cos x + 1 &= 0\\
(2\cos x - 1)(2\cos x - 1) &= 0\\
2\cos x - 1 &= 0\\
2\cos x &= 1\\
\cos x &= \frac{1}{2}\\
x = \frac{\pi}{3} \quad x &= \frac{5\pi}{3}
\end{aligned}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

67.

$$\begin{aligned}
4\cos^2 x &= 5 - 4\sin x\\
4\cos^2 x + 4\sin x - 5 &= 0\\
4\left(1-\sin^2 x\right) + 4\sin x - 5 &= 0\\
4 - 4\sin^2 x + 4\sin x - 5 &= 0\\
-4\sin^2 x + 4\sin x - 1 &= 0\\
4\sin^2 x - 4\sin x + 1 &= 0\\
(2\sin x - 1)(2\sin x - 1) &= 0\\
2\sin x - 1 &= 0\\
2\sin x &= 1\\
\sin x &= \frac{1}{2}\\
x = \frac{\pi}{6} \quad x &= \frac{5\pi}{6}
\end{aligned}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

68.

$$\begin{aligned}
3\cos^2 x &= \sin^2 x\\
3\left(1-\sin^2 x\right) &= \sin^2 x\\
3 - 3\sin^2 x - \sin^2 x &= 0\\
-4\sin^2 x &= -3\\
\sin^2 x &= \frac{3}{4}\\
\sin x &= \pm\sqrt{\frac{3}{4}}\\
\sin x &= \pm\frac{\sqrt{3}}{2}
\end{aligned}$$

$\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$ $\quad x = \frac{2\pi}{3}$ $\quad x = \frac{4\pi}{3}$ $\quad x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

69.

$$\begin{aligned}
\sin 2x &= \cos x\\
2\sin x\cos x &= \cos x\\
2\sin x\cos x - \cos x &= 0\\
\cos x(2\sin x - 1) &= 0
\end{aligned}$$

$\cos x = 0$ or $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}$ $\quad x = \frac{3\pi}{2}$ $\quad x = \frac{\pi}{6}$ $\quad x = \frac{5\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

70.
$$\sin 2x = \sin x$$
$$2\sin x\cos x = \sin x$$
$$2\sin x\cos x - \sin x = 0$$
$$\sin x(2\cos x - 1) = 0$$
$\sin x = 0$ or $2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = 0 \quad x = \pi \quad x = \frac{\pi}{3} \quad x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{3}, \pi,$ and $\frac{5\pi}{3}.$

71.
$$\cos 2x = \cos x$$
$$2\cos^2 x - 1 = \cos x$$
$$2\cos^2 x - 1 - \cos x = 0$$
$$2\cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$
$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$2\cos x = -1 \qquad \cos x = 1$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3} \qquad x = 0$

The solutions in the interval $[0, 2\pi)$ are

$0, \frac{2\pi}{3},$ and $\frac{4\pi}{3}.$

72.
$$\cos 2x = \sin x$$
$$1 - 2\sin^2 x = \sin x$$
$$1 - 2\sin^2 x - \sin x = 0$$
$$-2\sin^2 x - \sin x + 1 = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1 \qquad \sin x = -1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{6}, \frac{5\pi}{6},$ and $\frac{3\pi}{2}.$

73.
$$\cos 2x + 5\cos x + 3 = 0$$
$$2\cos^2 x - 1 + 5\cos x + 3 = 0$$
$$2\cos^2 x + 5\cos x + 2 = 0$$
$$(2\cos x + 1)(\cos x + 2) = 0$$
$2\cos x + 1 = 0$ or $\cos x + 2 = 0$

$2\cos x = -1 \qquad \cos x = -2$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3}$ $\qquad$ $\cos x$ cannot be less than -1

The solutions in the interval $[0, 2\pi)$ are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}.$

74.
$$\cos 2x + \cos x + 1 = 0$$
$$2\cos^2 x - 1 + \cos x + 1 = 0$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x(2\cos x + 1) = 0$$
$\cos x = 0$ or $2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \quad x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3},$ and $\frac{3\pi}{2}.$

75.
$$\sin x\cos x = \frac{\sqrt{2}}{4}$$
$$2\sin x\cos x = \frac{\sqrt{2}}{2}$$
$$\sin 2x = \frac{\sqrt{2}}{2}$$
The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}.$

All the solutions to $\sin 2x = \frac{\sqrt{2}}{2}$ are given by

$2x = \frac{\pi}{4} + 2n\pi$ or $2x = \frac{3\pi}{4} + 2n\pi$

$x = \frac{\pi}{8} + n\pi \qquad x = \frac{3\pi}{8} + n\pi$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8},$ and $\frac{11\pi}{8}.$

76. $\sin x\cos x=\frac{\sqrt{3}}{4}$

$2\sin x\cos x=\frac{\sqrt{3}}{2}$

$\sin 2x=\frac{\sqrt{3}}{2}$

The period of the sine function is 2π. In the interval $[0,2\pi)$, there are two values at which the sine function is $\frac{\sqrt{3}}{2}$. One is $\frac{\pi}{3}$. The sine is positive in quadrant II; thus, the other value is $\pi-\frac{\pi}{3}=\frac{2\pi}{3}$.

All the solutions to $\sin 2x=\frac{\sqrt{3}}{2}$ are given by

$2x=\frac{\pi}{3}+2n\pi$ or $2x=\frac{2\pi}{3}+2n\pi$

$x=\frac{\pi}{6}+n\pi$ $\quad$ $x=\frac{\pi}{3}+n\pi$

where *n* is any integer. The solutions in the interval $[0,2\pi)$ are obtained by letting *n* = 0 and *n* = 1.

The solutions are $\frac{\pi}{6},\frac{\pi}{3},\frac{7\pi}{6}$, and $\frac{4\pi}{3}$.

77. $\sin x+\cos x=1$

$(\sin x+\cos x)^2=1^2$

$\sin^2 x+2\sin x\cos x+\cos^2 x=1$

$\sin^2 x+\cos^2 x+2\sin x\cos x=1$

$1+2\sin x\cos x=1$

$2\sin x\cos x=0$

$\sin x\cos x=0$

$\sin x=0$ or $\cos x=0$

$x=0$ $\quad$ $x=\frac{\pi}{2}$

$x=\pi$ $\quad$ $x=\frac{3\pi}{2}$

After checking these proposed solutions, the actual solutions in the interval $[0,2\pi)$ are 0 and $\frac{\pi}{2}$.

78. $\sin x+\cos x=-1$

$(\sin x+\cos x)^2=(-1)^2$

$\sin^2 x+2\sin x\cos x+\cos^2 x=1$

$\sin^2 x+\cos^2 x+2\sin x\cos x=1$

$1+2\sin x\cos x=1$

$2\sin x\cos x=0$

$\sin x\cos x=0$

$\sin x=0$ or $\cos x=0$

$x=0$ $\quad$ $x=\frac{\pi}{2}$

$x=\pi$ $\quad$ $x=\frac{3\pi}{2}$

After checking these proposed solutions, the actual solutions in the interval $[0,2\pi)$ are π and $\frac{3\pi}{2}$.

79. $\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)=1$

$\frac{1}{2}\left[\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)\right]=1\cdot\frac{1}{2}$

$\sin x\cos\frac{\pi}{4}=\frac{1}{2}$

$\sin x\cdot\frac{\sqrt{2}}{2}=\frac{1}{2}$

$\sin x=\frac{1}{\sqrt{2}}$

$\sin x=\frac{\sqrt{2}}{2}$

$x=\frac{\pi}{4}$ or $x=\frac{3\pi}{4}$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

80. $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$

$\frac{1}{2}\left[\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)\right]=1\cdot\frac{1}{2}$

$\sin x\cos\frac{\pi}{3}=\frac{1}{2}$

$\sin x\cdot\frac{1}{2}=\frac{1}{2}$

$\sin x=1$

$x=\frac{\pi}{2}$

The solution in the interval $[0,2\pi)$ is $\frac{\pi}{2}$.

81. $\sin 2x\cos x+\cos 2x\sin x=\dfrac{\sqrt{2}}{2}$

$$\sin(2x+x)=\frac{\sqrt{2}}{2}$$

$$\sin 3x=\frac{\sqrt{2}}{2}$$

The period of the sine function is 2π. In the interval $[0,2\pi)$, there are two values at which the sine function is $\dfrac{\sqrt{2}}{2}$. One is $\dfrac{\pi}{4}$. The sine function is positive in quadrant II; thus, the other value is $\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$.

All the solutions to $\sin 3x=\dfrac{\sqrt{2}}{2}$ are given by

$3x=\dfrac{\pi}{4}+2n\pi$ or $3x=\dfrac{3\pi}{4}+2n\pi$

$x=\dfrac{\pi}{12}+\dfrac{2n\pi}{3}$ $\quad x=\dfrac{\pi}{4}+\dfrac{2n\pi}{3}$

where n is any integer. The solutions in the interval $[0,2\pi)$ are obtained by letting $n=0$, $n=1$, and $n=2$.

The solutions are $\dfrac{\pi}{12},\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{11\pi}{12},\dfrac{17\pi}{12}$, and $\dfrac{19\pi}{12}$.

82. $\sin 3x\cos 2x+\cos 3x\sin 2x=1$

$$\sin(3x+2x)=1$$

$$\sin 5x=1$$

The period of the sine function is 2π. In the interval $[0,2\pi)$, the only value at which the sine function is 1 is $\dfrac{\pi}{2}$. All the solutions to $\sin 5x=1$ are given by

$5x=\dfrac{\pi}{2}+2n\pi$

$x=\dfrac{\pi}{10}+\dfrac{2n\pi}{5}$

where n is any integer.

The solutions in the interval $[0,2\pi)$ are obtained by letting $n=0$, $n=1$, $n=2$, $n=3$, and $n=4$.

The solutions are $\dfrac{\pi}{10},\dfrac{\pi}{2},\dfrac{9\pi}{10},\dfrac{13\pi}{10}$, and $\dfrac{17\pi}{10}$.

83. $\tan x+\sec x=1$

$$\tan x-1=-\sec x$$

$$(\tan x-1)^2=(-\sec x)^2$$

$$\tan^2 x-2\tan x+1=\sec^2 x$$

$$\tan^2 x-2\tan x+1=1+\tan^2 x$$

$$-2\tan x=0$$

$$\tan x=0$$

$x=0\quad x=\pi$

We check these proposed solutions to see if any are extraneous.

Check 0: $\tan 0 + \sec 0$ 0 1

$0 + 1$ 0 1 True

Check π: $\tan\pi + \sec\pi$ 0 1

$0 + (-1)$ 0 1 False

The actual solution in the interval $[0,2\pi)$ is 0.

84. $\tan x-\sec x=1$

$$\tan x-1=\sec x$$

$$(\tan x-1)^2=\sec^2 x$$

$$\tan^2 x-2\tan x+1=\sec^2 x$$

$$\tan^2 x-2\tan x+1=1+\tan^2 x$$

$$-2\tan x=0$$

$$\tan x=0$$

$x=0\quad x=\pi$

We check these proposed solutions to see if any are extraneous.

Check 0: $\tan 0-\sec 0$ 0 1

$0-1$ 0 1

False

Check π: $\tan\pi-\sec\pi$ 0 1

$0-(-1)$ 0 1

True

The actual solution in the interval $[0,2\pi)$ is π.

85. $\sin x=0.8246$

Be sure calculator is in radian mode and find the inverse sine of 0.8246. This gives the first quadrant reference angle.

$\theta=\sin^{-1}0.8246\approx 0.9695$

The sine is positive in quadrants I and II thus,

$x\approx 0.9695$ or $x\approx\pi-0.9695$

$x\approx 2.1721$

86. $\sin x=0.7392$

Be sure calculator is in radian mode and find the inverse sine of 0.7392. This gives the first quadrant reference angle.

$\theta=\sin^{-1}0.7392\approx 0.8319$

The sine is positive in quadrants I and II thus,

$x\approx 0.8319$ or $x\approx\pi-0.8319$

$x\approx 2.3097$

87. $\cos x = -\frac{2}{5}$

Be sure calculator is in radian mode and find the inverse cosine of $+\frac{2}{5}$. This gives the first quadrant reference angle.

$\theta = \cos^{-1}\frac{2}{5} \approx 1.1593$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 1.1593$ or $x \approx \pi + 1.1593$
$x \approx 1.9823$ $\quad x \approx 4.3009$

88. $\cos x = -\frac{4}{7}$

Be sure calculator is in radian mode and find the inverse cosine of $+\frac{4}{7}$. This gives the first quadrant reference angle.

$\theta = \cos^{-1}\frac{4}{7} \approx 0.9626$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.9626$ or $x \approx \pi + 0.9626$
$x \approx 2.1790$ $\quad x \approx 4.1041$

89. $\tan x = -3$

Be sure calculator is in radian mode and find the inverse tangent of +3. This gives the first quadrant reference angle.

$\theta = \tan^{-1} 3 \approx 1.2490$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.2490$ or $x \approx 2\pi - 1.2490$
$x \approx 1.8925$ $\quad x \approx 5.0341$

90. $\tan x = -5$

Be sure calculator is in radian mode and find the inverse tangent of +5. This gives the first quadrant reference angle.

$\theta = \tan^{-1} 5 \approx 1.3734$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.3734$ or $x \approx 2\pi - 1.3734$
$x \approx 1.7682$ $\quad x \approx 4.9098$

91. $\cos^2 x - \cos x - 1 = 0$

Use the quadratic formula to solve for cos*x*.

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\cos x = \frac{1 \pm \sqrt{5}}{2}$$

$\cos x \approx -0.6180$ or $\cos x \approx 1.6180$

~~$\cos x \approx 1.6180$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of +0.6180. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.6180 \approx 0.9046$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.9046$ or $x \approx \pi + 0.9046$
$x \approx 2.2370$ $\quad x \approx 4.0462$

92. $3\cos^2 x - 8\cos x - 3 = 0$

$(3\cos x + 1)(\cos x - 3) = 0$

$3\cos x + 1 = 0$ or $\cos x - 3 = 0$

$\cos x = -\frac{1}{3}$ $\quad \cos x = 3$

~~$\cos x = 3$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of $+\frac{1}{3}$. This gives the first quadrant reference angle.

$\theta = \cos^{-1}\frac{1}{3} \approx 1.2310$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 1.2310$ or $x \approx \pi + 1.2310$
$x \approx 1.9106$ $\quad x \approx 4.3726$

93. $4\tan^2 x - 8\tan x + 3 = 0$

$(2\tan x - 1)(2\tan x - 3) = 0$

$2\tan x - 1 = 0$ or $2\tan x - 3 = 0$

$\tan x = \frac{1}{2}$ $\quad \tan x = \frac{3}{2}$

$x \approx 0.4636,\ 3.6052$ $\quad x \approx 0.9828,\ 4.1244$

94. $\tan^2 x - 3\cos x + 1 = 0$

$$\tan x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\tan x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$\tan x = \frac{3 \pm \sqrt{5}}{2}$$

$\tan x \approx 0.3820$ or $\tan x \approx 2.6180$

$x \approx 0.3649,\ 3.5065$ $\quad x \approx 1.2059,\ 4.3475$

95. $7\sin^2 x - 1 = 0$

$$\sin^2 x = \frac{1}{7}$$

$$\sin x = \pm\sqrt{\frac{1}{7}}$$

$$\sin x = \pm\frac{\sqrt{7}}{7}$$

$\sin x \approx 0.3780$ or $\sin x \approx -0.3780$

$x \approx 0.3876,\ 2.7540$ $\quad x \approx 3.5292,\ 5.8956$

96. $5\sin^2 x - 1 = 0$

$$\sin^2 x = \frac{1}{5}$$

$$\sin x = \pm\sqrt{\frac{1}{5}}$$

$$\sin x = \pm\frac{\sqrt{5}}{5}$$

$\sin x \approx 0.4472$ or $\sin x \approx -0.4472$

$x \approx 0.4636,\ 2.6780$ $\quad x \approx 3.6052,\ 5.8195$

97. $2\cos 2x + 1 = 0$

$$\cos 2x = -\frac{1}{2}$$

The period of the cosine function is 2π. On the interval $[0, 2\pi)$ the cosine function equals $-\frac{1}{2}$ at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. This means that $2x = \frac{2\pi}{3}$ or $2x = \frac{4\pi}{3}$. Because the period is 2π, all the solutions of the equation are given by

$$2x = \frac{2\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{4\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{3} + n\pi \qquad x = \frac{2\pi}{3} + n\pi$$

Use all values of n that result in x values on the interval $[0, 2\pi)$. Thus,

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{or} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

98. $2\sin 3x + \sqrt{3} = 0$

$$\sin 3x = \frac{-\sqrt{3}}{2}$$

The period of the sine function is 2π. On the interval $[0, 2\pi)$ the sine function equals $\frac{-\sqrt{3}}{2}$ at $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$. This means that $3x = \frac{4\pi}{3}$ or $3x = \frac{5\pi}{3}$. Because the period is 2π, all the solutions of the equation are given by

$$3x = \frac{4\pi}{3} + 2n\pi \quad \text{or} \quad 3x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{4\pi}{9} + \frac{2n\pi}{3} \qquad x = \frac{5\pi}{9} + \frac{2n\pi}{3}$$

Use all values of n that result in x values on the interval $[0, 2\pi)$. Thus,

$$x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$$

99. $\sin 2x + \sin x = 0$

$$2\sin x\cos x + \sin x = 0$$

$$\sin x(2\cos x + 1) = 0$$

$\sin x = 0$ or $2\cos x + 1 = 0$

$x = 0, \pi$ $\qquad \cos x = -\frac{1}{2}$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Thus, $x = 0, \frac{2\pi}{3}, \pi$, and $\frac{4\pi}{3}$.

100. $\sin 2x + \cos x = 0$

$$2\sin x\cos x + \cos x = 0$$

$$\cos x(2\sin x + 1) = 0$$

$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \sin x = -\frac{1}{2}$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Thus, $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$, and $\frac{11\pi}{6}$.

101. $3\cos x - 6\sqrt{3} = \cos x - 5\sqrt{3}$

$$2\cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

102. $\cos x - 5 = 3\cos x + 6$

$-2\cos x = 11$

$\cos x = -\frac{11}{2}$

The cosine function cannot be less than –1, thus the equation has no solution.

103. $\tan x = -4.7143$

Be sure calculator is in radian mode and find the inverse tangent of +4.7143. This gives the first quadrant reference angle. $\theta = \tan^{-1} 4.7143 \approx 1.3618$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.3618$ or $x \approx 2\pi - 1.3618$

$x \approx 1.7798$ $\quad x \approx 4.9214$

104. $\tan x = -6.2154$

Be sure calculator is in radian mode and find the inverse tangent of +6.2154. This gives the first quadrant reference angle. $\theta = \tan^{-1} 6.2154 \approx 1.4113$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.4113$ or $x \approx 2\pi - 1.4113$

$x \approx 1.7303$ $\quad x \approx 4.8719$

105. $2\sin^2 x = 3 - \sin x$

$2\sin^2 x + \sin x - 3 = 0$

$(\sin x - 1)(2\sin x + 3) = 0$

$\sin x - 1 = 0$ or $2\sin x + 3 = 0$

$\sin x = 1$ $\quad \sin x = -\frac{3}{2}$

$x = \frac{\pi}{2}$ $\quad$ ~~$\sin x = -\frac{3}{2}$~~

106. $2\sin^2 x = 2 - 3\sin x$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$2\sin x - 1 = 0$ or $\sin x + 2 = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = -2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ ~~$\sin x = -2$~~

107. $\cos x \csc x = 2\cos x$

$\cos x \csc x - 2\cos x = 0$

$\cos x(\csc x - 2) = 0$

$\cos x = 0$ or $\csc x - 2 = 0$

$\sin x = \frac{1}{2}$ $\quad \csc x = 2$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Thus, $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

108. $\tan x \sec x = 2\tan x$

$\tan x \sec x - 2\tan x = 0$

$\tan x(\sec x - 2) = 0$

$\tan x = 0$ or $\sec x - 2 = 0$

$x = 0, \pi$ $\quad \sec x = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

Thus, $x = 0, \frac{\pi}{3}, \pi$, and $\frac{5\pi}{3}$.

109. $5\cot^2 x - 15 = 0$

$\cot^2 x = 3$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$\tan x = \frac{\sqrt{3}}{3}$ or $\tan x = -\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$ $\quad x = \frac{5\pi}{6}, \frac{11\pi}{6}$

Thus, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

110. $5\sec^2 x - 10 = 0$

$\sec^2 x = 2$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\cos x = \frac{\sqrt{2}}{2}$ or $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$ $\quad x = \frac{3\pi}{4}, \frac{5\pi}{4}$

Thus, $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$.

111. $\cos^2 x + 2\cos x - 2 = 0$

Use the quadratic formula to solve for cos*x*.

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$

$\cos x = \frac{-2 \pm \sqrt{12}}{2}$

$\cos x = \frac{-2 \pm 2\sqrt{3}}{2}$

$\cos x = -1 \pm \sqrt{3}$

$\cos x \approx 0.7321$ or $\cos x \approx -2.7321$

~~$\cos x \approx -2.7321$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of 0.7321. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.7321 \approx 0.7494$

The cosine is positive in quadrants I and IV thus,

$x \approx 0.7494$ or $x \approx 2\pi - 0.7494$

$x \approx 5.5338$

112. $\cos^2 x + 5\cos x - 1 = 0$

Use the quadratic formula to solve for cos*x*.

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-1)}}{2(1)}$

$\cos x = \frac{-5 \pm \sqrt{29}}{2}$

$\cos x \approx 0.1926$ or $\cos x \approx -5.1926$

~~$\cos x \approx -5.1926$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of 0.1926. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.1926 \approx 1.3770$

The cosine is positive in quadrants I and IV thus,

$x \approx 1.3770$ or $x \approx 2\pi - 1.3770$

$x \approx 4.9062$

113.

$5\sin x = 2\cos^2 x - 4$

$5\sin x = 2\left(1 - \sin^2 x\right) - 4$

$5\sin x = 2 - 2\sin^2 x - 4$

$2\sin^2 x + 5\sin x + 2 = 0$

$(2\sin x + 1)(\sin x + 2) = 0$

$2\sin x + 1 = 0$ or $\sin x + 2 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin x = -2$

~~$\sin x = -2$~~

114.

$7\cos x = 4 - 2\sin^2 x$

$7\cos x = 4 - 2\left(1 - \cos^2 x\right)$

$7\cos x = 4 - 2 + 2\cos^2 x$

$-2\cos^2 x + 7\cos x - 2 = 0$

$2\cos^2 x - 7\cos x + 2 = 0$

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)}$

$\cos x = \frac{7 \pm \sqrt{33}}{4}$

$\cos x \approx 0.3139$ or $\cos x \approx 3.1861$

~~$\cos x \approx 3.1861$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of 0.3139. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.3139 \approx 1.2515$

The cosine is positive in quadrants I and IV thus,

$x \approx 1.2515$ or $x \approx 2\pi - 1.2515$

$x \approx 5.0317$

115. $2\tan^2 x + 5\tan x + 3 = 0$

$(\tan x + 1)(2\tan x + 3) = 0$

$\tan x + 1 = 0$ or $2\tan x + 3 = 0$

$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\tan x = -\frac{3}{2}$

$x \approx 2.1588,\ 5.3004$

116. $3\tan^2 x - \tan x - 2 = 0$

$(\tan x - 1)(3\tan x + 2) = 0$

$\tan x - 1 = 0$ or $3\tan x + 2 = 0$

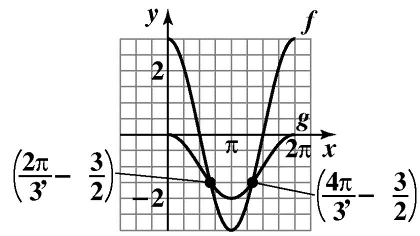
$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan x = -\frac{2}{3}$

$x \approx 2.5536,\ 5.6952$

117.

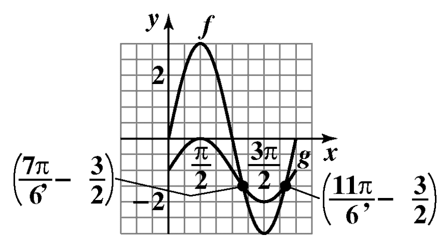


f(x) = 3 cos x

g(x) = cos x − 1

$$3\cos x = \cos x - 1$$
$$2\cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$
$$\left(\frac{2\pi}{3}, -\frac{3}{2}\right), \left(\frac{4\pi}{3}, -\frac{3}{2}\right)$$

118.

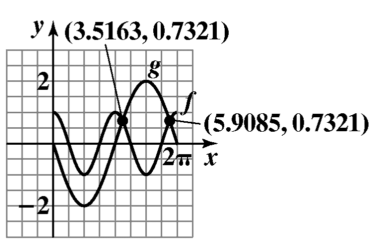


y = 3 sin x

y = sin x − 1

$$3\sin x = \sin x - 1$$
$$2\sin x = -1$$
$$\sin x = -\frac{1}{2}$$
$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$
$$\left(\frac{7\pi}{6}, -\frac{3}{2}\right), \left(\frac{11\pi}{6}, -\frac{3}{2}\right)$$

119.



f(x) = cos 2x

g(x) = −2 sin x

$$\cos 2x = -2\sin x$$
$$1 - 2\sin^2 x = -2\sin x$$
$$-2\sin^2 x + 2\sin x + 1 = 0$$
$$2\sin^2 x - 2\sin x - 1 = 0$$
$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$
$$\sin x = \frac{2 \pm \sqrt{12}}{4}$$
$$\sin x = \frac{2 \pm 2\sqrt{3}}{4}$$
$$\sin x = \frac{1 \pm \sqrt{3}}{2}$$

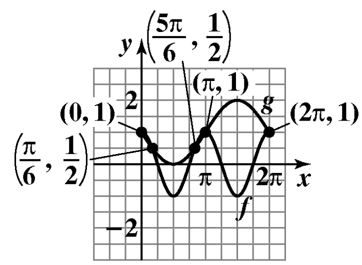
$\sin x \approx -0.3660$ or $\sin x \approx 1.3660$

~~$\sin x \approx 1.3660$~~

$x \approx 3.5163,\ 5.9085$ This equation has no solution.

$$(3.5163, 0.7321), (5.9085, 0.7321)$$

120.



f(x) = cos 2x

g(x) = 1 − sin x

$$\cos 2x = 1 - \sin x$$
$$1 - 2\sin^2 x = 1 - \sin x$$
$$-2\sin^2 x + \sin x = 0$$
$$\sin x(-2\sin x + 1) = 0$$

$\sin x = 0$ or $-2\sin x + 1 = 0$

$x = 0, \pi, 2\pi$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$(0,1), \left(\frac{\pi}{6}, \frac{1}{2}\right), \left(\frac{5\pi}{6}, \frac{1}{2}\right), (\pi, 1), (2\pi, 1)$$

121. $|\cos x| = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$ or $\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ $\qquad x = \frac{5\pi}{6}, \frac{7\pi}{6}$

122. $|\sin x| = \frac{1}{2}$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

123.
$$10\cos^2 x + 3\sin x - 9 = 0$$
$$10\left(1-\sin^2 x\right) + 3\sin x - 9 = 0$$
$$10 - 10\sin^2 x + 3\sin x - 9 = 0$$
$$-10\sin^2 x + 3\sin x + 1 = 0$$
$$10\sin^2 x - 3\sin x - 1 = 0$$
$$(2\sin x - 1)(5\sin x + 1) = 0$$

$2\sin x - 1 = 0$ or $5\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -\frac{1}{5}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = 3.3430, 6.0818$

124.
$$3\cos^2 x - \sin x = \cos^2 x$$
$$2\cos^2 x - \sin x = 0$$
$$2\left(1-\sin^2 x\right) - \sin x = 0$$
$$2 - 2\sin^2 x - \sin x = 0$$
$$2\sin^2 x + \sin x - 2 = 0$$
$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$
$$\sin x = \frac{-1 \pm \sqrt{17}}{4}$$

$\sin x = 0.7808$ or $\sin x = -1.2808$

$x = 0.8959, 2.2457$ $\quad$ ~~$\sin x = -1.2808$~~

125. $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$
$$\cos^2 x(2\cos x + 1) - 1(2\cos x + 1) = 0$$
$$(2\cos x + 1)\left(\cos^2 x - 1\right) = 0$$
$$(2\cos x + 1)(\cos x + 1)(\cos x - 1) = 0$$

$\cos x = -\frac{1}{2}$ or $\cos x = -1$ or $\cos x = 1$

$x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

126. $2\sin^3 x - \sin^2 x - 2\sin x + 1 = 0$
$$\sin^2 x(2\sin x - 1) - 1(2\sin x - 1) = 0$$
$$(2\sin x - 1)\left(\sin^2 x - 1\right) = 0$$
$$(2\sin x - 1)(\sin x + 1)(\sin x - 1) = 0$$

$\sin x = \frac{1}{2}$ or $\sin x = -1$ or $\sin x = 1$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

127. 0.3649, 1.2059, 3.5065, 4.3475
This matches graph a.

128. 0.4636, 0.9828, 3.6052, 4.1244
This matches graph b.

129. Substitute $y = 0.3$ into the equation and solve for x:
$$0.3 = 0.6\sin\frac{2\pi}{5}x$$
$$\frac{0.3}{0.6} = \frac{0.6\sin\frac{2\pi}{5}x}{0.6}$$
$$\frac{1}{2} = \sin\frac{2\pi}{5}x$$
$$\sin\frac{2\pi}{5}x = \frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$. All of the solutions to $\sin\frac{2\pi}{5}x = \frac{1}{2}$ are given by
$$\frac{2\pi}{5}x = \frac{\pi}{6} + 2n\pi$$
$$x = \frac{5}{12} + 5n$$
or
$$\frac{2\pi}{5}x = \frac{5\pi}{6} + 2n\pi$$
$$x = \frac{25}{12} + 5n$$

where n is any integer. In the interval $[0, 5]$ we obtain solutions when $n = 0$. The solutions are $\frac{5}{12}$ and $\frac{25}{12}$.

Therefore, we are inhaling at 0.3 liter per second at $x = \frac{5}{12} \approx 0.4$ second and at $x = \frac{25}{12} \approx 2.1$ seconds.

130. When we exhale, velocity of air flow is negative, so $y = -0.3$ liters per second.
Solve:

$$-0.3 = 0.6\sin\frac{2\pi}{5}x$$

$$\frac{-0.3}{0.6} = \frac{0.6\sin\frac{2\pi}{5}x}{0.6}$$

$$-\frac{1}{2} = \sin\frac{2\pi}{5}x$$

$$\sin\frac{2\pi}{5}x = -\frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$. One is $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. The other is $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$. All the solutions to $\sin\frac{2\pi}{5}x = -\frac{1}{2}$ are given by

$$\frac{2\pi}{5}x = \frac{7\pi}{6} + 2n\pi$$

$$x = \frac{35}{12} + 5n$$

or

$$\frac{2\pi}{5}x = \frac{11\pi}{6} + 2n\pi$$

$$x = \frac{55}{12} + 5n$$

where n is any integer.
In the interval $[0, 5]$ we obtain solutions when $n = 0$. The solutions are $\frac{35}{12}$ and $\frac{55}{12}$.
Therefore, we are exhaling at 0.3 liters per second at $x = \frac{35}{12} \approx 2.9$ seconds and at $x = \frac{55}{12} \approx 4.6$ seconds.

131. Substitute $y = 10.5$ into the equation and solve for x:

$$10.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right] + 12$$

$$-1.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\frac{-1.5}{3} = \frac{3\sin\left[\frac{2\pi}{365}(x-79)\right]}{3}$$

$$-\frac{1}{2} = \sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\sin\left[\frac{2\pi}{365}(x-79)\right] = -\frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$. One is $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. The other is $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$. All the solutions to $\sin\left[\frac{2\pi}{365}(x-79)\right] = -\frac{1}{2}$ are given by

$$\frac{2\pi}{365}(x-79) = \frac{7\pi}{6} + 2n\pi$$

$$x - 79 = \frac{2555}{12} + 365n$$

$$x = \frac{3503}{12} + 365n$$

or

$$\frac{2\pi}{365}(x-79) = \frac{11\pi}{6} + 2n\pi$$

$$x - 79 = \frac{4015}{12} + 365n$$

$$x = \frac{4963}{12} + 365n$$

where n is any integer.
Substitute various integers for n in the two equations. In the interval $[0, 365]$ we obtain values of 49 and 292 days. Thus, Boston has 10.5 hours of daylight 49 and 292 days after January 1.

132. Substitute $y = 13.5$ into the equation and solve for x:

$$13.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right] + 12$$

$$1.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\frac{1.5}{3} = \frac{3\sin\left[\frac{2\pi}{365}(x-79)\right]}{3}$$

$$\frac{1}{2} = \sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\sin\left[\frac{2\pi}{365}(x-79)\right] = \frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine function is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$. All the solutions to $\sin\left[\frac{2\pi}{365}(x-79)\right] = \frac{1}{2}$ are given by

$$\frac{2\pi}{365}(x-79) = \frac{\pi}{6} + 2n\pi$$

$$x - 79 = \frac{365}{12} + 365n$$

$$x = \frac{1313}{12} + 365n$$

or

$$\frac{2\pi}{365}(x-79)=\frac{5\pi}{6}+2n\pi$$
$$x-79=\frac{1825}{12}+365n$$
$$x=\frac{2773}{12}+365n$$

where n is any integer.
Substitute various integers for n in the two equations. In the interval $[0,365]$ we obtain values of 109 and 231 days. Thus, Boston has 13.5 hours of daylight 109 and 231 days after January 1.

133. Substitute $d = 2$ into the equation and solve for t:

$$2=-4\cos\frac{\pi}{3}t$$
$$\frac{2}{-4}=\frac{-4\cos\frac{\pi}{3}t}{-4}$$
$$-\frac{1}{2}=\cos\frac{\pi}{3}t$$
$$\cos\frac{\pi}{3}t=-\frac{1}{2}$$

The period of the cosine function is 2π. In the interval $[0,2\pi)$, there are two values at which the cosine function is $-\frac{1}{2}$. One is $\frac{2\pi}{3}$. The cosine function is negative in quadrant III; thus, the other value is $2\pi-\frac{2\pi}{3}=\frac{4\pi}{3}$. All solutions to $\cos\frac{\pi}{3}t=-\frac{1}{2}$ are given by

$$\frac{\pi}{3}t=\frac{2\pi}{3}+2n\pi$$
$$t=2+6n$$

or

$$\frac{\pi}{3}t=\frac{4\pi}{3}+2n\pi$$
$$t=4+6n$$

where n is any nonnegative integer.

134. Substitute $d = -2$ into the equation and solve for t:

$$-2=-4\cos\frac{\pi}{3}t$$
$$\frac{-2}{-4}=\frac{-4\cos\frac{\pi}{3}t}{-4}$$
$$\frac{1}{2}=\cos\frac{\pi}{3}t$$
$$\cos\frac{\pi}{3}t=\frac{1}{2}$$

The period of the cosine function is 2π. In the interval $[0,2\pi)$, there are two values at which the cosine function is $\frac{1}{2}$. One is $\frac{\pi}{3}$. The cosine function is positive in quadrant IV; thus, the other value is $2\pi-\frac{\pi}{3}=\frac{5\pi}{3}$. All the solutions to $\cos\frac{\pi}{3}t=\frac{1}{2}$ are given by

$$\frac{\pi}{3}t=\frac{\pi}{3}+2n\pi$$
$$t=1+6n$$

or

$$\frac{\pi}{3}t=\frac{5\pi}{3}+2n\pi$$
$$t=5+6n$$

where n is any nonnegative integer.

135. Substitute $v_0=90$ and $d = 170$, and solve for θ:

$$170=\frac{90^2}{16}\sin\theta\cos\theta$$
$$\frac{136}{405}=\sin\theta\cos\theta$$
$$2\cdot\frac{136}{405}=2\sin\theta\cos\theta$$
$$\frac{272}{405}=\sin 2\theta$$
$$\sin 2\theta=\frac{272}{405}$$

The period of the sine function is 360°. In the interval $[0,360°]$, there are two values at which the sine function is $\frac{272}{405}$.

One is $\sin^{-1}\left(\frac{272}{405}\right)\approx 42.19°$. The sine function is positive in quadrant II; Thus, the other value is $180°-42.19°=137.81°$. All solutions to $\sin 2\theta=\frac{272}{405}$ are given by

$$2\theta=42.19°+360°n$$
$$\theta=21.095°+180°n$$

or

$$2\theta=137.81°+360°n$$
$$\theta=68.905°+180°n$$

where n is any integer.
In the interval $[0,90°)$ we obtain the solutions by letting $n = 0$. The solutions are approximately 21° and 69°.
Therefore, the angle of elevation should be 21° or 69°.

136. Substitute $v_o = 90$ and $d = 200$, and solve for θ:

$$200 = \frac{90^2}{16}\sin\theta\cos\theta$$
$$\frac{32}{81} = \sin\theta\cos\theta$$
$$2\cdot\frac{32}{81} = 2\sin\theta\cos\theta$$
$$\frac{64}{81} = \sin 2\theta$$
$$\sin 2\theta = \frac{64}{81}$$

The period of the sine function is 360°. In the interval $[0, 360°]$, there are two values at which the sine function is $\frac{64}{81}$. One is $\sin^{-1}\left(\frac{64}{81}\right) \approx 52.20°$.

The sine function is positive in quadrant II; thus, the other value is $180° - 52.20° = 127.80°$. All solutions to $\sin 2\theta = \frac{64}{81}$ are given by

$$2\theta = 52.20° + 360°n$$
$$\theta = 26.10° + 180°n$$

or

$$2\theta = 127.80° + 360°n$$
$$\theta = 63.90° + 180°n$$

where n is any integer.

In the interval $[0, 90°)$ we obtain the solutions by letting $n = 0$. The solutions are approximately 26° and 64°. Therefore, the angle of elevation should be 26° or 64°.

137. – 146. Answers may vary.

147. $x \approx 1.37,\ 2.30,\ 3.98,\ 4.91$

148. $x \approx 0.74$

149. $x \approx 0.37,\ 2.77$

150. $x \approx 1.08$

151. $x \approx 0,\ 1.57,\ 2.09,\ 3.14,\ 4.71$

152. does not make sense; Explanations will vary. Sample explanation: You did not attempt to "solve." You attempted to "verifiy" as if this was an identity.

153. makes sense

154. does not make sense; Explanations will vary. Sample explanation: It is more efficient to solve as follows.

$$\cos\left(x - \frac{\pi}{3}\right) = -1$$
$$x - \frac{\pi}{3} = \cos^{-1}(-1)$$
$$x - \frac{\pi}{3} = \pi$$
$$x = \pi + \frac{\pi}{3}$$
$$x = \frac{4\pi}{3}$$

155. does not make sense; Explanations will vary. Sample explanation: You do not need to solve a trigonometric equation. You nee to find a trigonometric value of an angle ang simplify using arithmetic.

156. true

157. false; Changes to make the statement true will vary. A sample change is: The equation has an infinite number of solutions

158. false; Changes to make the statement true will vary. A sample change is: To be an identity, the equation must be true for all defined values of the variable.

159. false; Changes to make the statement true will vary. A sample change is: Over this interval, the first equation has two solutions and the second equation has 4 solutions.

160.

$$2\cos x - 1 + 3\sec x = 0$$
$$2\cos x - 1 = -3\sec x$$
$$2\cos x - 1 = \frac{-3}{\cos x}$$
$$\cos x\cdot(2\cos x - 1) = \frac{-3}{\cos x}$$
$$2\cos^2 x - \cos x = -3$$
$$2\cos^2 x - \cos x + 3 = 0$$

The equation is now in quadratic form $2t^2 - t + 3 = 0$. Use the quadratic formula to solve.

$$\cos x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(3)}}{2(2)}$$
$$\cos x = \frac{1 \pm \sqrt{1 - 24}}{4}$$
$$\cos x = \frac{1 \pm \sqrt{-23}}{4}$$

Since $\frac{1 \pm \sqrt{-23}}{4}$ are not real numbers, the equation has no solution.

161. $\sin 3x + \sin x + \cos x = 0$

$$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + \cos x = 0$$
$$2\sin 2x\cos x + \cos x = 0$$
$$\cos x(2\sin 2x + 1) = 0$$

$\cos x = 0$ or $2\sin 2x + 1 = 0$

$x = \frac{\pi}{2}$ $x = \frac{3\pi}{2}$ $2\sin 2x = -1$

$\sin 2x = -\frac{1}{2}$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$. One is $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. The other is $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$. All the solutions to $\sin 2x = -\frac{1}{2}$ are given by

$2x = \frac{7\pi}{6} + 2n\pi$ or $2x = \frac{11\pi}{6} + 2n\pi$

$x = \frac{7\pi}{12} + n\pi$ $x = \frac{11\pi}{12} + n\pi$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting n = 0 and n = 1.The solutions are $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}$, and $\frac{23\pi}{12}$.

162. $\sin x + 2\sin\frac{x}{2} = \cos\frac{x}{2} + 1$

$$\sin\left(2\cdot\frac{x}{2}\right) + 2\sin\frac{x}{2} = \cos\frac{x}{2} + 1$$
$$2\sin\frac{x}{2}\cos\frac{x}{2} + 2\sin\frac{x}{2} = \cos\frac{x}{2} + 1$$
$$2\sin\frac{x}{2}\cos\frac{x}{2} + 2\sin\frac{x}{2} - \cos\frac{x}{2} - 1 = 0$$
$$2\sin\frac{x}{2}\left(\cos\frac{x}{2} + 1\right) - \left(\cos\frac{x}{2} + 1\right) = 0$$
$$\left(\cos\frac{x}{2} + 1\right)\left(2\sin\frac{x}{2} - 1\right) = 0$$

$\cos\frac{x}{2} + 1 = 0$ or $2\sin\frac{x}{2} - 1 = 0$

$\cos\frac{x}{2} = -1$ $2\sin\frac{x}{2} = 1$

$\sin\frac{x}{2} = \frac{1}{2}$

The period of the sine function and cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The other is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

In the interval $[0, 2\pi)$, the only value at which the cosine function is –1 is π. All of the solutions to $\cos\frac{x}{2} = -1$ are given by

$$\frac{x}{2} = \pi + 2n\pi$$
$$x = 2\pi + 4n\pi$$

where n is any integer. All of the solutions to $\sin\frac{x}{2} = \frac{1}{2}$ are given by

$$\frac{x}{2} = \frac{\pi}{6} + 2n\pi$$
$$x = \frac{\pi}{3} + 4n\pi$$

or

$$\frac{x}{2} = \frac{5\pi}{6} + 2n\pi$$
$$x = \frac{5\pi}{3} + 4n\pi$$

where n is any integer.

The solutions in the interval $[0, 2\pi)$, are obtained by letting n = 0. The solutions are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

163. $\frac{4\pi}{3}$ lies in quadrant III. The reference angle is

$$\theta' = \frac{4\pi}{3} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}.$$

$$\tan\frac{\pi}{3} = \sqrt{3}$$

Because the tangent is positive in quadrant III,

$$\tan\frac{5\pi}{4} = +\tan\frac{\pi}{4} = \sqrt{3}$$

164. $y = 4\sin(2\pi x + 2) = 4\sin(2\pi x - (-2))$

The equation $y = 4\sin(2\pi x - (-2))$ is of the form $y = A\sin(Bx - C)$ with $A = 4$, $B = 2\pi$, and $C = -2$. The amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-2}{2\pi} = -\frac{1}{\pi}$.

The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -\frac{1}{\pi}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{1}{\pi}$

$x = -\frac{1}{\pi} + \frac{1}{4} = \frac{\pi - 4}{4\pi}$

$x = \frac{\pi - 4}{4\pi} + \frac{1}{4} = \frac{\pi - 2}{2\pi}$

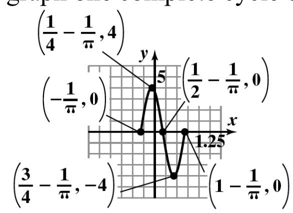
$x = \frac{\pi - 2}{2\pi} + \frac{1}{4} = \frac{3\pi - 4}{4\pi}$

$x = \frac{3\pi - 4}{4\pi} + \frac{1}{4} = \frac{\pi - 1}{\pi}$

Evaluate the function at each value of x.

x	$y = 4\sin(2\pi x + 2)$	coordinates
$-\frac{1}{\pi}$	$y = 4\sin\left(2\pi\left(-\frac{1}{\pi}\right) + 2\right)$ $= 4\sin(-2 + 2)$ $= 4\sin 0 = 4 \cdot 0 = 0$	$\left(-\frac{1}{\pi}, 0\right)$
$\frac{\pi - 4}{4\pi}$	$y = 4\sin\left(2\pi\left(\frac{\pi - 4}{4\pi}\right) + 2\right)$ $= 4\sin\left(\frac{\pi - 4}{2} + 2\right)$ $= 4\sin\left(\frac{\pi}{2} - 2 + 2\right)$ $= 4\sin\frac{\pi}{2} = 4 \cdot 1 = 4$	$\left(\frac{\pi - 4}{4\pi}, 4\right)$
$\frac{\pi - 2}{2\pi}$	$y = 4\sin\left(2\pi\left(\frac{\pi - 2}{2\pi}\right) + 2\right)$ $= 4\sin(\pi - 2 + 2)$ $= 4\sin\pi = 4 \cdot 0 = 0$	$\left(\frac{\pi - 2}{2\pi}, 0\right)$
$\frac{3\pi - 4}{4\pi}$	$y = 4\sin\left(2\pi\left(\frac{3\pi - 4}{4\pi}\right) + 2\right)$ $= 4\sin\left(\frac{3\pi - 4}{2} + 2\right)$ $= 4\sin\left(\frac{3\pi}{2} - 2 + 2\right)$ $= 4\sin\frac{3\pi}{2} = 4(-1) = -4$	$\left(\frac{3\pi - 4}{4\pi}, -4\right)$
$\frac{\pi - 1}{\pi}$	$y = 4\sin\left(2\pi\left(\frac{\pi - 1}{\pi}\right) + 2\right)$ $= 4\sin(2\pi - 2 + 2)$ $= 4\sin 2\pi = 4 \cdot 0 = 0$	$\left(\frac{\pi - 1}{\pi}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 4\sin(2\pi x + 2)$

165. $\log x + \log(x+1) = \log 12$

$\log\left(x(x+1)\right) = \log 12$

$x(x+1) = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, 3$

Reject $x = -4$. The solution set is $\{3\}$.

166. $\frac{a}{\sin 46^\circ} = \frac{56}{\sin 63^\circ}$

$a\sin 63^\circ = 56\sin 46^\circ$

$a = \frac{56\sin 46^\circ}{\sin 63^\circ}$

$a \approx 45.2^\circ$

167. $\frac{81}{\sin 43^\circ} = \frac{62}{\sin B}$

$81\sin B = 62\sin 43^\circ$

$\sin B = \frac{62\sin 43^\circ}{81}$

$\sin B \approx 0.522023436$

$B \approx \sin^{-1}(0.522023436)$

$B \approx 31.5^\circ$

168. $\frac{51}{\sin 75^\circ} = \frac{71}{\sin B}$

$51\sin B = 71\sin 75^\circ$

$\sin B = \frac{71\sin 75^\circ}{51}$

$\sin B \approx 1.344720268$

No solution.

Chapter 5 Review Exercises

1. $\sec x - \cos x = \frac{1}{\cos x} - \cos x$
$= \frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}$
$= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$
$= \frac{1 - \cos^2 x}{\cos x}$
$= \frac{\sin^2 x}{\cos x}$
$= \frac{\sin x}{\cos x} \cdot \sin x$
$= \tan x \sin x$

2. $\cos x + \sin x \tan x$
$= \frac{\cos x}{\cos x} \cdot \cos x + \sin x \cdot \frac{\sin x}{\cos x}$
$= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$
$= \frac{\cos^2 x + \sin^2 x}{\cos x}$
$= \frac{1}{\cos x} = \sec x$

3. $\sin^2 \theta (1 + \cot^2 \theta) = \sin^2 \theta + \sin^2 \theta \cot^2 \theta$
$= \sin^2 \theta + \sin^2 \theta \cdot \frac{\cos^2 \theta}{\sin^2 \theta}$
$= \sin^2 \theta + \cos^2 \theta$
$= 1$

4. $(\sec \theta - 1)(\sec \theta + 1) = \sec^2 \theta - 1$
$= 1 + \tan^2 \theta - 1$
$= \tan^2 \theta$

5. $\frac{1 - \tan x}{\sin x} = \frac{1}{\sin x} - \frac{\tan x}{\sin x}$
$= \csc x - \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$
$= \csc x - \frac{1}{\cos x}$
$= \csc x - \sec x$

6. $\frac{1}{\sin t - 1} + \frac{1}{\sin t + 1}$
$= \frac{1}{\sin t - 1} \cdot \frac{\sin t + 1}{\sin t + 1} + \frac{1}{\sin t + 1} \cdot \frac{\sin t - 1}{\sin t - 1}$
$= \frac{\sin t + 1}{\sin^2 t - 1} + \frac{\sin t - 1}{\sin^2 t - 1}$
$= \frac{\sin t + 1 + \sin t - 1}{\sin^2 t - 1}$
$= \frac{2 \sin t}{\sin^2 t - 1}$
$= \frac{2 \sin t}{-\cos^2 t}$
$= -2 \cdot \frac{\sin t}{\cos t} \cdot \frac{1}{\cos t}$
$= -2 \tan t \sec t$

7. $\frac{1 + \sin t}{\cos^2 t} = \frac{1}{\cos^2 t} + \frac{\sin t}{\cos^2 t}$
$= \sec^2 t + \frac{\sin t}{\cos t} \cdot \frac{1}{\cos t}$
$= \tan^2 t + 1 + \tan t \sec t$

8. $\frac{\cos x}{1 - \sin x} = \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x}$
$= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x}$
$= \frac{\cos x(1 + \sin x)}{\cos^2 x}$
$= \frac{1 + \sin x}{\cos x}$

9. $1 - \frac{\sin^2 x}{1 + \cos x} = 1 - \frac{1 - \cos^2 x}{1 + \cos x}$
$= 1 - \frac{(1 + \cos x)(1 - \cos x)}{1 + \cos x}$
$= 1 - (1 - \cos x)$
$= 1 - 1 + \cos x$
$= \cos x$

10. $(\tan \theta + \cot \theta)^2$
$= \tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta$
$= \sec^2 \theta - 1 + 2 \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} + \csc^2 \theta - 1$
$= \sec^2 \theta - 1 + 2 + \csc^2 \theta - 1$
$= \sec^2 \theta + \csc^2 \theta$

11. $\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{1}{\sin\theta-\cos\theta}$
$=\dfrac{\sin\theta-\cos\theta}{\sin\theta-\cos\theta}\cdot\dfrac{1}{\sin\theta+\cos\theta}+\dfrac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}\cdot\dfrac{1}{\sin\theta-\cos\theta}$
$=\dfrac{\sin\theta-\cos\theta}{\sin^2\theta-\cos^2\theta}+\dfrac{\sin\theta+\cos\theta}{\sin^2\theta-\cos^2\theta}$
$=\dfrac{2\sin\theta}{\sin^2\theta-\cos^2\theta}$
$=\dfrac{2\sin\theta}{\sin^2\theta-\cos^2\theta}\cdot\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+\cos^2\theta}$
$=\dfrac{2\sin\theta\cdot 1}{\sin^4\theta-\cos^4\theta}$
$=\dfrac{2\sin\theta}{\sin^4\theta-\cos^4\theta}$

12. $\dfrac{\cos t}{\cot t-5\cos t}=\dfrac{\cos t}{\cot t-5\cos t}\cdot\dfrac{\frac{1}{\cos t}}{\frac{1}{\cos t}}$
$=\dfrac{\frac{\cos t}{\cos t}}{\frac{\cot t-5\cos t}{\cos t}}$
$=\dfrac{1}{\frac{\cot t}{\cos t}-5}$
$=\dfrac{1}{\frac{\frac{\cos t}{\sin t}}{\cos t}-5}$
$=\dfrac{1}{\frac{\cos t}{\sin t}\cdot\frac{1}{\cos t}-5}$
$=\dfrac{1}{\frac{1}{\sin t}-5}$
$=\dfrac{1}{\csc t-5}$

13. $\dfrac{1-\cos t}{1+\cos t}=\dfrac{1-\cos t}{1+\cos t}\cdot\dfrac{1-\cos t}{1-\cos t}$
$=\dfrac{(1-\cos t)^2}{1-\cos^2 t}$
$=\dfrac{(1-\cos t)^2}{\sin^2 t}$
$=\left(\dfrac{1-\cos t}{\sin t}\right)^2$
$=\left(\dfrac{1}{\sin t}-\dfrac{\cos t}{\sin t}\right)^2$
$=(\csc t-\cot t)^2$

14. $\cos(45°+30°)$
$=\cos 45°\cos 30°-\sin 45°\sin 30°$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}$
$=\dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}$
$=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

15. $\sin 195°=\sin(135°+60°)$
$=\sin 135°\cos 60°+\cos 135°\sin 60°$
$=\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}+\left(-\dfrac{\sqrt{2}}{2}\right)\cdot\dfrac{\sqrt{3}}{2}$
$=\dfrac{\sqrt{2}}{4}-\dfrac{\sqrt{6}}{4}$
$=\dfrac{\sqrt{2}-\sqrt{6}}{4}$

16. $\tan\left(\dfrac{4\pi}{3}-\dfrac{\pi}{4}\right)=\dfrac{\tan\frac{4\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{4\pi}{3}\cdot\tan\frac{\pi}{4}}=\dfrac{\sqrt{3}-1}{1+\sqrt{3}\cdot(1)}$
$=\dfrac{\left(\sqrt{3}-1\right)\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}=\dfrac{-\left(1-\sqrt{3}\right)^2}{1-3}$
$\dfrac{-\left(1-2\sqrt{3}+3\right)}{-2}=\dfrac{1-2\sqrt{3}+3}{2}$
$=\dfrac{4-2\sqrt{3}}{2}=\dfrac{2\left(2-\sqrt{3}\right)}{2}=2-\sqrt{3}$

17. $\tan\dfrac{5\pi}{12}=\tan\left(\dfrac{2\pi}{12}+\dfrac{3\pi}{12}\right)$
$=\tan\left(\dfrac{\pi}{6}+\dfrac{\pi}{4}\right)$
$=\dfrac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$
$=\dfrac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1}=\dfrac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}}\cdot\dfrac{\left(1+\frac{\sqrt{3}}{3}\right)}{\left(1+\frac{\sqrt{3}}{3}\right)}$
$=\dfrac{\frac{2\sqrt{3}}{3}+1+\frac{1}{3}}{1-\frac{1}{3}}=\dfrac{\frac{2\sqrt{3}}{3}+\frac{4}{3}}{\frac{2}{3}}$
$=\left(\dfrac{2\sqrt{3}}{3}+\dfrac{4}{3}\right)\cdot\dfrac{3}{2}=\sqrt{3}+2$

18. $\cos 65°\cos 5°+\sin 65°\sin 5°$
$=\cos(65°-5°)$
$=\cos 60°$
$=\dfrac{1}{2}$

19. $\sin 80° \cos 50° - \cos 80° \sin 50°$
$= \sin(80° - 50°)$
$= \sin 30°$
$= \dfrac{1}{2}$

20. $\sin\left(x + \dfrac{\pi}{6}\right) - \cos\left(x + \dfrac{\pi}{3}\right)$
$= \sin x \cos\dfrac{\pi}{6} + \cos x \sin\dfrac{\pi}{6} - \left(\cos x \cos\dfrac{\pi}{3} - \sin x \sin\dfrac{\pi}{3}\right)$
$= \sin x \cdot \dfrac{\sqrt{3}}{2} + \cos x \cdot \dfrac{1}{2} - \left(\cos x \cdot \dfrac{1}{2} - \sin x \cdot \dfrac{\sqrt{3}}{2}\right)$
$= 2 \cdot \dfrac{\sqrt{3}}{2} \cdot \sin x$
$= \sqrt{3} \sin x$

21. $\tan\left(x + \dfrac{3\pi}{4}\right) = \dfrac{\tan x + \tan \frac{3\pi}{4}}{1 - \tan x \tan \frac{3\pi}{4}}$
$= \dfrac{\tan x + (-1)}{1 - \tan x(-1)}$
$= \dfrac{\tan x - 1}{1 + \tan x}$

22. $\sec(\alpha + \beta) = \dfrac{1}{\cos(\alpha + \beta)}$
$= \dfrac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$
$= \dfrac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} \cdot \dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$
$= \dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$= \dfrac{\frac{1}{\cos\alpha} \cdot \frac{1}{\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$= \dfrac{\sec\alpha\sec\beta}{1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$= \dfrac{\sec\alpha\sec\beta}{1 - \tan\alpha\tan\beta}$

23. $\dfrac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= \dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= 1 + \tan\alpha\tan\beta$

24. $\cos^4 t - \sin^4 t = \left(\cos^2 t - \sin^2 t\right)\left(\cos^2 t + \sin^2 t\right)$
$= (\cos 2t) \cdot (1)$
$= \cos 2t$

25. $\sin t - \cos 2t = \sin t - \left(1 - 2\sin^2 t\right)$
$= \sin t - 1 + 2\sin^2 t$
$= 2\sin^2 t + \sin t - 1$
$= (2\sin t - 1)(\sin t + 1)$

26. $\dfrac{\sin 2\theta - \sin\theta}{\cos 2\theta + \cos\theta} = \dfrac{2\sin\theta\cos\theta - \sin\theta}{2\cos^2\theta - 1 + \cos\theta}$
$= \dfrac{\sin\theta(2\cos\theta - 1)}{2\cos^2\theta + \cos\theta - 1}$
$= \dfrac{\sin\theta(2\cos\theta - 1)}{(2\cos\theta - 1)(\cos\theta + 1)}$
$= \dfrac{\sin\theta}{\cos\theta + 1}$
$= \dfrac{\sin\theta}{\cos\theta + 1} \dfrac{\cos\theta - 1}{\cos\theta - 1}$
$= \dfrac{\sin\theta(\cos\theta - 1)}{\cos^2\theta - 1}$
$= \dfrac{\sin\theta(\cos\theta - 1)}{-\sin^2\theta}$
$= \dfrac{-(\cos\theta - 1)}{\sin\theta}$
$= \dfrac{1 - \cos\theta}{\sin\theta}$

27. $\dfrac{\sin 2\theta}{1 - \sin^2\theta} = \dfrac{2\sin\theta\cos\theta}{\cos^2\theta}$
$= \dfrac{2\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{\cos\theta}$
$= 2\tan\theta$

28. $2\sin t\cos t\sec 2t = \sin 2t \cdot \sec 2t$
$= \sin 2t \cdot \dfrac{1}{\cos 2t}$
$= \dfrac{\sin 2t}{\cos 2t}$
$= \tan 2t$

29. $\cos 4t = \cos(2 \cdot 2t)$.
$= 1 - 2\sin^2 2t$
$= 1 - 2(\sin 2t)^2$
$= 1 - 2 \cdot (2\sin t\cos t)^2$
$= 1 - 2 \cdot 4\sin^2 t\cos^2 t$
$= 1 - 8\sin^2 t\cos^2 t$

30. $\tan\dfrac{x}{2}(1 + \cos x) = \dfrac{\sin x}{1 + \cos x} \cdot (1 + \cos x) = \sin x$

31. $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$= \frac{1-\cos x}{\sin x} \cdot \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$

$= \frac{\frac{1-\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \frac{\frac{1}{\cos x} - \frac{\cos x}{\cos x}}{\tan x}$

$= \frac{\sec x - 1}{\tan x}$

32. **a.** The graph appears to be the cosine curve, $y = \cos x.$ It cycles through maximum, intercept, minimum, intercept and back to maximum. Thus, $y = \cos x$ also describes the graph.

b. $\sin\left(x - \frac{3\pi}{2}\right) = \sin x \cos\frac{3\pi}{2} - \cos x \sin\frac{3\pi}{2}$
$= \sin x \cdot 0 - \cos x \cdot (-1)$
$= \cos x$

33. **a.** The graph appears to be the negative of the sine curve, $y = -\sin x.$ It cycles through intercept, minimum, intercept, maximum and back to intercept. Thus, $y = -\sin x$ also describes the graph.

b. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$
$= \cos x \cdot 0 - \sin x \cdot 1$
$= -\sin x$

34. **a.** The graph appears to be the tangent curve, $y = \tan x.$ It cycles through intercept to positive infinity, then from negative infinity through the intercept. Thus, $y = \tan x$ also describes the graph.

b. $y = \frac{\tan x - 1}{1 - \cot x}$

$= \frac{\frac{\sin x}{\cos x} - 1}{1 - \frac{\cos x}{\sin x}}$

$= \frac{\frac{\sin x - \cos x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}}$

$= \frac{\sin x - \cos x}{\cos x} \cdot \frac{\sin x}{\sin x - \cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

35. $\sin\alpha = \frac{3}{5} = \frac{y}{r}$

Because α lies in quadrant I, x is positive.

$x^2 + 3^2 = 5^2$
$x^2 = 5^2 - 3^2 = 16$
$x = \sqrt{16} = 4$

Thus, $\cos\alpha = \frac{x}{r} = \frac{4}{5},$ and $\tan\alpha = \frac{y}{x} = \frac{3}{4}.$

$\sin\beta = \frac{12}{13} = \frac{y}{r}$

Because β lies in quadrant II, x is negative.

$x^2 + 12^2 = 13^2$
$x^2 = 13^2 - 12^2 = 25$
$x = -\sqrt{25} = -5$

Thus, $\cos\beta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13},$ and

$\tan\beta = \frac{y}{x} = \frac{12}{-5} = -\frac{12}{5}.$

a. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= \frac{3}{5} \cdot \left(-\frac{5}{13}\right) + \frac{4}{5} \cdot \frac{12}{13} = \frac{33}{65}$

b. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= \frac{4}{5} \cdot \left(-\frac{5}{13}\right) + \frac{3}{5} \cdot \frac{12}{13} = \frac{16}{65}$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
$=\dfrac{\frac{3}{4}+\left(-\frac{12}{5}\right)}{1-\frac{3}{4}\left(-\frac{12}{5}\right)}$
$=\dfrac{-\frac{33}{20}}{1+\frac{36}{20}}=\dfrac{-\frac{33}{20}}{\frac{56}{20}}$
$=-\dfrac{33}{56}$

d. $\sin 2\alpha=2\sin\alpha\cos\alpha=2\cdot\dfrac{3}{5}\cdot\dfrac{4}{5}=\dfrac{24}{25}$

e. $\cos\dfrac{\beta}{2}=\sqrt{\frac{1+\cos\beta}{2}}=\sqrt{\frac{1-\frac{5}{13}}{2}}$
$=\sqrt{\frac{8}{26}}=\sqrt{\frac{4}{13}}=\dfrac{2}{\sqrt{13}}=\dfrac{2\sqrt{13}}{13}$

36. $\tan\alpha=\dfrac{4}{3}=\dfrac{-4}{-3}=\dfrac{y}{x}$
Because r is a distance, it is positive.
$r^2=(-4)^2+(-3)^2=25$
$r=\sqrt{25}=5$

Thus, $\sin\alpha=\dfrac{y}{r}=\dfrac{-4}{5}=-\dfrac{4}{5}$, and

$\cos\alpha=\dfrac{x}{r}=\dfrac{-3}{5}=-\dfrac{3}{5}.$

$\tan\beta=\dfrac{5}{12}=\dfrac{y}{x}$
Because r is a distance, it is positive.
$r^2=5^2+12^2=169$
$r=\sqrt{169}=13$

Thus, $\sin\beta=\dfrac{y}{r}=\dfrac{5}{13}$, and $\cos\beta=\dfrac{x}{r}=\dfrac{12}{13}.$

a. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=-\dfrac{4}{5}\cdot\dfrac{12}{13}+\left(-\dfrac{3}{5}\right)\cdot\dfrac{5}{13}$
$=-\dfrac{63}{65}$

b. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$=-\dfrac{3}{5}\cdot\dfrac{12}{13}+\left(-\dfrac{4}{5}\right)\cdot\dfrac{5}{13}$
$=-\dfrac{56}{65}$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
$=\dfrac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}}=\dfrac{\frac{21}{12}}{1-\frac{20}{36}}=\dfrac{\frac{21}{12}}{\frac{16}{36}}$
$=\dfrac{21}{12}\cdot\dfrac{36}{16}=\dfrac{63}{16}$

d. $\sin 2\alpha=2\sin\alpha\sin\alpha$
$=2\left(-\dfrac{4}{5}\right)\left(-\dfrac{4}{5}\right)=\dfrac{24}{25}$

e. $\cos\dfrac{\beta}{2}=\sqrt{\dfrac{1+\cos\beta}{2}}=\sqrt{\dfrac{1+\frac{12}{13}}{2}}$
$=\sqrt{\dfrac{25}{26}}=\dfrac{5}{\sqrt{26}}=\dfrac{5\sqrt{26}}{26}$

37. $\tan\alpha=-3=\dfrac{3}{-1}=\dfrac{y}{x}$
Because r is a distance, it is positive.
$r^2=3^2+(-1)^2$
$r^2=10$
$r=\sqrt{10}$

$\sin\alpha=\dfrac{3}{\sqrt{10}}=\dfrac{3\sqrt{10}}{10}$
$\cos\alpha=\dfrac{-1}{\sqrt{10}}=-\dfrac{\sqrt{10}}{10}$
$\cot\beta=-3=\dfrac{3}{-1}=\dfrac{x}{y}$
Because r is a distance, it is positive.
$r^2=3^2+(-1)^2$
$r^2=10$
$r=\sqrt{10}$
$\sin\beta=\dfrac{-1}{\sqrt{10}}=-\dfrac{\sqrt{10}}{10}$
$\cos\beta=\dfrac{3}{\sqrt{10}}=\dfrac{3\sqrt{10}}{10}$

a. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=\dfrac{3\sqrt{10}}{10}\cdot\dfrac{3\sqrt{10}}{10}+\left(-\dfrac{\sqrt{10}}{10}\right)\left(-\dfrac{\sqrt{10}}{10}\right)$
$=\dfrac{90}{100}+\dfrac{10}{100}$
$=\dfrac{100}{100}$
$=1$

b. $\cos(\alpha-\beta)$
$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= \left(-\frac{\sqrt{10}}{10}\right)\left(\frac{3\sqrt{10}}{10}\right) + \frac{3\sqrt{10}}{10}\left(\frac{-\sqrt{10}}{10}\right)$
$= -\frac{60}{100}$
$= -\frac{3}{5}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{-3+\left(\frac{-1}{3}\right)}{1-(-3)\left(-\frac{1}{3}\right)} = \frac{\frac{-10}{3}}{0}$

Since this value is undefined, the tangent function is undefined at $\alpha+\beta$.

d. $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{3\sqrt{10}}{10}\right)\left(\frac{-\sqrt{10}}{10}\right) = -\frac{3}{5}$

e. $\cos\frac{\beta}{2} = \sqrt{\frac{1+\cos\beta}{2}} = \sqrt{\frac{1+\frac{3\sqrt{10}}{10}}{2}} = \sqrt{\frac{10+3\sqrt{10}}{20}} = \frac{\sqrt{10+3\sqrt{10}}}{2\sqrt{5}}$

38. $\sin\alpha = -\frac{1}{3} = \frac{-1}{3} = \frac{y}{r}$

Because α is in quadrant II, *x* is negative.

$x^2 + (-1)^2 = 3^2$
$x^2 + 1 = 9$
$x^2 = 8$
$x = -\sqrt{8} = -2\sqrt{2}$

$\cos\alpha = \frac{-2\sqrt{2}}{3}$

$\tan\alpha = \frac{-1}{-2\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\cos\beta = -\frac{1}{3} = \frac{-1}{3} = \frac{x}{r}$

Because β is in quadrant III, *y* is negative.

$(-1)^2 + y^2 = 3^2$
$y^2 = 8$
$y = -\sqrt{8} = -2\sqrt{2}$

$\sin\beta = \frac{-2\sqrt{2}}{3}$

$\tan\beta = \frac{-2\sqrt{2}}{-1} = 2\sqrt{2}$

a. $\sin(\alpha+\beta)$
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$= -\frac{1}{3}\cdot -\frac{1}{3} + \left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{2\sqrt{2}}{3}\right)$
$= \frac{9}{9} = 1$

b. $\cos(\alpha-\beta)$
$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= -\frac{2\sqrt{2}}{3}\cdot\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)\cdot\left(-\frac{2\sqrt{2}}{3}\right)$
$= \frac{4\sqrt{2}}{9}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan B} = \frac{\frac{\sqrt{2}}{4}+2\sqrt{2}}{1-\left(\frac{\sqrt{2}}{4}\right)\left(2\sqrt{2}\right)} = \frac{\frac{9\sqrt{2}}{4}}{0}$

Since this value is undefined, the tangent function is undefined at $\alpha+\beta$.

d. $\sin 2\alpha = 2\sin\alpha\cos\alpha$
$= 2\left(-\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right) = \frac{4\sqrt{2}}{9}$

e. $\cos\frac{\beta}{2} = -\sqrt{\frac{1+\cos\beta}{2}} = -\sqrt{\frac{1+\left(-\frac{1}{3}\right)}{2}} = -\sqrt{\frac{\frac{2}{3}}{2}} = -\sqrt{\frac{1}{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

39. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = 15°$.

$$\cos^2 15° - \sin^2 15° = \cos(2 \cdot 15°) = \cos 30° = \frac{\sqrt{3}}{2}$$

40. The given expression is the right side of the formula for $\tan 2\theta$ with $\theta = \frac{5\pi}{12}$.

$$\frac{2\tan\frac{5\pi}{12}}{1-\tan^2\frac{5\pi}{12}} = \tan\left(2 \cdot \frac{5\pi}{12}\right) = \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$$

41. Because 22.5° lies in quadrant I, $\sin 22.5° > 0$.

$$\sin 22.5° = \sin\frac{45°}{2} = \sqrt{\frac{1-\cos 45°}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

42. Because $\frac{\pi}{12}$ lies in quadrant I, $\tan\frac{\pi}{12} > 0$.

$$\tan\frac{\pi}{12} = \tan\frac{\frac{\pi}{6}}{2} = \frac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2-\sqrt{3}$$

43. $\sin 6x \sin 4x$

$$= \frac{1}{2}[\cos(6x-4x) - \cos(6x+4x)] = \frac{1}{2}[\cos 2x - \cos 10x]$$

44. $\sin 7x \cos 3x$

$$= \frac{1}{2}[\sin(7x+3x) + \sin(7x-3x)] = \frac{1}{2}[\sin 10x + \sin 4x]$$

45. $\sin 2x - \sin 4x$

$$= 2\sin\left(\frac{2x-4x}{2}\right)\cos\left(\frac{2x+4x}{2}\right) = 2\sin(-x)\cos 3x = -2\sin x \cos 3x$$

46. $\cos 75° + \cos 15°$

$$= 2\cos\left(\frac{75°+15°}{2}\right)\cos\left(\frac{75°-15°}{2}\right) = 2\cos 45° \cos 30° = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$$

47.

$$\frac{\cos 3x + \cos 5x}{\cos 3x - \cos 5x} = \frac{2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)}{-2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right)}$$
$$= \frac{2\cos\left(\frac{8x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{-2\sin\left(\frac{8x}{2}\right)\sin\left(\frac{-2x}{2}\right)} = \frac{2\cos 4x\cos(-x)}{-2\sin 4x\sin(-x)} = \frac{2\cos 4x\cos x}{2\sin 4x\sin x} = \frac{\cos 4x}{\sin 4x}\cdot\frac{\cos x}{\sin x} = \cot 4x\cot x = \cot x\cot 4x$$

48.

$$\frac{\sin 2x + \sin 6x}{\sin 2x - \sin 6x} = \frac{2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)}{2\sin\left(\frac{2x-6x}{2}\right)\cos\left(\frac{2x+6x}{2}\right)}$$
$$= \frac{2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{-4x}{2}\right)}{2\sin\left(\frac{-4x}{2}\right)\cos\left(\frac{8x}{2}\right)} = \frac{\sin 4x\cos(-2x)}{\sin(-2x)\cos 4x} = -\frac{\sin 4x\cos 2x}{\sin 2x\cos 4x} = -\frac{\sin 4x}{\cos 4x}\cdot\frac{\cos 2x}{\sin 2x} = -\tan 4x\cot 2x$$

49. **a.** The graph appears to be the cotangent curve, $y = \cot x$. It cycles from positive infinity through the intercept to negative infinity. Thus, $y = \cot x$ also describes the graph.

b. $$\frac{\cos 3x+\cos x}{\sin 3x-\sin x}=\frac{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{2\sin\left(\frac{3x-x}{2}\right)\cos\left(\frac{3x+x}{2}\right)}$$
$$=\frac{2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)}{2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{4x}{2}\right)}$$
$$=\frac{2\cos 2x\cos x}{2\sin x\cos 2x}$$
$$=\frac{\cos x}{\sin x}$$
$$=\cot x$$

This verifies our observation that $y=\frac{\cos 3x+\cos x}{\sin 3x-\sin x}$ and $y=\cot x$ describe the same graph.

50. $\cos x=-\frac{1}{2}$

Because $\cos\frac{\pi}{3}=\frac{1}{2}$, the solutions for $\cos x=-\frac{1}{2}$ in $[0, 2\pi)$ are $x=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ $x=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$.

Because the period of the cosine function is 2π, the solutions are given by

$x=\frac{2\pi}{3}+2n\pi$ or $x=\frac{4\pi}{3}+2n\pi$ where n is any integer.

51. $\sin x=\frac{\sqrt{2}}{2}$

Because $\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, the solutions for $\sin x=\frac{\sqrt{2}}{2}$ in $[0, 2\pi)$ are $x=\frac{\pi}{4}$ and $x=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$. Because the period of the cosine function is 2π, the solutions are given by $x=\frac{\pi}{4}+2n\pi$ or $x=\frac{3\pi}{4}+2n\pi$ where n is any integer.

52. $2\sin x+1=0$
$2\sin x=-1$
$\sin x=-\frac{1}{2}$

Because $\sin\frac{\pi}{6}=\frac{1}{2}$, the solutions for $\sin x=-\frac{1}{2}$ in $[0, 2\pi)$ are

$x=\pi+\frac{\pi}{6}=\frac{7\pi}{6}$
$x=2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$.

Because the period of the sine function is 2π, the solutions are given by

$x=\frac{7\pi}{6}+2n\pi$ or $x=\frac{11\pi}{6}+2n\pi$

where n is any integer.

53. $\sqrt{3}\tan x-1=0$
$\sqrt{3}\tan x=1$
$\tan x=\frac{1}{\sqrt{3}}$

Because $\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$, the solution for $\tan x=\frac{1}{\sqrt{3}}$ in $[0, \pi)$ is $x=\frac{\pi}{6}$.

Because the period of the tangent function is π, the solutions are given by

$x=\frac{\pi}{6}+n\pi$ where n is any integer.

54. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, the only value at which the cosine function is –1 is π. All the solutions to $\cos 2x=-1$ are given by

$2x=\pi+2n\pi$
$x=\frac{\pi}{2}+n\pi$ where n is any integer.

The solutions in the interval $[0, 2\pi)$ are obtained by letting n = 0 and n = 1.

The solutions are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

55. The period of the sine function is 2π. In the interval $[0, 2\pi)$, the only value at which the sine function is 1 is $\frac{\pi}{2}$. All the solutions to $\sin 3x=1$ are given by

$3x=\frac{\pi}{2}+2n\pi$
$x=\frac{\pi}{6}+\frac{2n\pi}{3}$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting n = 0, n = 1, and n =2.

The solutions are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{9\pi}{6}$.

56. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is -1 is $\frac{3\pi}{4}$. All the solutions to $\tan\frac{x}{2} = -1$ are given by

$$\frac{x}{2} = \frac{3\pi}{4} + n\pi$$
$$x = \frac{3\pi}{2} + 2n\pi$$

where n is any integer. The solution in the interval $[0, 2\pi)$ is obtained by letting $n = 0$.

The solution is $\frac{3\pi}{2}$.

57.
$$\tan x = 2\cos x \tan x$$
$$\tan x - 2\cos x \tan x = 0$$
$$\tan x(1 - 2\cos x) = 0$$

$\tan x = 0$ or $1 - 2\cos x = 0$

$x = 0 \quad x = \pi \qquad -2\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} \quad x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are 0, $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$.

58. The given equation is in quadratic form $t^2 - 2t = 3$ with $t = \cos x$.

$$\cos^2 x - 2\cos x = 3$$
$$\cos^2 x - 2\cos x - 3 = 0$$
$$(\cos x + 1)(\cos x - 3) = 0$$

$\cos x + 1 = 0$ or $\cos x - 3 = 0$

$\cos x = -1 \qquad \cos x = 3$

$x = \pi \qquad \cos x$ cannot be greater than 1.

The solution in the interval $[0, 2\pi)$ is π.

59.
$$2\cos^2 x - \sin x = 1$$
$$2\left(1 - \sin^2 x\right) - \sin x = 1$$
$$2 - 2\sin^2 x - \sin x - 1 = 0$$
$$-2\sin^2 x - \sin x + 1 = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1 \qquad \sin x = -1$

$\sin x = \frac{1}{2} \qquad x = \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

60. The given equation is in quadratic form $4t^2 = 1$ with $t = \sin x$.

$$4\sin^2 x = 1$$
$$4\sin^2 x - 1 = 0$$
$$(2\sin x - 1)(2\sin x + 1) = 0$$

$2\sin x - 1 = 0$ or $2\sin x + 1 = 0$

$2\sin x = 1 \qquad 2\sin x = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \qquad x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

61.
$$\cos 2x - \sin x = 1$$
$$2\cos^2 x - 1 - \sin x = 1$$
$$2\left(1 - \sin^2 x\right) - \sin x - 2 = 0$$
$$2 - 2\sin^2 x - \sin x - 2 = 0$$
$$-2\sin^2 x - \sin x = 0$$
$$2\sin^2 x + \sin x = 0$$
$$\sin x\left(2\sin x + 1\right) = 0$$

$\sin x = 0 \qquad 2\sin x + 1 = 0$

$x = 0, \pi \qquad \sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $0, \pi, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$..

62.
$$\sin 2x = \sqrt{3}\sin x$$
$$2\sin x\cos x = \sqrt{3}\sin x$$
$$2\sin x\cos x - \sqrt{3}\sin x = 0$$
$$\sin x\left(2\cos x - \sqrt{3}\right) = 0$$

$\sin x = 0$ or $2\cos x - \sqrt{3} = 0$

$x = 0 \quad x = \pi \qquad 2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} \quad x = \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are 0, $\frac{\pi}{6}$, π, and $\frac{11\pi}{6}$.

63.
$$\sin x = \tan x$$
$$\sin x = \frac{\sin x}{\cos x}$$
$$\sin x \bullet \cos x = \sin x$$
$$\sin x\cos x - \sin x = 0$$
$$\sin x(\cos x - 1) = 0$$

$\sin x = 0$ or $\cos x - 1 = 0$

$x = 0 \quad x = \pi \qquad \cos x = 1$

$x = 0$

The solutions in the interval $[0, 2\pi)$ are 0 and π.

64. $\sin x = -0.6031$

Be sure calculator is in radian mode and find the inverse sine of +0.6031. This gives the first quadrant reference angle.

$\theta = \sin^{-1}(0.6031) \approx 0.6474$

The sine is negative in quadrants III and IV thus,

$x \approx \pi + 0.6474$ or $x \approx 2\pi - 0.6474$

$x \approx 3.7890 \qquad x \approx 5.6358$

65. $5\cos^2 x - 3 = 0$
$$\cos^2 x = \frac{3}{5}$$
$$\cos x = \pm\sqrt{\frac{3}{5}}$$
$$\cos x = \pm\frac{\sqrt{15}}{5}$$

$\cos x \approx 0.7746$ or $\cos x \approx -0.7746$

$x \approx 0.6847, \ 5.5985 \qquad x \approx 2.4569, \ 3.8263$

66.
$$1 + \tan^2 x = 4\tan x - 2$$
$$\tan^2 x - 4\tan x + 3 = 0$$
$$(\tan x - 1)(\tan x - 3) = 0$$

$\tan x = 1$ or $\tan x = 3$

$x = \frac{\pi}{4}, \frac{5\pi}{4} \qquad x \approx 1.2490, \ 4.3906$

67. $2\sin^2 x + \sin x - 2 = 0$
$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$
$$\sin x = \frac{-1 \pm \sqrt{17}}{4}$$

$\sin x = 0.7808$ or $\sin x = -1.2808$

$x = 0.8959, \ 2.2457 \qquad$ ~~$\sin x = -1.2808$~~

68. Substitute $d = -3$ into the equation and solve for *t:*
$$-3 = -6\cos\frac{\pi}{2}t$$
$$\frac{-3}{-6} = \frac{-6\cos\frac{\pi}{2}t}{-6}$$
$$\frac{1}{2} = \cos\frac{\pi}{2}t$$
$$\cos\frac{\pi}{2}t = \frac{1}{2}$$

The period of the cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the cosine function is $\frac{1}{2}$. One is $\frac{\pi}{3}$. The cosine function is positive in quadrant IV. Thus, the other value is $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

All solutions to $\cos\frac{\pi}{2}t = \frac{1}{2}$ are given by
$$\frac{\pi}{2}t = \frac{\pi}{3} + 2n\pi$$
$$\frac{\pi}{2}t = \frac{5\pi}{3} + 2n\pi$$
$$t = \frac{2}{3} + 4n \text{ or}$$
$$t = \frac{10}{3} + 4n$$

where *n* is any integer.

69. Substitute $v_0 = 90$ and $d = 100$, and solve for θ:
$$100 = \frac{90^2}{16}\sin\theta\cos\theta$$
$$\frac{16}{81} = \sin\theta\cos\theta$$
$$2\cdot\frac{16}{81} = 2\sin\theta\cos\theta$$
$$\frac{32}{81} = \sin 2\theta$$
$$\sin 2\theta = \frac{32}{81}$$

The period of the sine function is 360°. In the interval $[0, 360°)$, there are two values at which the sine function is $\frac{32}{81}$. One is $\sin^{-1}\left(\frac{32}{81}\right) \approx 23.27°$. The sine function is positive in quadrant II. Thus, the other value is 180° – 23.27° = 156.73°. All solutions to $\sin 2\theta = \frac{32}{81}$ are given by

$$2\theta = 23.27° + 360°n$$
$$\theta = 11.635° + 180°n$$

or

$$2\theta = 156.73° + 360°n$$
$$\theta = 78.365° + 180°n$$

where n is any integer.

In the interval $[0, 90°)$ we obtain the solutions by letting $n = 0$. The solutions are approximately 12° and 78°. Therefore, the angle of elevation should be 12° or 78°.

Chapter 5 Test

For exercises 1–4: $\sin\alpha = \frac{4}{5} = \frac{y}{r}$

Because α lies in quadrant II, x is negative.

$$x^2 + 4^2 = 5^2$$
$$x^2 = 5^2 - 4^2 = 9$$
$$x = -\sqrt{9} = -3$$

Thus, $\cos\alpha = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$, and

$\tan\alpha = \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3}$.

$\cos\beta = \frac{5}{13} = \frac{x}{r}$

Because β lies in quadrant I, y is positive.

$$5^2 + y^2 = 13^2$$
$$y^2 = 13^2 - 5^2 = 144$$
$$y = \sqrt{144} = 12$$

Thus, $\sin\beta = \frac{y}{r} = \frac{12}{13}$, and $\tan\beta = \frac{y}{x} = \frac{12}{5}$.

1. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= -\frac{3}{5}\cdot\frac{5}{13} - \frac{4}{5}\cdot\frac{12}{13} = -\frac{63}{65}$$

2. $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$$= \frac{-\frac{4}{3}-\frac{12}{5}}{1+\left(-\frac{4}{3}\right)\cdot\frac{12}{5}} = \frac{-\frac{56}{15}}{-\frac{33}{15}} = \frac{56}{33}$$

3. $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$

4. $\cos\frac{\beta}{2} = \sqrt{\frac{1+\cos\beta}{2}} = \sqrt{\frac{1+\frac{5}{13}}{2}} = \sqrt{\frac{18}{26}}$

$$= \frac{3\sqrt{2}}{\sqrt{26}} = \frac{3\sqrt{52}}{26} = \frac{3\cdot 2\sqrt{13}}{26}$$
$$= \frac{3\sqrt{13}}{13}$$

5. $\sin 105° = \sin(135° - 30°)$

$$= \sin 135°\cos 30° - \cos 135°\sin 30°$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

6. $\cos x\csc x = \cos x\cdot\frac{1}{\sin x} = \frac{\cos x}{\sin x} = \cot x$

7. $\frac{\sec x}{\cot x + \tan x} = \frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$

$$= \frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}$$
$$= \frac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x\cos x}} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x\cos x}}$$
$$= \frac{1}{\cos x}\cdot\frac{\sin x\cos x}{1}$$
$$= \sin x$$

8. $1 - \frac{\cos^2 x}{1+\sin x} = 1 - \frac{\left(1-\sin^2 x\right)}{1+\sin x}$

$$= 1 - \frac{(1+\sin x)(1-\sin x)}{1+\sin x}$$
$$= 1 - (1-\sin x)$$
$$= \sin x$$

9. $\cos\left(\theta+\frac{\pi}{2}\right) = \cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}$

$$= \cos\theta\cdot 0 - \sin\theta\cdot 1$$
$$= -\sin\theta$$

10. $\frac{\sin(\alpha-\beta)}{\sin\alpha\cos\beta} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$$= \frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} - \frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$$
$$= 1 - \cot\alpha\tan\beta$$

11. $\sin t\cos t(\tan t+\cot t) = \sin t\cos t\left(\dfrac{\sin t}{\cos t}+\dfrac{\cos t}{\sin t}\right)$

$$= \frac{\sin^2 t\cos t}{\cos t}+\frac{\sin t\cos^2 t}{\sin t}$$
$$= \sin^2 t+\cos^2 t$$
$$= 1$$

12. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$.

One is $\pi+\frac{\pi}{6}=\frac{7\pi}{6}$. The other is $2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$. All the solutions to $\sin 3x=-\frac{1}{2}$ are given by

$$3x=\frac{7\pi}{6}+2n\pi$$
$$x=\frac{7\pi}{18}+\frac{2n\pi}{3}$$

or

$$3x=\frac{11\pi}{6}+2n\pi$$
$$x=\frac{11\pi}{18}+\frac{2n\pi}{3}$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, and $n = 2$.

The solutions are $\frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}$, and $\frac{35\pi}{18}$.

13.
$$\sin 2x+\cos x=0$$
$$2\sin x\cos x+\cos x=0$$
$$\cos x(2\sin x+1)=0$$

$\cos x=0$ or $2\sin x+1=0$

$x=\frac{\pi}{2}$, $x=\frac{3\pi}{2}$

$2\sin x=-1$

$\sin x=-\frac{1}{2}$

$x=\frac{7\pi}{6}$, $x=\frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$, and $\frac{11\pi}{6}$.

14.
$$2\cos^2 x-3\cos x+1=0$$
$$(2\cos x-1)(\cos x-1)=0$$

$2\cos x-1=0$ or $\cos x-1=0$

$2\cos x=1$ $\quad$ $\cos x=1$

$\cos x=\frac{1}{2}$ $\quad$ $x=0$

$x=\frac{\pi}{3}$, $x=\frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{3}$, and $\frac{5\pi}{3}$.

15.
$$2\sin^2 x+\cos x=1$$
$$2\left(1-\cos^2 x\right)+\cos x-1=0$$
$$2-2\cos^2 x+\cos x-1=0$$
$$-2\cos^2 x+\cos x+1=0$$
$$2\cos^2 x-\cos x-1=0$$
$$(2\cos x+1)(\cos x-1)=0$$

$2\cos x+1=0$ or $\cos x-1=0$

$2\cos x=-1$ $\quad$ $\cos x=1$

$\cos x=-\frac{1}{2}$ $\quad$ $x=0$

$x=\frac{2\pi}{3}$, $x=\frac{4\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{2\pi}{3}$, and $\frac{4\pi}{3}$.

16. $\cos x=-0.8092$

Be sure calculator is in radian mode and find the inverse cosine of +0.8092. This gives the first quadrant reference angle.

$\theta=\cos^{-1}(0.8092)\approx 0.6280$

The cosine is negative in quadrants II and III thus,

$x\approx\pi-0.6280$ or $x\approx\pi+0.6280$

$x\approx 2.5136$ $\quad$ $x\approx 3.7696$

17.
$$\tan x\sec x=3\tan x$$
$$\tan x\sec x-3\tan x=0$$
$$\tan x\left(\sec x-3\right)=0$$

$\tan x=0$ or $\sec x-3=0$

$\sec x=3$

$x=0,\ \pi$

$\cos x=\frac{1}{3}$

$x\approx 1.2310,\ 5.0522$

18. $\tan^2 x - 3\tan x - 2 = 0$

$\tan x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\tan x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$

$\tan x = \dfrac{3 \pm \sqrt{17}}{2}$

$\tan x \approx 3.5616$ or $\tan x \approx -0.5616$

$x \approx 1.2971,\ 4.4387$ $\quad x \approx 2.6299,\ 5.7715$

Cumulative Review Exercises (Chapters P–5)

1. $x^3 + x^2 - x + 15 = 0$

The possible rational zeros are: ±1, ±3, ±5, ±15.

Synthetic division shows that –3 is a zero:

–3	1	1	–1	15
		–3	6	–15
	1	–2	5	0

The quotient is $x^2 - 2x + 5$. The remaining zeros are found using the quadratic formula:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4-20}}{2}$$
$$= \frac{2 \pm \sqrt{-16}}{2}$$
$$= \frac{2 \pm 4i}{2}$$
$$= 1 \pm 2i$$

All solutions are: –3, 1 + 2*i* and 1 – 2*i*.

2.

$$11^{x-1} = 125$$
$$\log 11^{x-1} = \log 125$$
$$(x-1)\log 11 = \log 125$$
$$x - 1 = \frac{\log 125}{\log 11}$$
$$x = \frac{\log 125}{\log 11} + 1$$

or $x \approx 3.01$

3. $x^2+2x-8>0$
$(x-2)(x+4)>0$
zero points are $x = 2$ and $x = –4$.

Test Interval	Representative Number	Substitute into $x^2+2x-8>0$	Conclusion
$(-\infty, -4)$	–5	$(-5)^2+2(-5)-8=25-10-8=7>0$	$(-\infty, -4)$ belongs to thesolution set.
$(-4, 2)$	0	$0^2+2(0)-8=-8>0$	$(-4, 2)$ does not belong to the solution set.
$(2, \infty)$	3	$3^2+2(3)-8=9+6-8=7>0$	$(2, \infty)$ belongs to the solution set.

The solution intervals are $(-\infty, -4)\cup(2, \infty)$.

4. $\cos 2x+3=5\cos x$
$2\cos^2 x-1+3=5\cos x$
$2\cos^2 x-5\cos x+2=0$
$(2\cos x-1)(\cos x-2)=0$
$2\cos x-1=0$ or $\cos x-2=0$
$2\cos x=1$
$\cos x=\frac{1}{2}$

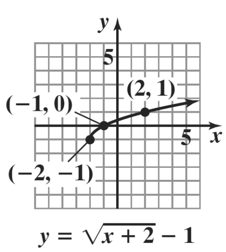
$\cos x=2$
$\cos x$ cannot be greater than 1.

$x=\frac{\pi}{3}$ $x=\frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

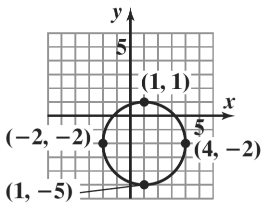
5. $\tan x+\sec^2 x=3$
$\tan x+1+\tan^2 x=3$
$\tan^2 x+\tan x-2=0$
$(\tan x-1)(\tan x+2)=0$
$\tan x-1=0$ or $\tan x+2=0$
$\tan x=1$ $\tan x=-2$
$x=\frac{\pi}{4}, \frac{5\pi}{4}$ $x\approx 2.0344, 5.1761$

6.



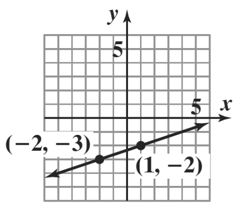
Shift the graph of $y=\sqrt{x}$ left 2 units and down 1 unit.

7.



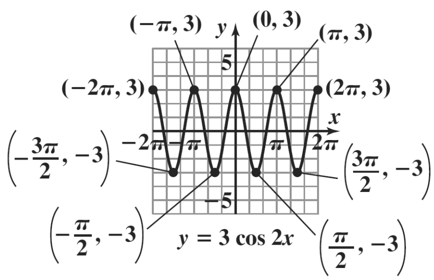
$(x-1)^2+(y+2)^2=9$

8.

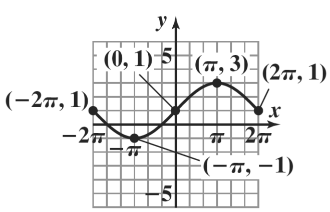


$y+2=\frac{1}{3}(x-1)$

9.

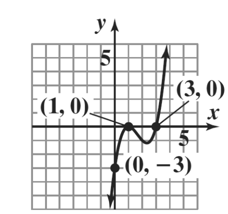


10.



$y=2\sin\frac{x}{2}+1$

11.



$f(x)=(x-1)^2(x-3)$

12. $f(x)=x^2+3x-1$

$$\frac{f(a+h)-f(a)}{h}$$
$$=\frac{(a+h)^2+3(a+h)-1-\left(a^2+3a-1\right)}{h}$$
$$=\frac{a^2+2ah+h^2+3a+3h-1-a^2-3a+1}{h}$$
$$=\frac{2ah+h^2+3h}{h}$$
$$=2a+h+3$$

13.
$$\sin 225^\circ=\sin(180^\circ+45^\circ)$$
$$=\sin 180^\circ\cos 45^\circ+\cos 180^\circ\sin 45^\circ$$
$$=0\cdot\frac{\sqrt{2}}{2}+(-1)\cdot\frac{\sqrt{2}}{2}$$
$$=-\frac{\sqrt{2}}{2}$$

14. $\sec^4 x-\sec^2 x$
$$=\sec^2 x\cdot\sec^2 x-\sec^2 x$$
$$=\left(1+\tan^2 x\right)\left(1+\tan^2 x\right)-\left(1+\tan^2 x\right)$$
$$=1+2\tan^2 x+\tan^4 x-1-\tan^2 x$$
$$=\tan^4 x+\tan^2 x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

15. $320^\circ\times\frac{\pi}{180^\circ}=\frac{16}{9}\pi$ or 5.59 radians

16.
$$A=Pe^{rt}$$
$$3P=Pe^{0.0575t}$$
$$3=e^{0.0575t}$$
$$\ln 3=\ln e^{0.0575t}$$
$$\ln 3=0.0575t$$
$$\frac{\ln 3}{0.0575}=t$$
$$t\approx 19.1\text{ years}$$

17. $$f(x)=\frac{2x+1}{x-3}$$
$$y=\frac{2x+1}{x-3}$$
$$x=\frac{2y+1}{y-3}$$
$$x(y-3)=2y+1$$
$$xy-3x=2y+1$$
$$xy-2y=3x+1$$
$$y(x-2)=3x+1$$
$$y=\frac{3x+1}{x-2}$$
$$f^{-1}(x)=\frac{3x+1}{x-2}$$

18. The third angle is:
$B=180°-90°-23°=67°$.
Since $\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}}$,
$\sin A=\sin 23°=\frac{12}{c}$
$c=\frac{12}{\sin 23°}\approx 30.71$ and
$\sin B=\sin 67°=\frac{b}{30.71}$
$b=30.71\cdot\sin 67°\approx 28.27$
The angles are 90°, 23°, and 67°.
The sides are 12, 30.71, and 28.27.

19. Solve $8.5=\frac{12}{150}\cdot a$
where a is the adult dose.
$$a=\frac{(8.5)\cdot 150}{12}$$
$$=106.25\text{ mg}$$
$$a\approx 106\text{ mg}$$

20.. Let h be the height of the flagpole.
Then $\tan 53°=\frac{h}{12}$
$$h=12\cdot\tan 53°$$
$$h\approx 15.9\text{ feet}$$